CHAPTER 205

IRREGULAR WAVE TRANSFORMATION IN A BOUSSINESQ WAVE MODEL

H.-H. Prüser * / H. Schaper * / W. Zielke *

1. INTRODUCTION

Numerical wave models for shallow water waves are of particular importance for the calculation of the wave climate in harbours and coastal areas. Especially nonlinear time domain models, which are based on the Boussinesq-Wave-Equations, may be helpful in the future for simulating the interaction of currents with refraction, diffraction, reflection and for simulating shoaling of irregular waves in natural areas; a potential which has not yet been fully developed.

During the last ten years numerical models, based on these equations, have been published; such as ABBOTT et. al. , HAUGUEL and SCHAPER / ZIELKE . Research on this topic is currently being carried on.

Some efforts have been made to verify the capability of the models to describe the various physical phenomena. However, up to now, verification has been limited to regular waves. The aim of this paper therefore is, to consider questions concerning irregular, nonlinear waves.

*Zielke, Prof. of Fluid Mechanics
*Prüser, Research Assistant
*Schaper, Research Assistant

Inst. of Fluid Mechanics
University of Hannover
Callinstraße 32
3000 Hannover, FRG

A numerical model, based on the solution of the Boussinesq equations, was used to calculate the behaviour of wave spectra and bichromatic waves in a one dimensional wave flume with constant water depth or with mild slope. The generation of long waves and the energy transfer between individual frequency components in the numerical model was of particular interest. Comparisons with analytical solutions and hydraulic measurements have been made.

2. NUMERICAL MODEL

All computations have been done using the Boussinesq wave model described in detail by SCHAPER / ZIELKE, 1984. Experimental data from a hydraulic wave flume were available. Therefore the study has been restricted to the onedimensional case. The basic equations read:

$$\frac{\delta p}{\delta t} + \frac{\delta}{\delta x}\left(\frac{p^2}{h}\right) + gh\,\frac{\delta \zeta}{\delta x} = \frac{Dh}{2}\,\frac{\delta^3}{\delta x^2 \delta t}\left(\frac{Dp}{h}\right) - \frac{D^2 h}{6}\,\frac{\delta^3}{\delta x^2 \delta t}\left(\frac{p}{h}\right)$$

$$\frac{\delta \zeta}{\delta t} + \frac{\delta p}{\delta x} = 0$$

$h = D + \zeta$: total water depth D : mean water level
p : vertically integrated flux ζ : waterelevation
g : acceleration due to gravity

Solitary and cnoidal waves can be regarded as special solutions of the Boussinesq equations. They have been used in the initial stage of model development to investigate the accuracy of the numerical solution for waves up to limiting height.

In short shallow water waves, the nonlinear amplitude-dispersive terms on the left hand side and the nonlinear frequency-dispersive terms on the right hand side are of the same order. It is therefore necessary to include all these terms into the computation. In the numerical model, the Boussinesq equations are solved with a third order corrected, implicite finite difference method, using two time levels and central differences.

A nonreflecting boundary condition is used at the left end of the flume, instead of a model of the wave maker. Therefore, the re-reflection of waves returning from the right end of the channel is neglegted. The input waves can be represented as a precribed time serie $f(t, x=0)$, which have been obtained from measurements in the flume. The unknown values for the flux and water elevation, as the result of incoming and outgoing waves, can be calculated by using the following equation:

$$p - c\zeta = -2c\, f(t)$$

with: $c = \sqrt{gh - p^2/h^2}$ wave velocity

3. TRANSFORMATION OF PIERSON-MOSKOWITZ SPECTRA

3.1 Hydraulic Test Programme

Hydraulic measurements were taken by Daemrich and Götschenberg from the Franzius-Institute of the University of Hannover to verify the numerical model. In a flume with a constant water depth of 0.5 m, some Pierson-Moskowitz spectra were generated by the wave maker. The amplitudes have been reduced by a constant factor in order to avoid wave breaking, which does occur, if this deep water spectrum is used in shallow water. Several wave gauges were placed along the flume for measuring the time series of the water elevation. Table 1 displays the significant wave parameter of the test program.

Peak-Period [sec] / Peak-Frequency [Hz]			Significant Wave-Height [m]		
1.5	/	0.666	0.04	0.06	0.08
3.0	/	0.333	0.04	0.06	0.08
4.5	/	0.222	0.04	0.06	0.08

Table 1: Test Program

The water elevation, measured at the gauge nearest to the wavemaker, is used to feed the waves into the numerical model at the left boundary.

3.2 Range of Application

Boussinesq equations are long-wave-equations with additional third order terms. They are able to describe wave phenomena in shallow water and in intermediate depths but not in deep water. If one assumes a water depth of 0.5 m, then the deep water waves are shorter than 1.0 m and have a corresponding frequency of higher than 1.25 Hz. Therefore it is to be expected, that the smaller the "deep water" part of the spectrum, the better the calculation of the wave spectra.

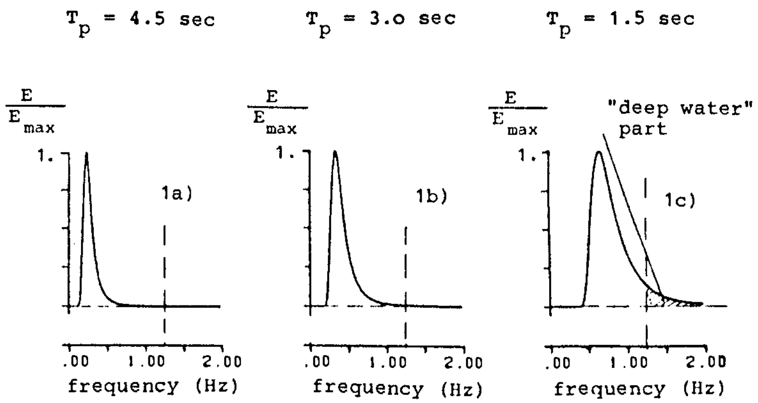


Figure 1: Normalized Pierson-Moskowitz Spectra

Figure 1 displays the normalized Pierson-Moskowitz spectra of the hydraulic test program. The deep water part of spectrum 1c is significant, thus the calculation cannot be accurate in this case. And indeed, verification shows that the Boussinesq wave model reproduces the time series of the water elevation for the spectra 1a) and 1b) very well, but worse for spectrum 1c).

3.2 Calculation and Comparison

3.2.1 Pierson-Moskowitz Spectra in a Flume with even Bottom and a mild Slope

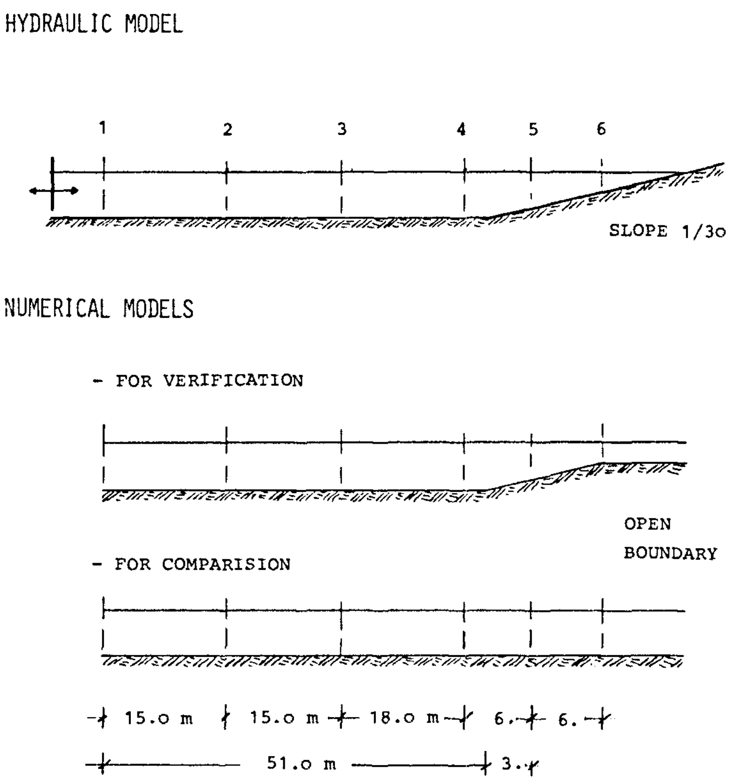
3.2.1.1 Experimental Set-Up

For the hydraulic model, a 54.0 m long flume with even bottom and a slope 1/30 at the right side was used. Two numerical models were created:

The first model was used for verification. The Boussinesq wave model does not contain the possibility of simulating breaking waves, which occur on the upper part of the slope. Instead a nonreflecting boundary was applied at that point of the slope, where the water depth was 0.2 m.

In the second model no slope is taken into account. Therefore, there is a possibility to extract the calculated slope influence by comparing these two numerical models.

The measured time series of the water elevation at gauge 1 is used as wave input for the numerical model. Figure 2 shows the complete experimental set-up.



1, 2, 3, 4	GAUGES	WATER DEPTH = .5o m	
5	GAUGE	WATER DEPTH = .4o m	(.5o m)
6	GAUGE	WATER DEPTH = .2o m	(.5o m)

Figure 2: Experimental set-up / flume with slope

3.2.1.2 Results

Figure 3 displays a comparision between measured and calculated time series at different gauges for a Pierson-Moskowitz spectrum (peakperiod period = 3. sec, significant wave height = 0.08m). It can be seen that the numerical model describes the water elevation with a high degree of similarity in the range of horizontal bottom as well as on the slope.

Calculations with both numerical models (with and without a slope) indicate, that the influence of reflection at the mild slope is negligible for the chosen spectra.

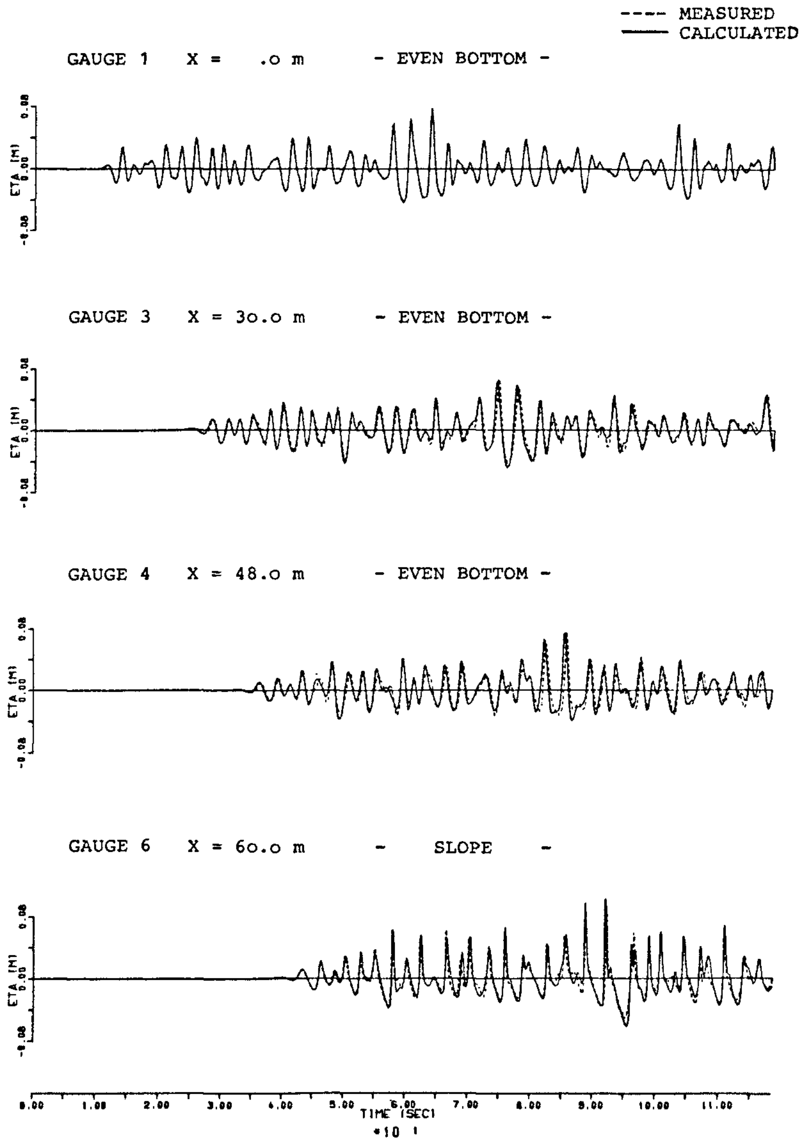


Figure 3: Transformation of wave spectra
even bottom / slope, time domain

A better interpretation of the experimental and calculated results is possible by transforming the time series into the frequency domain.

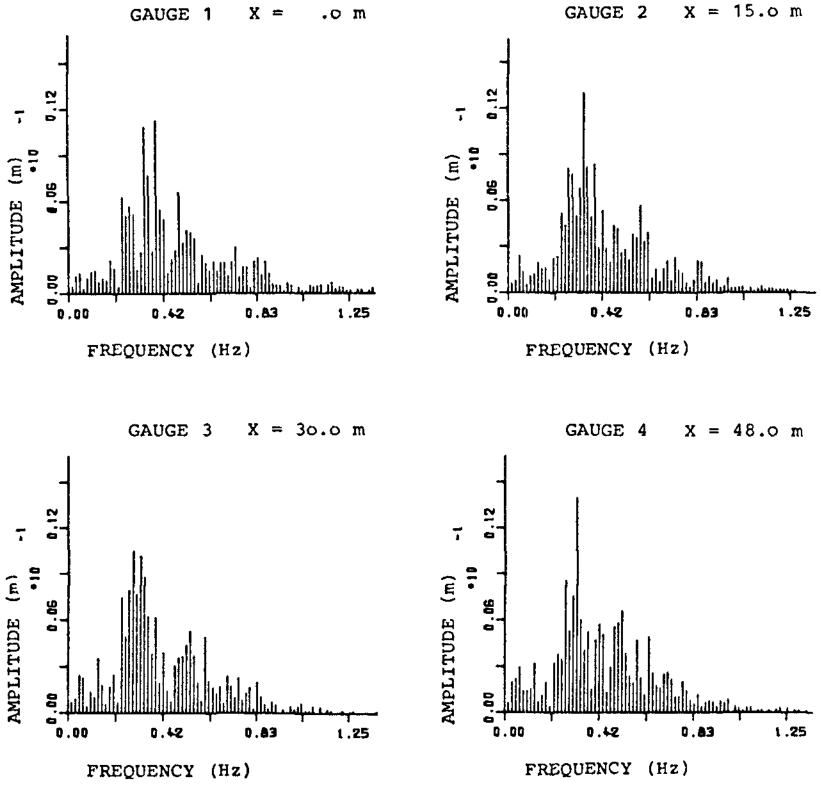


Figure 4: Transformation of wave spectra
even bottom / slope, frequency domain

In linear wave theory, the shape of the spectrum, generated at the wave maker, should not change along the flume if the water depth is constant. However, comparing the amplitude spectra at different locations (see figure 4), an energy transfer between individuel frequency components and the creation and increase of lower frequencies, not included in the original Pierson-Moskowitz spectrum, can be observed in the hydraulic as well as in the numerical model.

At the slope the water depth decreases and the waves turn to the typical cnoidal shape: The crests are higher and shorter and the troughs are longer and less deep than would be predicted for constant water depth.

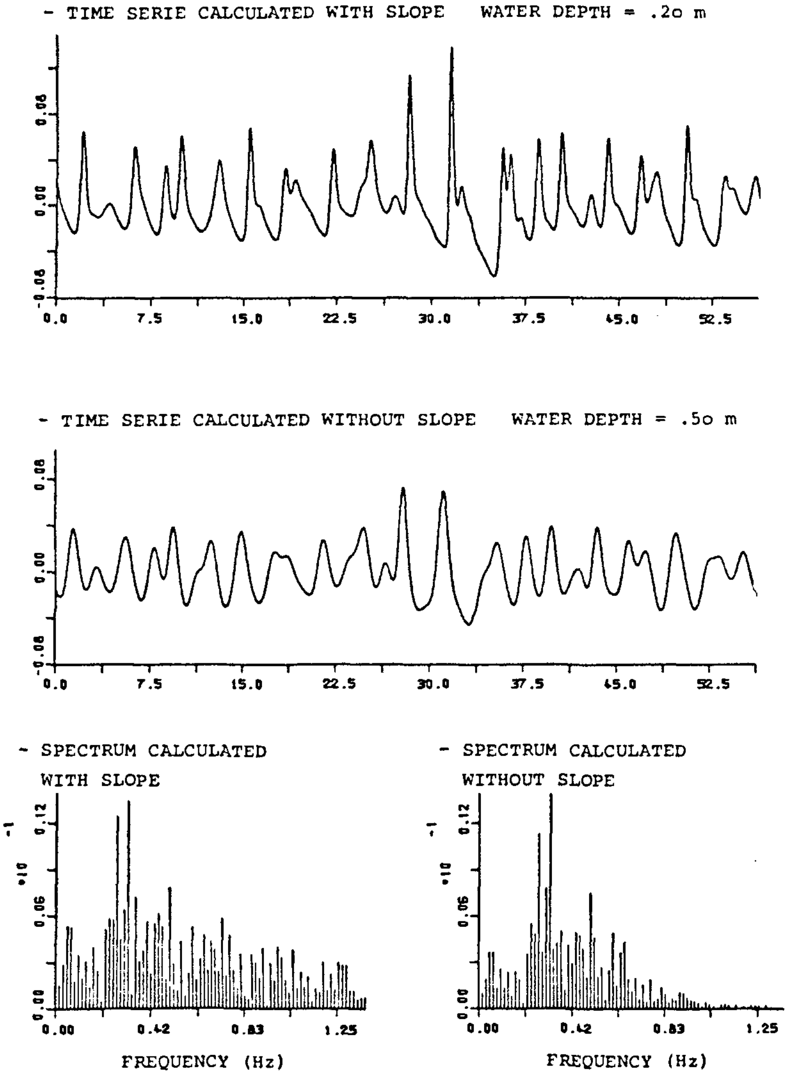


Figure 5: Slope influence calculated at gauge 6.
Comparison in time and frequency domain

The increase of the long wave components and the generation of higher frequencies on the slope are results which can be detected in the frequency domain. Figure 5 displays a comparison of the time series and amplitude spectra at gauge 6, calculated with and without a slope.

3.2.2 Pierson-Moskowitz Spectra Reflected at a Wall

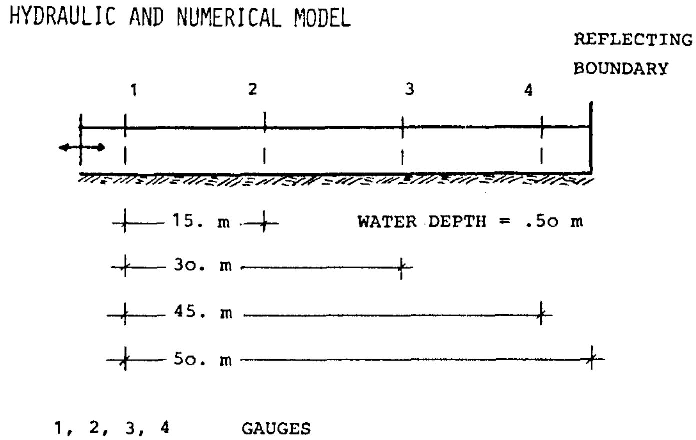


Figure 6: Experimental set-up

The test program was used to verify the reflecting boundary condition at the wall and the capability of the numerical model to calculate the nonlinear interaction of the initial and reflected waves. Figure 6 displays the experimental set-up, figure 7 compares measured and calculated time series of the water elevation at several gauges for the same spectrum used before. The distance between every gauge is 15.0 m. Therefore, it is possible to mark in figure 7, the front of the initial wave, as well as the first reflected wave and the first re-reflected wave. Also the time of the first reflection at the wall and the first re-reflection at the wave maker has been marked. The numerical model is able to describe the resulting wave climate very well. However a comparison between measurements and calculations is not valid after the reflected wave has reached gauge 1.

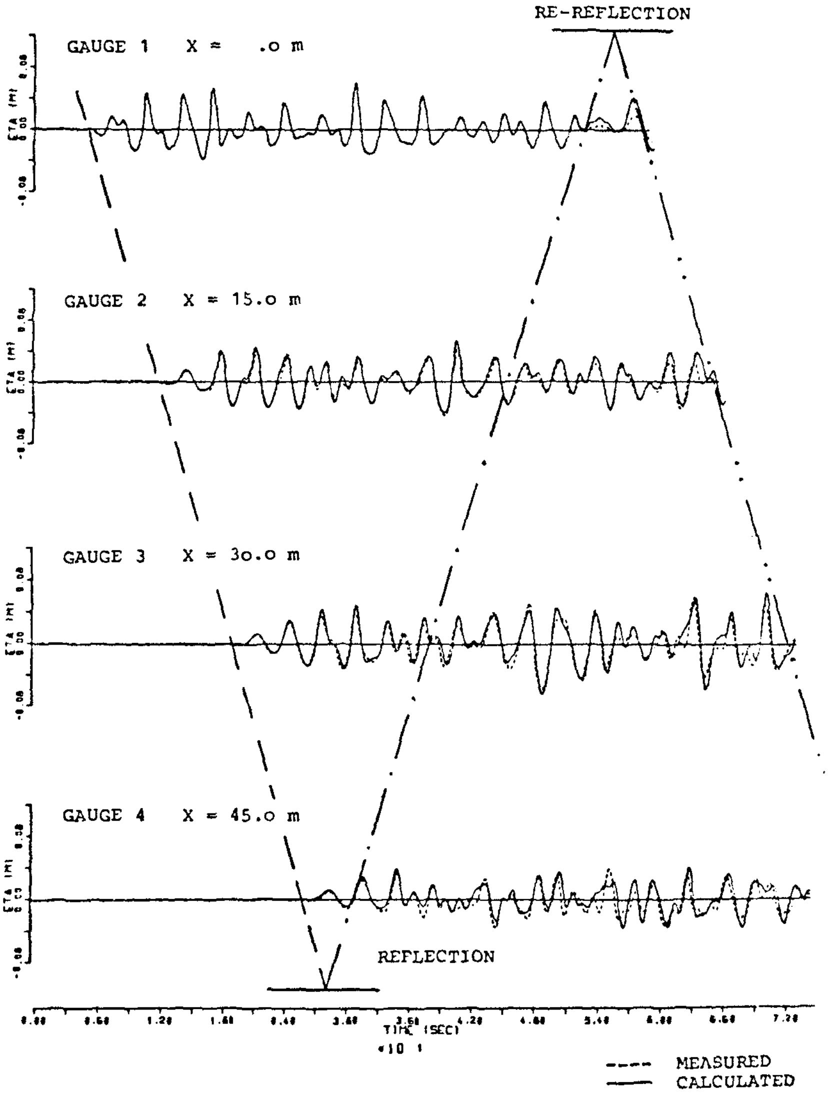


Figure 7: Comparison between measured and calculated time series of the water elevation in a flume with total reflection at a wall

The time series. measured in the hydraulic model at this gauge has been used as input data for the numerical model. which has a non-reflecting boundary at the wave maker. In reality the measurements include the reflection from the wave maker. This leads to significant differences after the first instance of re-reflection, as can be seen in figure 7.

4. LONG WAVES IN A BICHROMATIC WAVE SYSTEM

4.1 Long Wave Generation

OTTESEN-HANSEN pointed out, that two short period waves with the frequencies f_1 and f_2 create a bounded long wave with the frequency $\Delta f = f_1 - f_2$, which propagates with the group velocity. Bounded long waves are of second order with rather limited height. Resonance and shoaling effects can increase their influence.

In traditional first order wave generation, the boundary conditions at the wave board are not fulfilled and various free long waves with the same frequency Δf are generated. These waves appear also in the numerical model. Their velocity and wave length can be calculated by the dispersion relation (KOSTENSE).

Short waves		frequency	f_1 = 0.50 Hz
		amplitude	a_1 = 0.055 m
		frequency	f_2 = 0.40 Hz
		amplitude	a_2 = 0.011 m
Long Waves	Bounded:	frequency	f = 0.10 Hz
		amplitude	a_b = 0.0045 m
		wave length	L_b = 18.0 m
	Free :	frequency	f = 0.10 Hz
		wave length	L_f = 22.1 m

Table 2: Theoretical wave data

The numerical model was used to investigate the generation of bounded and free long waves in a flume with a constant water depth of 0.50 m. Initially a superposition of two harmonic waves was used. However, due to their inherent

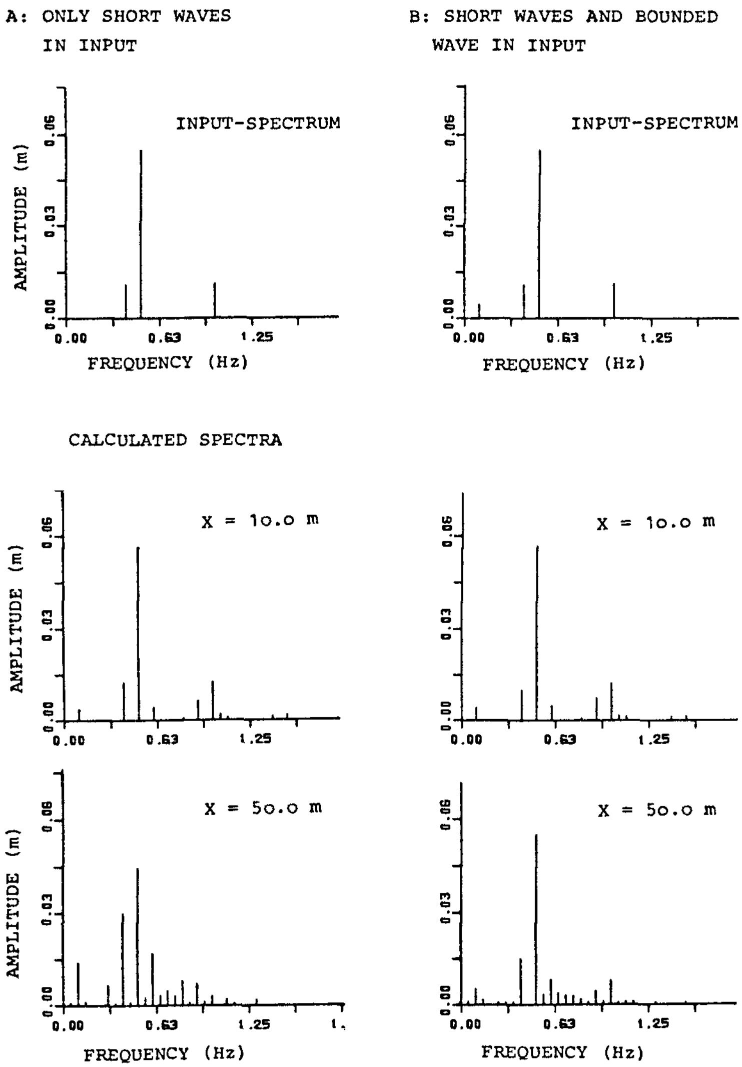


Figure 8: Long wave behaviour in a bichromatic wave system.
Flume with horizontal bottom.

instability, complicated transformations occurred, which made the interpretation of the results very difficult. Therefore, a superposition of two cnoidal waves, i.e. waves of permanent form, was used. Table 2 shows a list of the theoretical wave data. The difference in wave lengths of bounded and free long wave leads to a standing wave in the flume, which has been calculated in agreement with theoretical values.

Figure 8A displays wave spectra, calculated at different locations along the numerical flume. It can be seen, that a long period wave, which is not included in the input spectrum has been generated. The input spectrum is unstable and a energy transfer towards the lower frequencies takes place. If, instead the long wave component is added to the input spectrum, the spectrum becomes more stable (see Figure 8B).

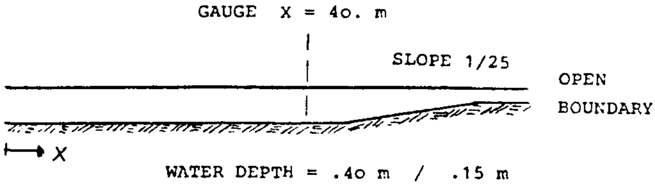
4.2 Reflection on a Slope

Investigation of the behaviour of a partial reflection on a slope has been carried out with an input of two short harmonic waves. The influence of the reflection is determined as the difference between calculations with and without a slope. Figure 9 shows the numerical model and the computed incoming and reflected spectra. The reflection coefficient of the long wave is about 3 times as high as that of the short waves. This result is qualitatively correct, but one should be aware that the wave breaking on a real beach has a significant influence on the amplitudes of the reflected waves.

5. CONCLUSIONS

The numerical model has been proven to be numerically stable for nearly breaking waves. The numerical diffusion appears to be insignificant, because cnoidal and solitary waves propagate with permanent form even after several wave lengths.

NUMERICAL MODEL



BICNOIDAL WAVE-INPUT:

1. AMPLITUDE = o.o3 m FREQUENCY = o.45 Hz

2. AMPLITUDE = o.o3 m FREQUENCY = o.5o Hz

CALCULATED SPECTRA

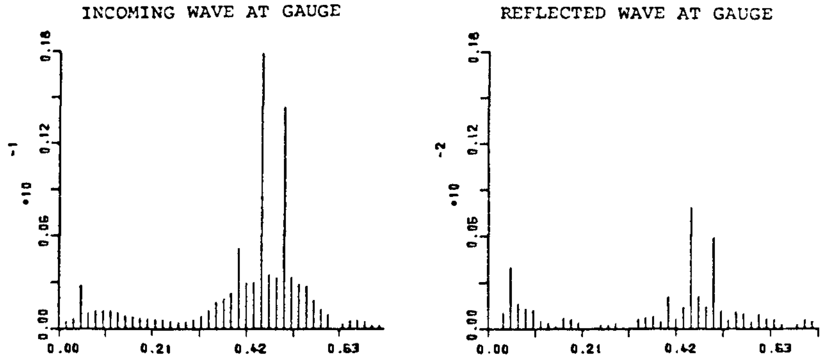


Figure 9: Reflection of bicnoidal waves on a slope

The nonlinear transformation of wave spectra has been studied for an even bottom , for a slope and for a reflection at a wall. A high degree of agreement between computations and measurements has been found, as long as the 'deep water' part of the spectrum is small; i.e. within the range of validity of the Boussinesq-wave-equations.

Long waves from a wave group (bicnoidal waves) are generated in the model in agreement with theoretical values. As should be expected, they are stronger reflected from a bottom slope than short waves.

6. REFERENCES

ABBOTT, M.B., PETERSON, H.M., and SKOOVGAARD, O. (1978): Computation of Short Waves in Shallow Water, Proc. 16th ICCE, Hamburg

HAUGUEL, A., (1980): Adaption of Tidal Numerical Models to Shallow Water Waves Problems, Proc. 18th ICCE, Sydney

KOSTENSE, J.K. (1984) Measurements of Surf Beat and Set-Down Beneath Wave Groups, Proc. 19th ICCE Houston

OTTESEN-HANSEN, N.E., (1978): Long Period Waves in Natural Wave Trains, Inst. Hydrodyn. and Hydraulic Eng., Technical University Denmark, Progress Rep. No. 46

SCHAPER, H., ZIELKE, W. (1984): A Numerical Solution of Boussinesq Type Wave Equations, Proc. 19th ICCE, Houston

Acknowledgements:

The authors express their appreciation to Dr. Daemrich and Dipl.-Ing. Götschenberg for supplying the valuable experimental data. They would also like to thank the German Research Society (Deutsche Forschungsgemeinschaft), which sponsored the research work as part of the scientific program of the "Sonderforschungsbereich" at the University of Hannover.