

CHAPTER 59

Practical Method For Evaluating Directional Spectra After Shoaling and Refraction

Frederick L.W. Tang* and C.F. Lin**

ABSTRACT

Different Methods have been developed to calculate the deformation of spectrum as waves advance to the shallow water. Wave ray method is a traditional way to compute the change of wave ray after refraction. It is restricted to the regular wave. After slicing the linear spectrum, a lot of components of spectrum can be gained, and each component can be considered as a monochromatic wave. And then after the deformed components are worked out at the same point in sea, a deformed spectrum in shallow water is gained by summing up them. Therefore the wave ray method could be extended to the irregular wave in ocean. In real bathymetry, the difficulty in applying such method is to solve the wave ray which must pass through the assigned point. In this paper, calculation approach as well as computer program are contrived to adopted in real bathymetry. The results of computation reveal some significant characteristics of directional spectra in shallow water. Furthermore, the statistical features of waves at the point of interest are to be evaluated.

Introduction

For design of offshore structures on continental shelf or shallow water area, it is necessary to evaluate the design spectrum at the location of construction. As waves propagate from deep water into shallow water, they are modified by their interaction with the bottom topography. Provided that the directional wave spectrum in deep water region is known by forecast or measurement, it can be sliced to rectangular columns of finite number as linear spectrum is assumed. Every column represents small amplitude monochromatic wave, its shoaling and refraction are to be calculated by traditional procedures, then the columns are changing their height, and shifting their directions. Summing up the new columns, a wave spectrum at the point of interest is worked out.

With a straight uniform parallel bottom contour model, such method has been used by others (Nagai, 1972, etc). In real bathymetry, more effort must be paid due to the unexpected wave direction as wave advanced into shallow water. A try and error method is adopted in this calculation.

Deformation of spectrum

* Professor, Dr. Eng., Department of Hydraulics & Ocean Eng., National Cheng Kung University, Tainan, Taiwan, 70101, R.O.C.

** Dr. Eng., Department of Hydraulics & Ocean Eng., National Cheng Kung University, Tainan, Taiwan, 70101, R.O.C.

The basic assumptions needed in this computation are:

- (1) Waves are generated in deep sea and diminished on shoreline.
- (2) Waves are in steady state, in which the time dependency which may result from wind generation is negligible.
- (3) Energy dissipation is disregarded due to wave-seafloor interaction, white cap and breakers.
- (4) The mathematical model implies no transfer of energy across orthogonals.

Ocean waves are composed of different frequency component waves in all direction. Its spectral density function can be defined as:

$$D(f, \theta) = S(f) F(f, \theta) \dots\dots\dots(1)$$

where $S(f)$: frequency spectrum
 $F(f, \theta)$: directional function

in which the directional function is such that

$$\int_{-\pi}^{\pi} F(f, \theta) d\theta = 1 \dots\dots\dots(2)$$

therefore, the one-dimensional frequency density function $S(f)$ results from the integration of:

$$s(f) = \int_{-\pi}^{\pi} F(f, \theta) d\theta \dots\dots\dots(3)$$

Linear spectrum can be sliced to numerous components. Each component has its representative frequency and can be considered as a regular wave. In practice, we divide the spectrum into finite components which depend on the accuracy of calculation and consumption of computer time.

There are three methods used in slicing the spectrum: (1) equal frequency interval method (2) equal energy interval method (3) combination of those mentioned above. Report indicated that over ninety percentage of spectrum energy concentrated in neighborhood of peak frequency f_{op} , bounded in $0.7 f_{op} \sim 1.6 f_{op}$, see Mitsuyasu, 1984. For this reason, it seems more desirable to adopt method(3), but it will take much computer time. In this calculation, we apply the equal energy interval method.

The sketch of sliced spectrum is shown on Fig.1. The representative frequency of every component can be calculated with its second moment as following:

$$f_{cn}^2 \cdot \Delta E = \int_{f_{n-1}}^{f_n} f^2 s(f) df \dots\dots\dots(4)$$

where f_{cn} : representative frequency of component spectrum
 f_{n-1}, f_n : Lower, upper bounded frequency of component spectrum
 spectral energy of component

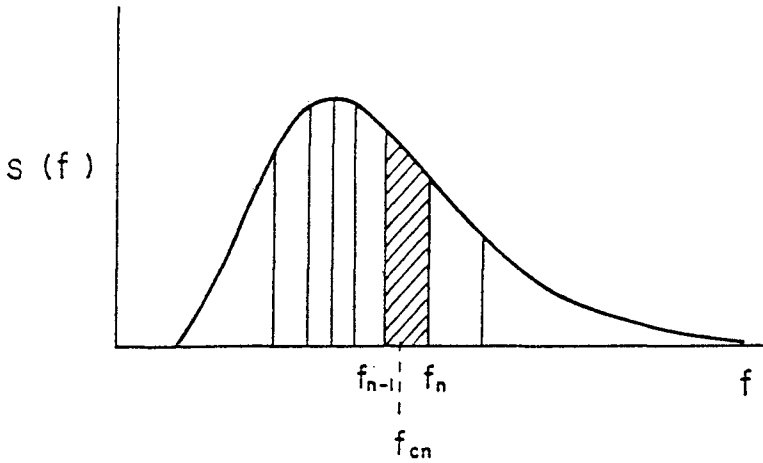


Fig.1 Sketch of sliced spectrum

Directional function can be divided by equal small angle $\Delta\theta$, as shown on Fig.2, then

$$\Delta\theta = \frac{|\theta_{max}| + |\theta_{min}|}{N} \dots\dots\dots(5)$$

N: total dividing number
 $\theta_{max}, \theta_{min}$: Max, Min. distribution angle
 with principal direction respectively.

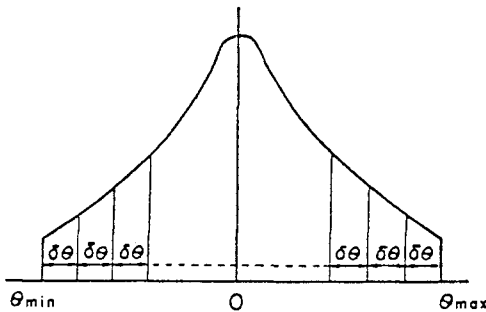


Fig.2 Sketch of sliced directional function

After slicing the known spectrum and directional function in deep sea, we can gain finite numbers of component spectrum with different incident directions and frequencies. Each component is a monochromatic wave, then the traditional wave ray method can be available for computing the deformation of component spectrum at the interesting point.

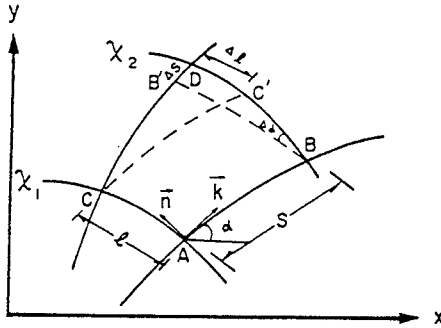


Fig.3 Sketch of wave ray and front

Fig.3 shows the sketch of wave ray and wave front, then the wave ray equation and wave intensity equation are given:

$$\left. \begin{aligned} \frac{d\alpha}{ds} &= \frac{1}{c} \left[\sin\alpha \frac{\partial c}{\partial x} - \cos\alpha \frac{\partial c}{\partial y} \right] \\ \frac{d^2\ell}{ds^2} - p(s) \frac{d\ell}{ds} + q(s)\ell &= 0 \end{aligned} \right\} \dots\dots\dots(6)$$

where

$$\left. \begin{aligned} p(s) &= \frac{1}{c} \left[\cos\alpha \frac{\partial c}{\partial x} + \sin\alpha \frac{\partial c}{\partial y} \right] \\ q(s) &= \frac{1}{c} \left[\sin^2\alpha \frac{\partial^2 c}{\partial x^2} - \sin 2\alpha \frac{\partial^2 c}{\partial x \partial y} \right. \\ &\quad \left. + \cos^2\alpha \frac{\partial^2 c}{\partial y^2} \right] \end{aligned} \right\} \dots\dots\dots(7)$$

C: wave celerity
 ds: increment along wave ray
 dl: increment along wave front

By solving equation (6), we can find width between two rays, ℓ , refracting angle α . Then the shoaling coefficient, refraction coefficient of component wave are calculated by definition:

$$\left. \begin{aligned} K_s &= \sqrt{\frac{C_o}{2nc}} \\ K_r &= \sqrt{\frac{\ell_o}{\ell}} \end{aligned} \right\} \dots\dots\dots(8)$$

where ℓ_o : initial value of width of two adjacent rays
 C_o : wave celerity in deep sea.

$$n: \text{factor} = \frac{1}{2} \left[1 + \frac{2kd}{\text{sinh}(2kd)} \right]$$

The shoaling coeff., K_s , and refraction coeff., K_r , of deformed spectra can be solved as following:
 For unidirectional spectrum:

$$\left. \begin{aligned} K_s &= \sqrt{\frac{\sum_{i=1}^M (K_s^2 \cdot S_o^*(f^*) \cdot \Delta f_n^*)_i / E_o^*}{\sum_{i=1}^M (K_s^2 K_r^2 \cdot S_o^*(f^*) \cdot \Delta f_n^*)_i}} \\ K_r &= \sqrt{\frac{\sum_{i=1}^M (K_s^2 K_r^2 \cdot S_o^*(f^*) \cdot \Delta f_n^*)_i}{\sum_{i=1}^M (K_s^2 \cdot S_o^*(f^*) \cdot \Delta f_n^*)_i}} \end{aligned} \right\} \dots\dots\dots(9)$$

For directional spectrum:

$$\begin{aligned} K_s &= \sqrt{\frac{\sum_{i=1}^M \sum_{j=1}^N (K_s^2)_{ij} \cdot (\Delta E_o^*)_i}{\sum_{i=1}^M (\Delta E_o^*)_i}} \\ K_s K_r &= \sqrt{\frac{\sum_{i=1}^M \sum_{j=1}^N (K_s^2 K_r^2)_{ij} \cdot (\Delta E_o^*)_i}{\sum_{i=1}^M (\Delta E_o^*)_i}} \end{aligned} \dots\dots(10)$$

Dimensionless analysis of spectrum

JONSWAP spectrum:

$$S(f) = \frac{\alpha'}{(2\pi)^4} \frac{g^2}{f^5} \exp\left[-\frac{5}{4} \left(\frac{f}{f_{op}}\right)^{-4}\right] \times \gamma^{exp\left[-\frac{1}{2\sigma_0^2} \left(\frac{f}{f_{op}} - 1\right)^2\right]} \dots\dots\dots(11)$$

SWOP directional function:

$$F(f, \theta) = \begin{cases} \frac{1}{\pi} \{ 1 + (0.5 + 0.82 \exp[-\frac{1}{2} (\frac{2\pi f}{\omega'})^4] \cos 2\theta) \\ + 0.32 \exp[-\frac{1}{2} (\frac{2\pi f}{\omega'})^4] \cos 4\theta \} & |\theta| \leq \frac{\pi}{2} \dots\dots(12) \\ 0 & |\theta| > \frac{\pi}{2} \end{cases}$$

where $\omega' = g / U_{5.0}$
 $U_{5.0}$: wind velocity at 5.0m above sea surface

For the convenience of calculation, we introduce a dimensionless analysis to the known spectrum and directional function. If significant wave height ($H_{1/3}$) and significant wave period ($T_{1/3}$) are chosen as parameters, then the dimensionless spectrum $S^*(f^*)$ is:

$$S^*(f^*) = \frac{S(f)}{H_{1/3}^2 T_{1/3}}$$

For JONSWAP spectrum, Wang et al (1976) derived an approximate value of zero-order moment of spectrum is:

$$m_0 \simeq 0.305 \cdot k_1 \cdot f_{op}^{-4} \dots\dots\dots(13)$$

$$k_1 = \frac{\alpha' g^2}{(2\pi)^4} \quad (\alpha' = 0.0081)$$

Introducing coefficients D_1, D_2 , then we can establish following expressions:

$$\left. \begin{aligned} H_{1/3} &= D_1 \sqrt{m_0} \\ f_{op} &= \frac{1}{D_2 T_{1/3}} \end{aligned} \right\} \dots\dots\dots(14)$$

From equation (11),(13) and (14), we can gain the dimensionless form:

$$S^*(f^*) = \frac{1}{0.305 \cdot D_1^2 \cdot D_2^4} \cdot f^{*-5} \cdot \exp [-1.25 (D_2 \cdot f^*)^{-4}] \\ \times \gamma^{\exp [-\frac{1}{2\sigma_0^2} (D_2 f^* - 1)^2]} \dots\dots\dots(15)$$

In West Taiwan Strait, Ou (1977) analyzed the measured data and deduced the spectrum of a JONSWAP type, those coefficients of spectrum are:

$$\left. \begin{aligned} H_{1/3} &= 3.8 \sqrt{m_0} \\ f_{op} &= \frac{1}{1.13 T_{1/3}} \\ \gamma &= 2.08 \\ \sigma_0 &= \begin{cases} 0.07 & f \leq f_{op} \\ \dots & \dots \\ 0.09 & f > f_{op} \end{cases} \end{aligned} \right\} \dots\dots\dots(16)$$

A dimensionless form of SWOP directional function is given by Nagai (1972):

$$F(f^*, \theta) = \frac{1}{\pi} \cdot \{ 1 + [0.5 + 0.82 \cdot \exp(-0.4416 f^{*4})] \times \cos 2\theta + 0.32 \cdot \exp(-0.4416 f^{*4}) \cdot \cos 4\theta \} \dots(17)$$

Numerical computation

Finite difference method is used in solving wave ray equation and intensity equation. Adopting central-difference method, hence

$$\left. \begin{aligned} \frac{d\ell}{ds} &= \frac{\ell_{n+1} - \ell_{n-1}}{2\Delta s} \\ \frac{d^2\ell}{ds^2} &= \frac{\ell_{n-1} - 2\ell_n + \ell_{n+1}}{(\Delta s)^2} \end{aligned} \right\} \dots\dots\dots(18)$$

And equation (6) yields:

$$\ell_{n+1} = \frac{4 - 2(\Delta s)^2 p_n}{2 - (\Delta s) p_n} \ell_n - \frac{2 + (\Delta s) p_n}{2 - (\Delta s) p_n} \ell_{n-1} \dots\dots\dots(19)$$

In regular wave, the relationship among wave length, period and water depth is:

$$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$$

Wave celerity $C=L/T$ angular frequency $\omega = \frac{2\pi}{T}$
 Then,

$$C = \frac{g}{\omega} \tanh \frac{\omega d}{C} \dots\dots\dots(20)$$

From which, following partial derivatives can be establish:

$$\begin{aligned}
 \frac{\partial C}{\partial x} &= H(C) \frac{\partial d}{\partial x} \\
 \frac{\partial C}{\partial y} &= H(C) \frac{\partial d}{\partial y} \\
 \frac{\partial^2 C}{\partial x^2} &= H(C) \left\{ \frac{\partial^2 d}{\partial x^2} + \left(\frac{\partial d}{\partial x} \right)^2 \frac{dH(C)}{dC} \right\} \\
 \frac{\partial^2 C}{\partial x \partial y} &= H(C) \left\{ \frac{\partial^2 d}{\partial x \partial y} + \frac{\partial d}{\partial x} \frac{\partial d}{\partial y} \frac{dH(C)}{dC} \right\} \\
 \frac{\partial^2 C}{\partial y^2} &= H(C) \left\{ \frac{\partial^2 d}{\partial y^2} + \left(\frac{\partial d}{\partial y} \right)^2 \frac{dH(C)}{dC} \right\}
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} \frac{\partial C}{\partial x} \\ \frac{\partial C}{\partial y} \\ \frac{\partial^2 C}{\partial x^2} \\ \frac{\partial^2 C}{\partial x \partial y} \\ \frac{\partial^2 C}{\partial y^2} \end{aligned}} \right\} \dots\dots\dots(21)$$

in which:

$$k_1 = \frac{1}{2\omega}$$

$$k_2 = \frac{\omega}{g}$$

$$H(C) = \left[2 k_1 C \frac{k_2}{1 - (k_2 C)^2} + 2 k_1 \tanh^{-1} (k_2 C) \right]^{-1}$$

$$\frac{dH(C)}{dC} = \frac{-4 k_1 k_2 [H(C)]^2}{[1 - (k_2 C)^2]^2}$$

With step-by-step procedure, the width of wave ray ℓ_n , refracting angle α_n can be solved. The wave ray and K_s , K_r of component spectrum will be traced and calculated. In uniform slope or parallel straight topography, all the deformed wave on contour line is the same. Therefore, the computation is rather simplified and shallow spectrum of interesting point can be solved easily. In real bathymetry, the deformation varies from point to point, a try and error technique is adopted ingeniously. Fig.4 shows the schematic grid points of topography, in which COMX, COMY represents X-, y-axis of interesting point respectively. 1~No, 1~Mo are node numbers.

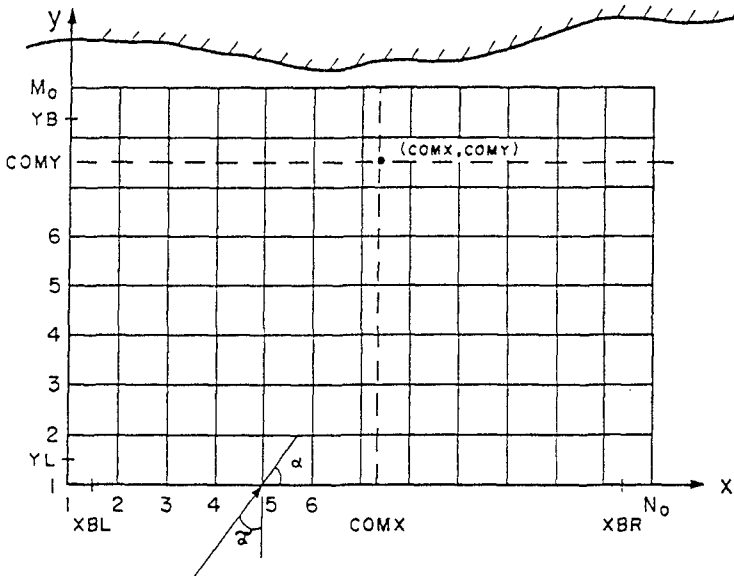


Fig.4 Schematic representations of coordinate

Try and error procedure:

First, setting the initial coordinate $(X_o)_1$ as starting point of computation in deep sea as following:

$$(X_o)_1 = X_o' \pm \left(\frac{90^\circ}{\alpha} - 1 \right)$$

where X_o' is a initial coordinate based on the assumption of no refraction of wave. Last term in above equation is a revised term of refraction, "+" is taken as $X_o' < \text{COMX}$, "-" as $X_o' > \text{COMX}$. When the computation comes to the neighborhood of $y = \text{COMY}$, the Lagrangian interpolation method is applied to obtain the K_s , K_r and α on $y = \text{COMY}$, then the corresponding $WXN(1)$ in X -axis is also obtained. If $(\text{COMX} - WXN(1)) < 0.1$, it is considered that the wave ray goes pass through the interesti: point and the computation of the component is completed. if not, revised the initial point for next step by:

$$(X_o)_2 = (X_o)_1 + (\text{COMX} - WXN) K_r^2$$

As repeating the same procedure only few times, the results could be gained. In few cases, a least square curve fitting may be avail-

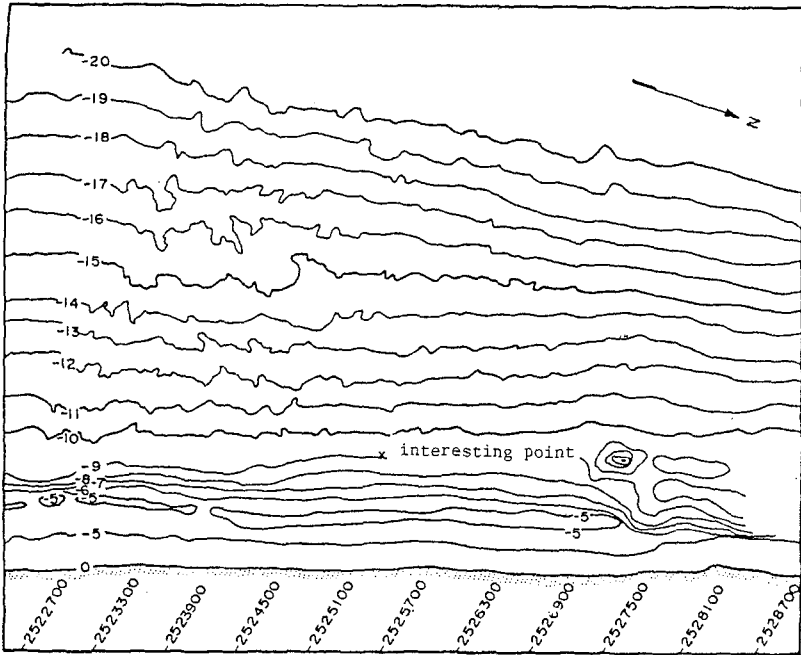


Fig.5 Topography off Hsing-Ta Harbour in West Taiwan Strait

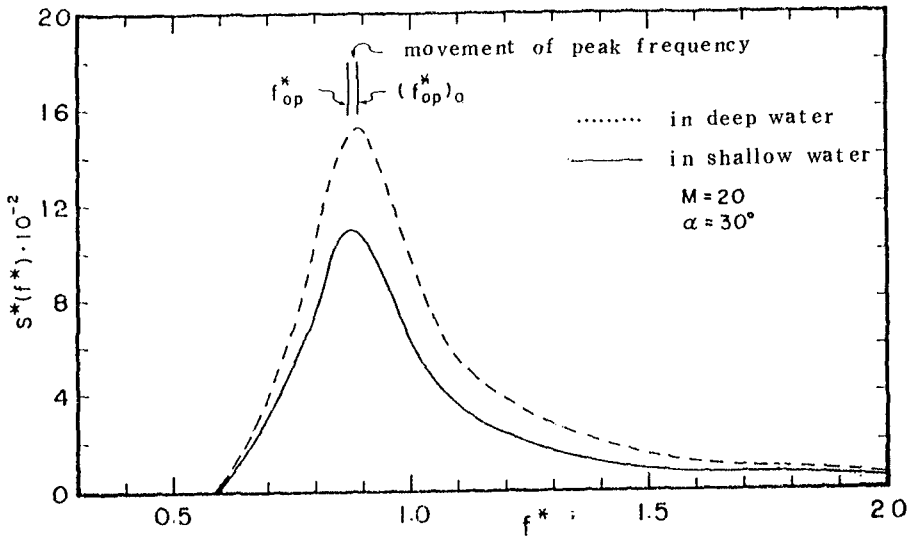


Fig.6 JONSWAP spectrum and its deformed spectrum in West Taiwan Strait

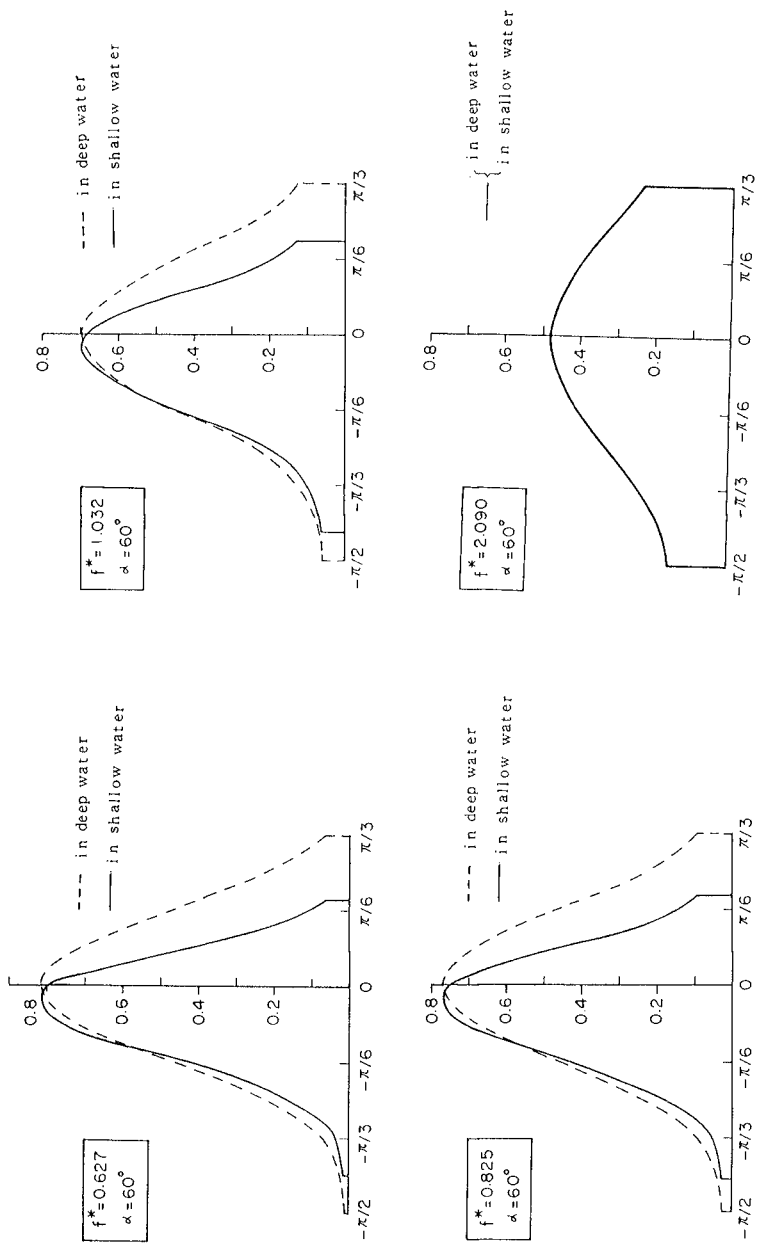


Fig.7 SWOP direction function and its deformed shape

Table 1 Comparison of results with regular wave and directional wave
($H = 7 \text{ m}$, $T = 10 \text{ sec}$)

incid. ang. (with X-axis)	sine wave		* P - M		* Bretschneider		* JONSWAP	
	H	T	$H_{1/3}$	$T_{1/3}$	$H_{1/3}$	$T_{1/3}$	$H_{1/3}$	$T_{1/3}$
0°	6.8	10	4.5	10.0	4.4	9.7	4.4	9.9
30°	5.7	10	5.9	10.3	5.6	10.0	5.8	10.2
60°	7.1	10	6.7	10.4	6.4	10.1	6.6	10.3
90°	7.2	10	7.1	10.4	6.8	10.1	7.0	10.3
120°	6.4	10	6.7	10.4	6.4	10.0	6.5	10.2
150°	5.6	10	6.0	10.4	5.8	10.1	5.9	10.2
180°	5.9	10	4.6	10.3	4.5	10.0	4.5	10.2

* SWOP directional function

able to solve the problems.

Due to the local drastic variation of topography or other reasons, there may occur the crossing of wave orthogonals. In such case, topographical smoothing is always used. However, A different way is adopted to treat this situation in this paper. It is by shifting the initial point several times to avoid or by-pass where the crossing is occurred, and a numerical approach is applied to get the solution at the interesting point with those results from different initial point.

Results and Conclusions

This model is applied to the slowly varying topography which ray theory is valid. A computer program is developed to fit this approach. As the coordinate of interesting point and wave spectrum in deep sea are given, the deformed spectrum at interesting point could be gained.

Fig.5 shows the topography off Hsing-Ta Harbour in West Taiwan Strait used in this calculation.

Considering the one-dimensional spectrum in deep sea, where incident angle of waves is 30 degree, and twenty dividing numbers are adopted, the deformed spectrum of interesting point as well as the original spectrum in deep sea is shown on Fig.6. It shows clearly that the peak frequency is different. SWOP directional is a distribution function related to frequency. Fig.7 shows the deformed directional function at various frequencies. The shapes of function in shallow water are more sharp. The energy of component spectrum is more concentrated to principle wave direction as waves advance to the shallow water when directional distribution of wave energy is considered. Table 1 is the comparison of results by applying regular wave and directional spectrum (SWOP directional function) There has a large difference at 0° and 180° between regular wave and directional spectrum. It is partly due to the effective income of energy when directional function is considered.

Reference

1. Mitsuyasu, H. "Recent Studies on Ocean Wave Spectra" Int. Congress of Theoretical and Applied Mechanics, Lyngby, Denmark, 1984.
2. Nagai, K. "Computation of Refraction and Diffraction of Irregular Sea" Report of the Port and Harbour Research Institute, Vol.11, No.2, June 1972.(in Japanese)
3. Ou, S. H. "Parametric Determination of Wave Statistics and Wave Spectrum of Gravity Waves" Doctor Dissertation, NCKU, 1977.
4. Wang, J. and Liang, N.K. "A Note on the Significant Wave derived from JONSWAP Spectrum" Acta Oceanogr. Taiwanica, N.T.U., 1976.
5. Bretschneider, C.L. "significant Waves and Wave Spectrum" Ocean Industry, pp.40-46, Feb. 1968.
6. Rice, S.O. "Mathematical Analysis of Random Noise" in Noise and Stochastic Process, N. Wax, ed., Dover Publication, New York, pp.133-294, 1954.