CHAPTER 52

WAVE HEIGHT DECAY MODEL WITHIN A SURF ZONE

Shigeki Sakai*, Kouetsu Hiyamizu**, Hiroshi Saeki***

ABSTRACT

A model for wave height decay of a spilling breaker is proposed. The energy dissipation of a breaking wave is approximated by that of a propagating bore. In order to explain the gentle decay of spilling breaker at the initial stage, a development of a foam region, which indicates the amount of foam on the wave profile and determines the rate of energy dissipation, is considered. In addition to this formulation, the energy and momentum balance equations are described by a linear wave theory in shallow water and are simultaneously solved. Comparisons with experimental results show that the model gives a good prediction in both inner and outer regions, and that two coefficients in the present model are related to the deep water wave steepness and the slope of beaches..

1 INTRODUCTION

The height of breaking wave and a rate of its decaying have very important influences on the nearshore wave-induced phenomena, such as the generation of nearshore currents, the transportation of sediments and so on. In order to discuss these phenomena, it is necessary to clarify the characteristics of breaking waves.

Many experimental investigations and several models for breaking wave have been presented. In these models, it is widely used to approximate the energy dissipation of the breaking wave by that of a propagating bore. Originally, this approach was proposed by Le Méhauté(1962). In his study, a conception named "non-saturated breaker" was introduced, and a formulation for the wave energy dissipation of the spilling breaker involving effects of the bottom friction was used to explain the

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* Instructor, Civil Engineering Department, Iwate University,
Ueda 4-3-5, Morioka 020, Japan
** Graduate Student, Civil Engineering Department, Iwate University,
Ueda 4-3-5, Morioka 020, Japan
*** Professor, Civil Engineering Department, Hokkaido University,
Kita 13, Nishi 8, Sapporo 060, Japan

wave height index ($H=0.78h$). Hwang and Divoky(1970) improved this approach by taking a wave set-up into considerations.

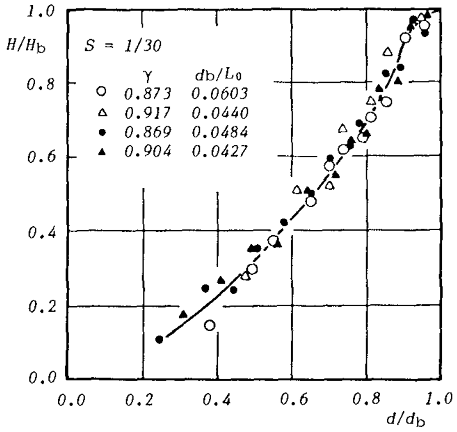
Battjes(1978) gave a model using a dissipation function for breaking waves depending on local wave properties and a local breakerheight (γh). This model showed an overall agreement with results of systematic experiments given by Horikawa and Kuo (1966). It was, however, pointed out that in most cases the initial decay was overestimated by this model.

Svendsen, Madsen and Hansen(1978) indicated that the surf zone is divided into inner and outer regions, and Svendsen (1984) showed that "surface roller" has an important role in the wave decay. The results of the model agreed with experiments well in the inner region.

While this approach has been improved by many studies, some problems still remain. The largest difference between calculations by the model and experiments appears in the outer region. Particularly for spilling breakers, models gave the larger energy dissipation than what were experimentally observed.

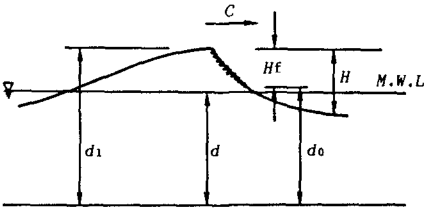
In the present study, experimental findings of the gentle decay of spilling breaker at the initial stage and a development of turbulent region, called "foam region", during the breaking are first explained. And a new model for the decay of a spilling breaker is proposed. In the model, the energy dissipation of the breaking wave is approximated by that of a propagating bore and the development of the foam region is mathematically expressed based on several assumptions. Energy and momentum equations are also simultaneously applied.

Finally, two coefficients in the model are determined by comparing with the experimental results.



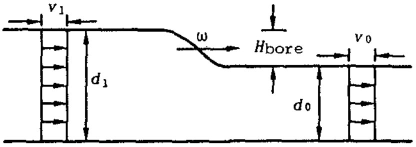
H : wave height, d : mean water depth
$\gamma = H_b/d_b$, L_0 : deep water wave length
subscript b denotes conditions at breaking point

Figure 1 Wave height decay of spilling breaker



$Hf = d_1 - d_0$: vertical length of foam region
C : wave celerity

(a) spilling breaker



$H\text{bore} = d_1 - d_0$: bore height
ω : propagation velocity of bore
v_0, v_1 : velocity of flow

(b) propagating bore

Figure 2 Comparison between spilling breaker and propagating bore

2 DEVELOPMENT OF FOAM REGION

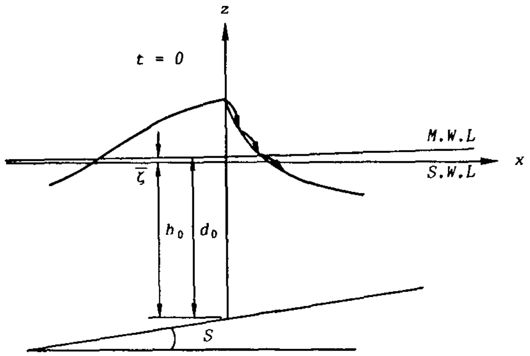
Examples of wave height decay of spilling breaker in a surf zone are shown in Figure 1. This figure shows that the wave height decay immediately after breaking points is rather gentle, and the change is rapid after the wave propagates some distance. This tendency was found in the experiments given by Horikawa and Kuo(1966), as well as in our experiments. This refers to a fact that a region of turbulence of the spilling breaker is restricted near the wave crest in the outer region and develops as the wave propagates. And the development of the turbulence region increases the rate of the wave energy dissipation.

The turbulence region in spilling breakers is called a "foam region" in this paper, and Hf indicates the vertical length of a foam region, as illustrated in Figure 2 (a). When the energy dissipation of spilling breakers is approximated by that of the propagating bore(Figure 2 (b)), the bore height Hbore corresponds to the length Hf of the spilling breaker, and therefore the change in the size of the foam region has an important influence on calculated results. Since the foam region immediately after breaking is narrower than that of well-developed breaker in the inner region, the increase of the energy dissipation rate of a spilling breaker can be explained by considering the development of the foam region.

3 WAVE HEIGHT DECAY MODEL

3.1 Foam Region

In order to explain the development of the foam region, it is assumed that a water particle is released from the crest of a wave at a breaking point, and the particle again meets the water surface after a very short time Δt, as shown in Figure 3. Now, the lower boundary of the foam region is defined by this point. Then, the water particle jumps again from this point, and consequently the foam region further develops.



d_0 : mean water depth at $t=0$
h_0 : still water depth at $t=0$
$\bar{\zeta}$: wave set-up at $t=0$

Figure 3 Development of foam region

In the calculations of the time interval and the distance for such action of a water particle, it is assumed that the wave celerity is given by a linear wave theory in shallow water and that the horizontal velocity of the released particle is identical to the wave celerity at the time when it was just released. Thus, the wave celerity C at $t=\Delta t$ and the horizontal velocity u_0 of the released water particle is given as follows;

$$C = \sqrt{gd_0} - \frac{Sg}{2}\Delta t \quad , \quad u_0 = C_0 = \sqrt{gd_0} \qquad (1)$$

where g : the gravitational acceleration, d_0 : a water depth at t=0, and S : a slope of beach.
Therefore, the relative velocities to the wave profile are described as;

$$u' = u_0 - C = \frac{Sg}{2}\Delta t \quad , \quad w' = -g\cdot\Delta t \qquad (2)$$

and consequently, the distances measured from the water profile are;

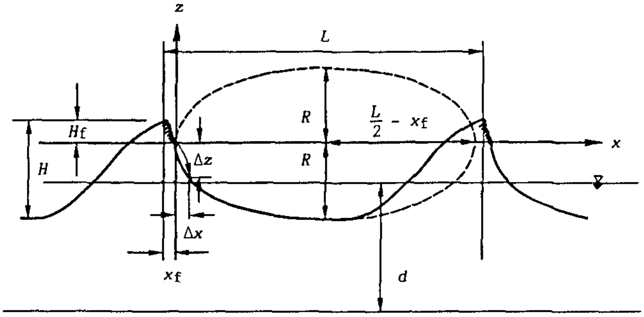
$$\Delta x = \frac{Sg}{4}\Delta t^2 \quad , \quad \Delta z = -\frac{g}{2}\Delta t^2 \qquad (3)$$

In addition, the front profile of the wave is approximated as an ellipse. At the time when the released water particle meet again the water surface, the following equation is satisfied.

$$\left(\frac{\Delta z}{R} \right)^2 + \left(\frac{\Delta x - \left(\frac{L}{2} - x_f \right)}{\frac{L}{2} - x_f} \right)^2 = 1 \qquad (4)$$

where notations are explained in Figure 4.
From Equations (3) and (4), the time interval is given, and then the distances are calculated by Equation (3). We further used a coefficient on the time interval, given by Equation (5).

$$\Delta t' = \alpha \cdot \Delta t \qquad (5)$$



X_f, H_f : horizontal and vertical length of foam region
Δx, Δz : horizontal and vertical increment of foam region
L : wave length, $R = H - H_f$

Figure 4 Approximation of wave profile by ellipse

3.2 Energy and Momentum Balance

In the present model, the energy dissipation of a spilling breaker is approximated by that of the propagating bore. As widely used in the previous studies, the energy dissipation per a unit length is described as a following equation with a coefficient B.

$$D' = -B \frac{\rho g}{4} \frac{1}{Td} H_f^3 \qquad (6)$$

where ρ is a density of the fluid, T is a wave period.
When the energy flux of the wave is calculated by a linear wave theory in shallow water, the energy balance equation is described as follows;

$$\frac{d}{dx} \left\{ \frac{1}{8} \rho g H^2 \left(gd \right)^{\frac{1}{2}} \right\} + B \frac{\rho g}{4} \frac{1}{Td} H_f^3 = 0 \qquad (7)$$

Since the development of the foam region is calculated descretely with the time interval Δt as shown in Figure 5, the energy balance is also considered within this interval. The time interval is so short that the foam region Hf within the interval can be regarded to be constant. Therefore, Equation(7) is integrated, and the change of wave height from t1 to t2 is given as follows;

$$\left(\frac{H_2}{H_1} \right)^{-1} = \left(1 - \frac{2}{3} K \right) \left(\frac{d_2}{d_1} \right)^{\frac{1}{4}} + \frac{2}{3} K \left(\frac{d_2}{d_1} \right)^{-\frac{1}{2}} \qquad (8)$$

$$K = \frac{2B}{S} \left(\frac{d_1}{gT^2} \right)^{\frac{1}{2}} \frac{H_1}{d_1}$$

In order to calculate correctly the wave height near a shore line, the wave set-up is included in calculations. In the present model, a following relationship given by Longuet-Higgins and Stewart(1964) is used to calculate the mean water depth.

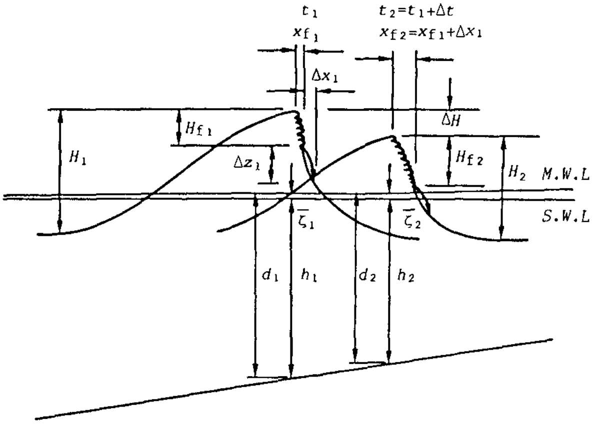
$$\frac{d}{dx} Sxx + \rho g \left(h + \bar{\zeta} \right) \frac{d\bar{\zeta}}{dx} = 0 \qquad (9)$$

Sxx : radiation stress

$\bar{\zeta}$: *wave set-up/down*

3.3 Procedure of Calculation

The time interval Δt and the increments of the lower boundary of the foam region Δx_1 and Δz_1, described in Figure 5, are calculated by Equations.(4), (5) and (3). And, the energy and momentum balance equations are simultaneously solved to give Hf2 and $\bar{\zeta}_2$.



Δt : time interval
ΔH : decrement of wave crest height

Figure 5 Description for development of foam region

Successively, the foam region at the next time step is given as;

$$H_{f2} = H_{f1} + \Delta z_1 - \Delta H$$

$$\doteqdot H_{f1} + \Delta z_1 - (H_1 - H_2) \frac{\eta_c}{H} \qquad (10)$$

where η_c is a wave crest height.
However, in order to calculate the decrement of the wave crest height, the relationship between the crest height and the wave height should be considered. Hansen's data quoted by Svendsen(1984) indicated that the ratios of the wave crest height to the wave height vary linearly with the change of the depth, while the magnitude of the ratios is different depending on the deep water wave steepness. This qualitative tendency is adopted to calculate the change of the wave crest height, and a following relationship is assumed.

$$\frac{\eta_c}{H} = 0.55 + 0.2 \frac{d}{d_b} \qquad (11)$$

where db is a breaking water depth.

4 VERIFICATION OF MODEL

Two coefficients α and B are used in the present model, to compensate slight uncertainties of the assumptions. In order to determine these coefficients, some experiments were conducted, and in these experiments the slopes of beaches were 1/15,1/30 and 1/50.

Comparisons between the calculations by the model and experimental results indicate that the optimum value of the coefficient B depends on the slope of beach. B has a larger value when the slope is steeper, and the relationship between the coefficient B and the slope of beach is shown in Figure 6. Experiments on the slope 1/65 given by Horikawa and Kuo(1966) were also explained by the present model with the value on the slope 1/50. This implies that this coefficient is constant on the slopes milder than 1/50.

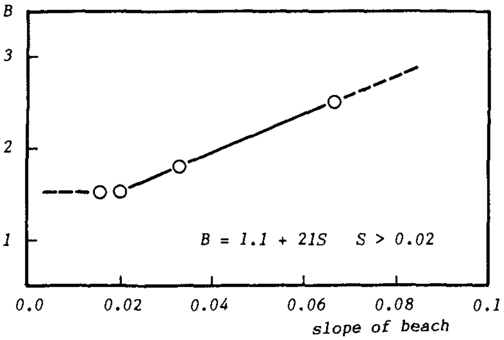


Figure 6 Relationship between coefficient B and slope of beach

For the other coefficient α, the optimum value is 2.5, in the cases where the deep water wave steepness is large and consequently the complete spilling breaker occurs. The decrease of the deep water wave steepness changes the breaker type from spilling to plunging. The spilling breaker with relatively small deep water wave steepness has a somewhat similar properties to plunging breaker. For such breaking waves, the coefficient should be magnified, in order to increase the energy dissipation in the outer region. Within our experiments, a relationship between the coefficient α and the deep water wave steepness is given in Figure 7.

Figure 8 (a),(b) and (c) show comparisons between the calculations by the present model and experimental results on the slope 1/15, 1/30 and 1/50, respectively, in which the deep water wave steepness is large. As shown in these figures, the calculations predict the experimental results fairly well. It is noteworthy that the present model is good, not only in the inner region but also in the outer region.

Figure 9 (a) shows a similar comparison for a spilling breaker on the slope 1/15 with a relatively small deep water wave steepness. The high accuracy of the prediction by the present model still remains. Figure 9 (b) gives a comparison on the slope 1/30. The calculation also agrees with experiments in this case.

Consequently, it was concluded that the present model gives good predictions of the wave height decay of the spilling breaker on the slopes from 1/15 to 1/50.

Figure 10 (a) and (b) show the results with respect to the wave set-up in the surf zone. The earlier models gave rapid increase of the mean water level in the outer region, since the wave energy dissipation was overestimated in this region. While the calculations by the present model still overestimates the wave set-up, the qualitative tendency of the change of the mean water level is explained.

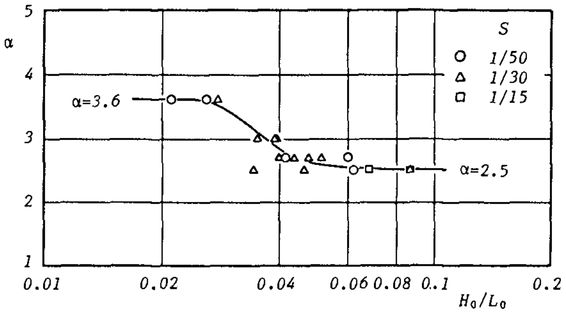
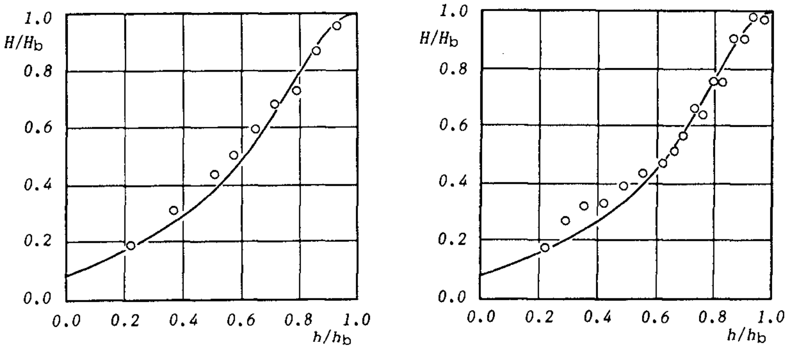


Figure 7 Relationship between coefficient α and deep water wave steepness



$S = 1/15$ $H_0/L_0 = 0.088$

$(H/h)_b = 0.754$ $h_b/L_0 = 0.112$

$(\bar{\zeta}/h)_b = -0.013$

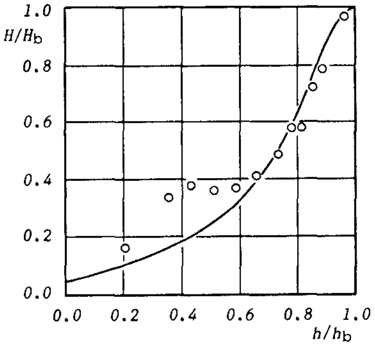
(a)

$S = 1/30$ $H_0/L_0 = 0.047$

$(H/h)_b = 0.793$ $h_b/L_0 = 0.065$

$(\bar{\zeta}/h)_b = -0.013$

(b)



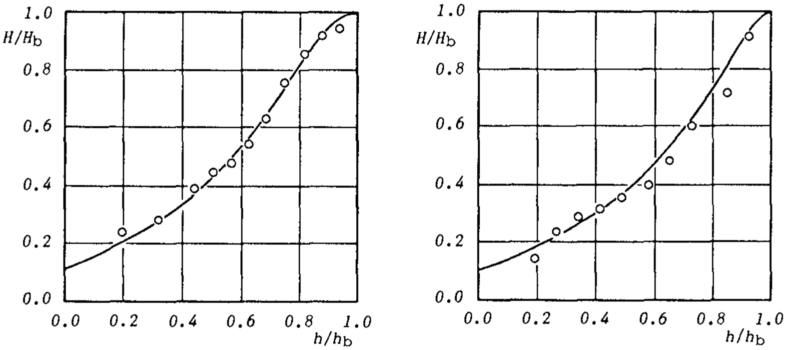
$S = 1/50$ $H_0/L_0 = 0.061$

$(H/h)_b = 0.708$ $h_b/L_0 = 0.085$

$(\bar{\zeta}/h)_b = -0.008$

(c)

Figure 8 Comparisons between calculated and experimental results of wave height decay



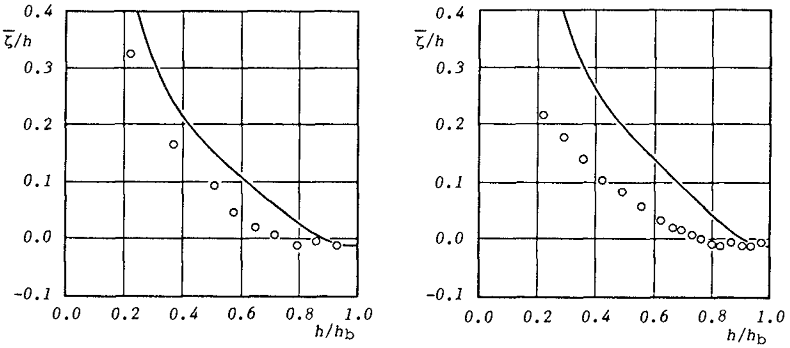
$S = 1/15$ $H_0/L_0 = 0.055$
$(H/h)_b = 0.826$ $h_b/L_0 = 0.072$
$(\bar{\zeta}/h)_b = -0.017$

(a)

$S = 1/30$ $H_0/L_0 = 0.028$
$(H/h)_b = 0.886$ $h_b/L_0 = 0.042$
$(\bar{\zeta}/h)_b = -0.013$

(b)

Figure 9 Comparisons between calculated and experimental results of wave height decay



$S = 1/15$ $H_0/L_0 = 0.088$
$(H/h)_b = 0.754$ $h_b/L_0 = 0.112$
$(\bar{\zeta}/h)_b = -0.013$

(a)

$S = 1/30$ $H_0/L_0 = 0.047$
$(H/h)_b = 0.793$ $h_b/L_0 = 0.065$
$(\bar{\zeta}/h)_b = -0.013$

(b)

Figure 10 Comparisons between calculated and experimental results of wave set-up

5 CONCLUSIONS

Experimental results show that the development of the foam region characterize the wave height decay of the spilling breaker in the outer region. This development and the energy and momentum balances were formulated by a linear wave theory in shallow water. This model explained the characteristics of wave height decay fairly well, even in the outer region. It was found that the coefficients used in the present model are determined by the deep water wave steepness and the slope of beaches.

REFERENCES

Battjes, J.A.(1978). Energy dissipation in breaking solitary and periodic waves. Manus. Delft Univ. of Tech.

Horikawa,K and C. T. Kuo (1964). A study on wave transformation inside surf zone. 10th I.C.C.E.,pp. 217-233.

Hwang, L.S. and D. Divoky (1970). Breaking wave setup and decay on gentle slopes. 12th I.C.C.E., pp. 377-389.

Le Méhauté, B. (1962). On non-saturated breakers and the wave set-up. 8th I.C.C.E., pp. 77-92.

Longuet-Higgins, M.S. and R.W. Stewart (1964). Radiation stresses in water waves; a physical discussion, with applications. Deep-Sea Research, Vol.11, pp. 529-562.

Svendsen, I.A., P.A. Madsen and J.B. Hansen (1978). Wave characteristics in the surf zone. 16th I.C.C.E., pp. 520-539.

Svendsen, I.A.(1984). Wave heights and set-up in a surf zone. Coastal Engineering, 8, pp. 303-329.