CHAPTER 9

INCOMING AND OUTGOING WAVE INTERACTIONS ON BEACHES

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A technique to decompose colocated random field measurements of wave elevation and current velocity into incoming (shoreward propagating) and outgoing (seaward propagating) components is presented. This decomposition technique, which is less sensitive to noise, enables us to determine the frequency dependent reflection coefficients and also the relative phase between the incoming and outgoing waves. The method is applied to C^2S^2 and NSTS data sets, from beaches with wide ranging characteristics and wave regimes. The results demonstrate the selective nature of beach absorption/reflection characteristics but are inconclusive in terms of a proper parameterization of reflections on natural beaches.

Introduction

Since the pioneering work of Miche (1951) coastal engineers have generally relied on empirical formulations to determine the reflectivity of a nearshore structure or a natural beach. These formulations, being based on monochromatic wave theories for planar beaches, were primarily designed to yield bulk reflection coefficients. In reality, however, we are faced with a spectrum of incident waves and complicated beach topographies, thus questioning the validity of bulk reflection coefficients. According to Miche's theory the reflected wave height is determined by the maximum wave steepness possible for a non breaking wave on a beach of linear slope. Therefore Miche's hypothesis suggests that there is a f^{-2} dependence of the reflection coefficient R, for monochromatic waves, where f is the wave frequency. This strong frequency dependence suggests that the use of a single bulk reflection coefficient for a *spectrum* of incident waves will be unsatisfactory. Carrier and Greenspan (1958) suggested that very small reflection coefficients are characteristic of natural beaches for waves of all but the very smallest amplitudes and that standing waves play no important role on such beaches. On the contrary there is ample evidence that standing waves are important on natural beaches (Suhayda,1974; Huntley, 1976; Holman, 1981; Guza and Thornton, 1982; Wright *et al.*, 1982; Bowen and Huntley, 1984; Elgar and Guza,1985) and that steeper beaches are reflective (Wright and Short, 1984).

As beaches have been used in hydraulic laboratories for over a century, it is but natural to focus atttention on the laboratory investigations for more information on wave reflections. However, it is surprising to note that these investigations are ambiguous due to a lack of consensus in defining the critical parameters affecting the determination of reflection coefficient, leading to differences in results (Battjes, 1974; Guza and Bowen, 1976) and confusion in understanding the physics. This confusion reflects the difficulty in obtaining reliable measurements of the wave reflections. Moreover there is an additional difficulty of extending these results obtained on the basis of monochromatic wave theories to a spectrum of incident waves naturally found on beaches.

Field investigations of low frequency waves have yielded equally confusing results. Munk (1949) and Tucker (1950) implied that there is a small nonlinear long wave correction under shoreward propagating wave groups and a larger seaward propagating low

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frequency wave released at breakpoint. In contradiction, Hasselman *et al.* (1963) presented evidence that shoreward propagating nonlinearly forced motion is larger than the seaward propagating component. Suhayda (1974) however, suggested that the incoming and outgoing components are of roughly equal magnitudes indicating that the concept of standing waves is important. The foregoing demonstrates the need to decompose random wave observations into incoming and outgoing components not only to understand clearly the nearshore wave dynamics and resulting sediment transport process but also to quantify the frequency dependent reflection coefficients.

The more recent studies on wave reflections utilize various analytical techniques to resolve a composite wave field into incident and reflected waves using measurements from wave sensors at adjacent locations (Thornton and Calhoun, 1972; Goda and Suzuki, 1976; Isobe and Kondo, 1984; Kim, 1985; Mansard and Funke, 1987).

The technique which is currently used by hydraulic laboratories for separating the incident and reflected spectra from the measured standing wave system in a laboratory is that of Goda and Suzuki (1976). The basic assumptions in this technique are that an irregular sea state can be described as a linear superposition of an infinite number of discrete frequency components, each with their own frequency, amplitude and phase, and that each frequency component travels with its own individual celerity described by the linear dispersion relationship. The technique consists of simultaneous measurements of the composite waves (superposition of incident and reflected) at two or three known positions in a line parallel to the direction of wave propagation. Fourier analysis of these measurements then provides the amplitudes and phases of the frequency components constituting the irregular sea state, on the basis of which the incident and reflected components can be resolved.

Although the technique is applicable to both regular and irregular trains of waves, the resolution into incident and reflected components is effective only outside the condition of the wave sensor spacing being an even integer of half wave length. In other words the resolution technique is strongly affected by the location of the sensors and the spacing between them. Also any nonlinear wave interaction affects the accuracy of the resolution technique.

Isobe and Kondo (1984) utilize the maximum likelihood method to measure the directional wave spectrum in a reflective system and determine the reflection coefficient of a structure in a directional sea. As the resolution power of the maximum likelihood method for a directional wave spectrum depends on the number and arrangement of wave sensors, this methodology cannot be applied where sufficient number of sensors are not deployed and also where the optimal arrangement of sensors is not made. A more serious limitation is the inaccuracy in computing reflectivities due to changing wave conditions with wave frequency and the wave direction.

Kim (1985) attempted to estimate the reflection coefficient of a natural beach by computing a cross correlation function for a wave field consisting of incoming and outgoing waves and comparing it with observed spatial correlation functions. The shallow water wave orbital velocities measured at two locations by current meters can be expressed as a combination of incoming and outgoing components. By analytically deriving an expression for the cross correlation function between the two measurements from the expressions for the combined wave fields at two locations, Kim suggests that cross correlation peaks appear at time lags associated with incoming and outgoing waves. Kim's study provides evidence for the existence of standing waves and seagoing waves at the expected time lags and allows a rough estimation of the relative magnitudes of incident and reflected waves. However, the major disadvantage of using the cross correlation function is that no frequency information can be obtained.

The most recent study on the problem of determining the reflection characteristics of test structures in laboratories was carried out independently by nine laboratories of the IAHR working group (Mansard and Funke, 1987). As the principle used to compute reflection coefficients is based essentially on Goda and Suzuki's theory, there are limitations in obtaining the true reflections. Mansard and Funke (1987), comparing results from various laboratory experiments under similar conditions using the same techniques, conclude that the estimated reflective properties of impervious sloped beaches vary appreciably between different laboratories.

The usual way of addressing the problem of wave reflection from beaches seems inevitably to imply perfect reflection, a direct consequence of the assumed form of the offshore wave field and the conservation of mechanical energy (Carrier and Greenspan, 1958; Meyer and Taylor, 1972). Energy considerations show that wave absorption must be associated with the degradation of mechanical energy through wave breaking. However, in practice it is not possible to make a direct determination of the energy absorption, so this has to be inferred from observations of the wave field. Also there are no rigorous theoretical models to describe the physics of wave breaking on beaches and it is customary to treat breaking as a specific event for each identifiable wave (some recent exceptions to this line of thought are Guza and Thornton, 1982; Symonds *et al.*, 1982, where random waves are considered). Hence we focussed our attention on field measurements in attempting to decompose a random wave field into incoming and outgoing components. Tatavarti and Huntley (1987) and Tatavarti (1987) demonstrate a number of different techniques attempted in order to compute frequency dependent reflections from field measurements and also highlight various advantages and limitations associated with each technique.

Over the past twenty years there has been a very significant increase in the number of field measurements of wave elevations and currents in the nearshore region. It has become customary in recent years to deploy colocated elevation and current meters in the field. Therefore, we decided to estimate frequency dependent reflections utilizing colocated elevation and current measurements.

Data For Analysis

Data collected as part of the C^2S^2 program on two maritime Canadian beaches at Pointe Sapin, New Brunswick, Stanhope Lane, P.E.I.; and NSTS data from Leadbetter beach at Santa Barbara, California (U.S.A) have been used to test the theoretical formulation. Measurements of the flow field were made using colocated Marsh- McBirney electromagnetic current meters and pressure transducers. Each current meter, measuring the two orthogonal axes of the flow with a response time of 0.2 *secs*, was aligned to measure the onshore/offshore and the alongshore components of the flow. The pressure transducers were designed to measure the wave elevation. The field environments comprise plane and barred beaches with wide ranging wave conditions (Table 1). Pointe Sapin (PS62) beach has a mild slope with a steep foreshore. Stanhope Lane (ST12) is a barred beach. Leadbetter (LB7, LB3) beach has a relatively steep slope (Fig.1).

Prior to the decomposition, corrections were made for the filter characteristics of the current meter electronics, the current meter axes have been aligned towards the predominant wave direction and the time series Fourier decomposed into frequency components and each Fourier amplitude multiplied by $(\tanh kh/\omega)$ factor, where k is the wave number, h the water depth and ω the radian frequency. The resulting Fourier amplitudes and the Fourier decomposed phases have been inverse Fourier transformed into the time domain. The output time series of the current measurements ($\hat{u}$) now have units of meters, consistent with the the units of elevation measurements.

TABLE 1

Run/Beach	Offshore Dist. from Shoreline (m)	Depth (m)	Beach slope	$H_s(m)$	$T_p(secs)$	$\langle v\rangle^2/\langle u\rangle^2$	Beach Profile
PS62 (Pointe Sapin)	58.0	1.95	0.05	0.90	8.4	0.034	concave
ST12 (Stanhope)	175.0	1.8	0.02*	0.87	5.6	0.323	Barred
LB7 (Leadbetter)	20.5	1.8	0.064	0.67	15.6	0.048	planar
LB3 (Leadbetter)	50.0	3.0	0.064	0.64	15.6	0.055	planar

*approximate slope

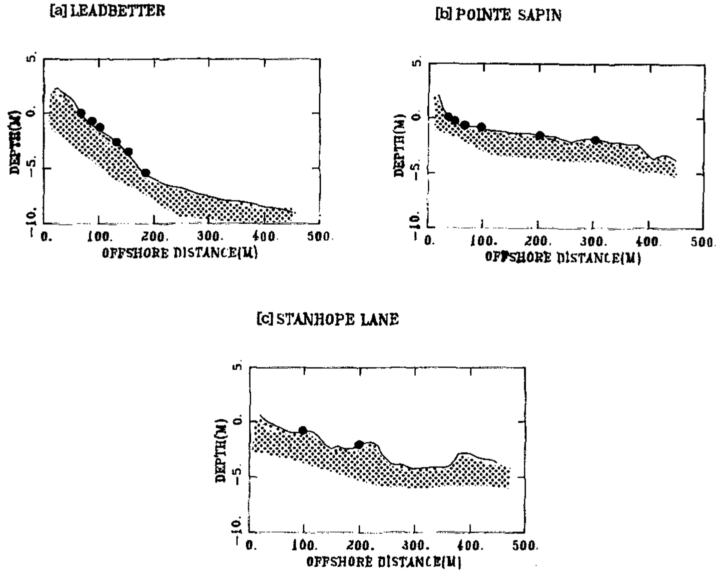

Figure 1. Beach profiles and relative locations of the instrument stations for (a) Leadbetter, California (U.S.A), (b) Pointe Sapin, New Brunswick (Canada) and (c) Stanhope Lane, P.E.I., (Canada).

Incoming and Outgoing Waves: Time Domain Analysis

Utilizing the linear wave thoery, the velocity potential Φ_{io} of near normal incidence shallow water waves may be expressed as a linear superposition of incoming(shoreward propagating) and outgoing waves(seaward propagating) over a flat bottom as follows,

$$\Phi_{io} = \frac{g}{\omega}[a\sin(kx - \omega t + \phi_i) + b\sin(kx + \omega t + \phi_o)] \tag{1}$$

where a and b are the amplitudes associated with incoming and outgoing waves respectively, ϕ_i and ϕ_o are the relative phases of incoming and outgoing waves respectively, x is the cross shore coordinate, positive onshore.

If we transform the cross shore velocity ($u = \frac{\partial\Phi_{io}}{\partial x}$) into elevation ($\eta = -g^{-1}\frac{\partial\Phi_{io}}{\partial t}$) using linear wave theory, then we can express the sum and difference series of elevation and the transformed velocity as incoming and outgoing components,

$$\frac{(\eta + \hat{u})}{2} = a\cos(kx - \omega t + \phi_i) \tag{2}$$

$$\frac{(\eta - \hat{u})}{2} = b\cos(kx + \omega t + \phi_o) \tag{3}$$

where $\hat{u}$ is the transformed u.

Defining the ratio of the amplitude of the outgoing wave to that of the incoming wave as the reflection coefficient R we have

$$R = \left[\frac{< (\eta - \hat{u})^2 >}{< (\eta + \hat{u})^2 >}\right]^{\frac{1}{2}} \tag{4}$$

where $< >$ denotes ensemble averaging. Thus, after obtaining the incoming and outgoing wave series the frequency dependent reflection coefficient can be determined using standard spectral techniques.

Guza *et al.* (1984) have shown that colocated elevation and current meters can be used to decompose measured waves into incoming and outgoing components based on the aforementioned theory. However, there is an important problem associated with this method of estimating the frequency dependent reflection coefficient. A close examination of the coherence estimates of η and u time series and the estimated frequency dependent reflection coefficient plots (Fig.2a and 2b) of the Stanhope Lane data set shows that the reflection plot is almost a mirror image of the coherence plot. At low coherences the reflection coefficient tends to unity. Clearly, if η and u are incoherent, their sum and difference time series, as in equations (2) and (3), will have the same variance so that the apparent reflection coefficient will tend to 1.0. Therefore there is a need to reduce the noise dependence of the reflection coefficient estimates. This led us to search for alternate techniques to decompose the field measurements into incoming and outgoing components.

Incoming and Outgoing Waves: Frequency Domain Analysis

Studies by Oppenheim and Lim (1981) have focussed on the importance of phase in signals. The phase between two signals, computed using the cross spectrum, is only related to the coherent part of the two series and hence should be independent of noise. So spectral phase information obtained from the η and u measurements may be used to obtain incoming and outgoing components and hence determine the frequency dependent reflection coefficients. As discussed in Tatavarti and Huntley (1987) this would require modelling waves over complex topographies naturally occuring on beaches. This is a significant complication. However, in principle one can avoid the necessity for modelling waves over complex topography by calculating, from the time series of η and u at a single location, the gain between the two series in addition to the phase.

Let us now express the time series of colocated measurements of wave elevation and on-offshore current velocity in terms of a modulation function and a carrier function.

$$\eta(x,t) = \Re\left\{A(\psi)e^{i\omega t}\right\} \tag{5}$$

$$\hat{u}(x,t) = \Re\left\{B(\psi)e^{i(\omega t+\theta(\psi))}\right\} \tag{6}$$

where $\eta(x,t)$, is the time series measurement of elevation (m) at an offshore location x, from shoreline.

$\hat{u}(x,t)$, is the time series measurement of the on-offshore velocity at an offshore location x, from the shoreline, normalized such that it has the units of elevation (m).

$A(\psi)$ and $B(\psi)$, are the slowly varying amplitude modulation functions associated with the time series of $\eta(x,t)$ and $\hat{u}(x,t)$ measurements, in which ψ is a function of beach slope, local wave amplitude, wave number and wave frequency.

$\theta(\psi)$ is the phase difference associated with the two time series measurements.

$e^{i\omega t}$, is the carrier wave in which ω is the radian frequency.

In practice the ensemble averaging is often estimated by a frequency band averaging, whose equivalence is supported by the ergodicity hypothesis. Therefore, from a spectral perspective we can rewrite the variance of the incoming waves and the variance of the outgoing waves respectively as per equations (2) and (3) as

$$E_{\text{in}}(\omega) = \tilde{A}^2(\omega) + \tilde{B}^2(\omega) + 2\tilde{A}(\omega)\tilde{B}(\omega)\cos\theta_{\eta\hat{u}}(\omega) \tag{7}$$

$$E_{\text{out}}(\omega) = \tilde{A}^2(\omega) + \tilde{B}^2(\omega) - 2\tilde{A}(\omega)\tilde{B}(\omega)\cos\theta_{\eta\hat{u}}(\omega) \tag{8}$$

where $\tilde{A}$ and $\tilde{B}$ are the average estimates of A and B over a realization (record length in practice). Writing the gain function as $G(\omega) = \frac{\tilde{A}(\omega)}{\tilde{B}(\omega)}$, we express the frequency dependent reflection coefficient $R(\omega)$ as

$$R^2(\omega) = \left[\frac{1 + G^2(\omega) - 2G(\omega)\cos\theta_{\eta\hat{u}}(\omega)}{1 + G^2(\omega) + 2G(\omega)\cos\theta_{\eta\hat{u}}(\omega)}\right] \tag{9}$$

Equation (9) therefore gives the frequency dependent reflection coefficient in terms of the gain and the phase relationships between η and u measurements.

The phase between two signals, computed using the cross spectrum, is only related to the coherent part of the two time series and hence should be independent of noise. *Hence the key to this method of estimating $R(\omega)$ is to find an estimator for gain, $G(\omega)$, which is insensitive to noise in both η and u measurements.*

Generally, in spectral analysis, the time series of one parameter is designated as a base series and coherence and phase relationships are computed between this series and those of other parameters. There are a number of problems associated with the use of base series. For example, when different waves are present in the same frequency band, there is considerable difficulty in interpreting the cross spectrum data as there is no way of determining how many wave structures are present and what is the relative contribution of each wave type to the variance spectra. Also the use of base series produces a bias in favour of the base series, when computing wave amplitudes. If the coherence between various parameters and the base series is not large, this can result in considerable distortion in the pattern of wave amplitudes. Moreover this does not exploit the information contained in the cross spectra between parameters other than the base series. In order to avoid these problems we resorted to the complex eigenvector analysis suggested by Wallace and Dickinson (1972), where the vector time series is expressed as a linear combination of eigenvectors of the cross spectrum matrix for frequency bands of interest. To increase the statistical confidence in E.O.F. analysis the colocated alongshore velocity (v) time series is also considered.

Employing complex empirical orthogonal function analysis to reduce noise sensitivity it is possible to determine the reflection coefficients from a spectral perspective utilizing the relation

$$\hat{R}(\omega) = \left[\frac{1+\hat{G}^2(\omega) - 2\hat{G}(\omega)\cos\hat{\theta}_{\eta\hat{u}}(\omega)}{1+\hat{G}^2(\omega) + 2\hat{G}(\omega)\cos\hat{\theta}_{\eta\hat{u}}(\omega)}\right]^{1/2} \tag{10}$$

where $\hat{G}(\omega)$ is the gain function, the ratio of the amplitude of the principal mode of the elevation to the amplitude of the principal mode of the on-offshore velocity at that frequency

$\hat{\theta}_{\eta\hat{u}}(\omega)$ is the phase difference between the principal modes of pressure and on-offshore velocity.

Figure 2c shows the estimated (using equation 9) frequency dependent reflection coefficients for the Stanhope Lane data set. A comparison of Fig. 2b and Fig. 2c demonstrates the significance of estimating reflection coefficients by the frequency domain analysis as opposed to that using the time domain analysis.

Determination Of The Phase Between Incoming and Outgoing Waves

Let us assume that the signal which a sensor measures at any time t, is a linear superposition of incoming wave and outgoing wave signals. One can always represent the incoming and outgoing wave signals as products of the modulation function and a carrier function. As in the previous section, let us consider a harmonic carrier system ($e^{i\omega t}$),

$$S(x,t) = \Re\left\{A(\psi)e^{i\omega t}\right\} = A_{\text{in}}(\psi)\cos\omega t + A_{\text{out}}(\psi)\cos(\omega t + \phi(\psi)) \tag{11}$$

where $S(x,t)$ is the signal the sensor measures at time t and location x.

$A_{\text{in}}(\psi)$ and $A_{\text{out}}(\psi)$ are the slowly varying amplitude modulation functions associated with incoming and outgoing wave signals, in which ψ is a function of beach slope, wave amplitude, wave number and wave frequency.

$\cos\omega t$ is the carrier function for the incoming wave signal

$\cos(\omega t + \phi(\psi))$ is the carrier function for the outgoing wave signal

$\phi(\psi)$ is the phase difference associated with the incoming and outgoing wave signals.

Hence, colocated pressure(elevation) and on-offshore current velocity measurements may be expressed as linear superpositions of incoming and outgoing signals,

$$\eta(x,t) = A_{\text{in}}(\psi)\cos\omega t + R(\psi)A_{\text{in}}(\psi)\cos(\omega t + \phi(\psi)) \tag{12}$$

$$\hat{u}(x,t) = B_{\text{in}}(\psi)\cos\omega t - R(\psi)B_{\text{in}}(\psi)\cos(\omega t + \phi(\psi)) \tag{13}$$

where $A(\psi)$ and $B(\psi)$ are the slowly varying amplitudes associated with $\eta(x,t)$ and $\hat{u}(x,t)$ measurements

$R(\psi)$ is the ratio of the outgoing to the incoming wave amplitudes, which can be approximated by the average value of the reflection coefficient for a given realization, usually the length of the record, estimated using equation (10)

Utilizing spectral techniques to compute Fourier coefficients, co spectrum, quadspectrum and phase spectrum the following expression for the phase associated with incoming and outgoing components is obtained,

$$\tan\phi(\omega) = \frac{2\hat{G}(\omega)}{1-\hat{G}^2(\omega)}\sin\hat{\theta}_{\eta\hat{u}}(\omega) \tag{14}$$

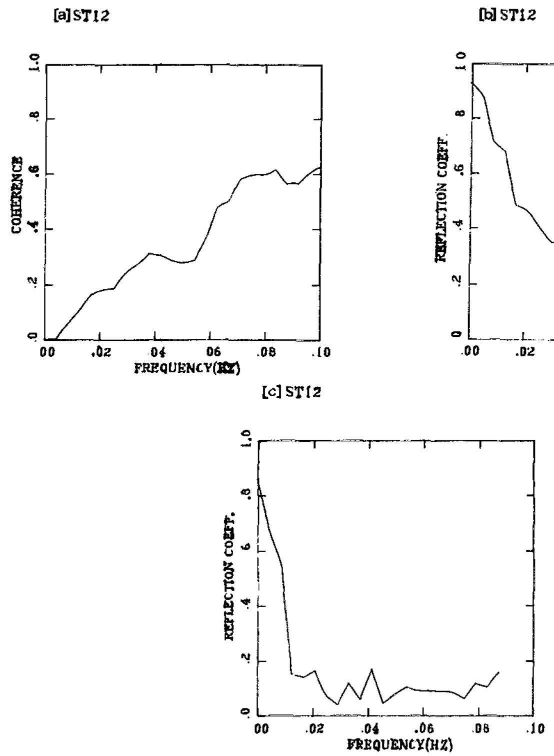

Figure 2. (**a**) Cross spectral coherence between the wave elevation η and the on/offshore velocity u for Stanhope Lane data (ST12). (**b**) Reflection coefficient as a function of the frequency using time domain analysis and (**c**) frequency domain analysis for the same data.

Decomposition Of Random Waves Into Incoming and Outgoing Components

Once the magnitude of the reflection, $R(\omega)$, and the phase associated with reflection, $\phi(\omega)$, are computed it becomes trivial to decompose the random wave field into incoming and outgoing components.

One can express the composite variance that any sensor measurement yields as

$$E(\omega) = A_{\text{in}}^2(\omega) + R^2(\omega)A_{\text{in}}^2(\omega) + 2A_{\text{in}}^2(\omega)R(\omega)\cos\phi(\omega) \tag{15}$$

In other words, the composite variance is the summation of the incoming variance, the outgoing variance and twice the co spectrum of incoming and outgoing components. Therefore, the incoming variance is given by

$$E_{\text{in}}(\omega) = \frac{E(\omega)}{\{1 + R^2(\omega) + 2R(\omega)\cos\phi(\omega)\}} \tag{16}$$

and the outgoing variance is given by

$$E_{\text{out}}(\omega) = \frac{E(\omega)}{\{1 + R^{-2}(\omega) + 2R^{-1}(\omega)\cos\phi(\omega)\}} \tag{17}$$

If the measured auto-spectral phase of the elevation sensor (computed by spectral analysis techniques) is denoted by θ_η, then assuming that the measured phase is actually a linear superposition of the incoming ($\phi_{\eta i}$) and the outgoing ($\phi_{\eta o}$) phases, it can be shown that

$$\phi_{\eta i}(\omega) = \theta_\eta(\omega) - \tan^{-1}\left[\frac{R(\omega)\sin\phi(\omega)}{1 + R(\omega)\cos\phi(\omega)}\right] \tag{18}$$

and

$$\phi_{\eta o}(\omega) = \phi(\omega) + \phi_{\eta i}(\omega) \tag{19}$$

Using equations (16), (17), (18) and (19) and employing inverse Fourier transform techniques the random series, i.e., the measured time series, can be decomposed into incoming time series and outgoing time series as shown in Figure 3.

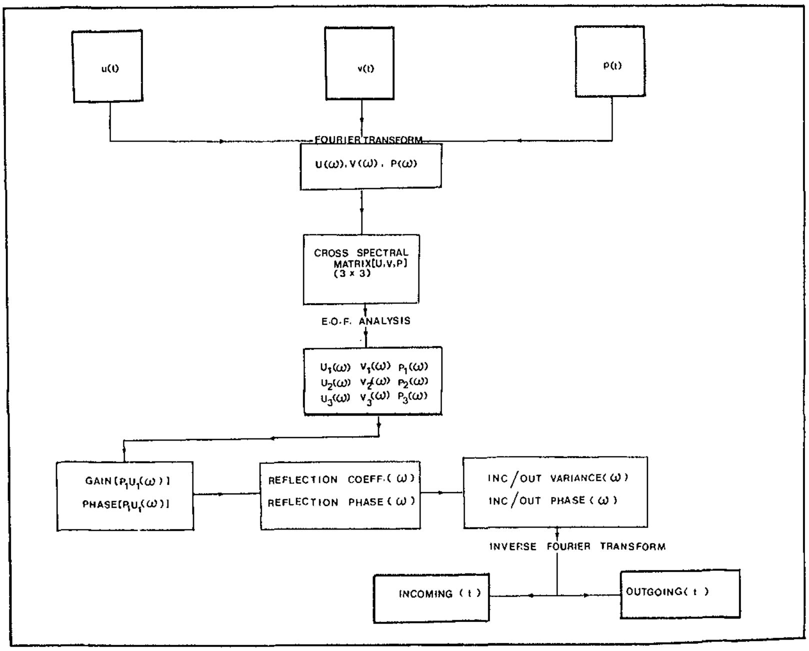

Figure 3. Flow chart demonstrating the technique employed to decompose random waves into incoming and outgoing components.

Observations and Discussion

Standing waves can either exist as leaky modes or trapped modes. Leaky modes are the two dimensional waves with the fluid motion normal to the shore and uniform alongshore. The trapped modes are the three dimensional edge waves trapped in the nearshore by refraction and periodic alongshore, with both shore normal and shore parallel components. The present technique of decomposition only considers the leaky waves. A review of the pertinent literature points out the difficulties previous researchers (Huntley, 1976; Holman *et al.*, 1978; Oltman-Shay and Guza, 1987) faced in distinguishing between the edge waves and leaky waves, even with observations from cross shore and longshore arrays of instruments located in the nearshore region. This is clearly indicative of how formidable, if not intractable, the problem of distinguishing edge waves from leaky waves can be. Utilizing the fact that the key distinction between edge waves and leaky waves lies in their directions of progress, with edge waves propagating alongshore and leaky waves propagating primarily on-offshore, the energy levels in the cross shore (u) and the alongshore (v) components of velocity measurement should provide an indication of the presence or absence of edge waves. The ratio of the energy in the v component to that of the u component of the horizontal velocity being larger in the presence of edge waves than in the presence of near normal incidence leaky waves alone. The ratios of variances [$< v^2 > / < u^2 >$] in Table 1 suggest that Pointe Sapin and Leadbetter beach data may not be seriously contaminated by edge wave motions. For Stanhope Lane beach data, however, the ratio is indicative of a significant contribution of energy from low mode edge waves.

Figure 4 shows the reflection coefficient estimates as functions of frequency for the different beaches. The most prominent feature that stands out in Fig. 4 is that the low frequency waves are strongly reflective while the wind wave frequencies are progressive onshore. This is consistent with earlier studies by Suhayda (1974), Huntley (1976), Bowen (1980), Holman (1981), Wright *et al.* (1982), Guza and Thornton (1982), Bowen and Huntley (1984), and Elgar and Guza (1985); who suggested that low frequency energy occurs in the form of cross shore leaky waves. The similar frequency dependent reflection coefficient plots for the data from different locations on the Leadbetter beach suggests the insignificant role of edge wave motions in these measurements and demonstrates the ability of this technique to minimize the influence of noise.

Figure 5 shows the relative phase between incoming and outgoing waves as a function of the frequency. For any incoming wave the outgoing wave can either be i) the reflected component, ii) any wave generated by some mechanism (for example, Symonds *et al.*, 1982) inside the zone between the sensor and the shoreline or iii) a combination of both. The more or less linear phase with frequency observed in Fig. 5 suggests that the outgoing waves are mostly the reflected waves. The linear phase is also consistent with the nondispersive time lag (considering only low frequencies) associated with a shoreline reflection for Pointe Sapin and Leadbetter beaches. Stanhope Lane beach of course, being a barred beach, further complicates the interpretation of the incoming/outgoing phase difference because of the distinct possibility of multiple reflectors (Tatavarti, 1987).

Figure 6 shows the decomposed incoming and outgoing spectra against the measured elevation spectra. As one would expect *a priori*, the outgoing variance is significantly smaller than the incoming variance, except in the low frequency region. It is clear that the low frequency outgoing energy is statistically significant. This feature has also been indicated in the frequency dependent reflection coefficient plots. Thus the primary region of interest for decomposition seems to be the low frequency band of the wave spectra.

The ability to decompose the observed wave field into incoming and outgoing components suggests that one can quantify the relative magnitudes of the incoming and outgoing components and determine whether the outgoing long wave is a simple reflection of the incoming long wave. Of future interest would be the question of the origin of the outgoing long waves. If the outgoing long waves are simple reflections of the incoming long waves then at what location on the beach are these waves being reflected? *i.e.*, Is there any particular region on the beach topography which effectively reflects the long waves?

Figure 7a shows the smoothed reflection coefficient as a function of the wave frequency for data from different beaches (Pointe Sapin, Stanhope Lane, Leadbetter) and at different locations on the same beach (Leadbetter). It is clear that the barred beach data set (ST12)

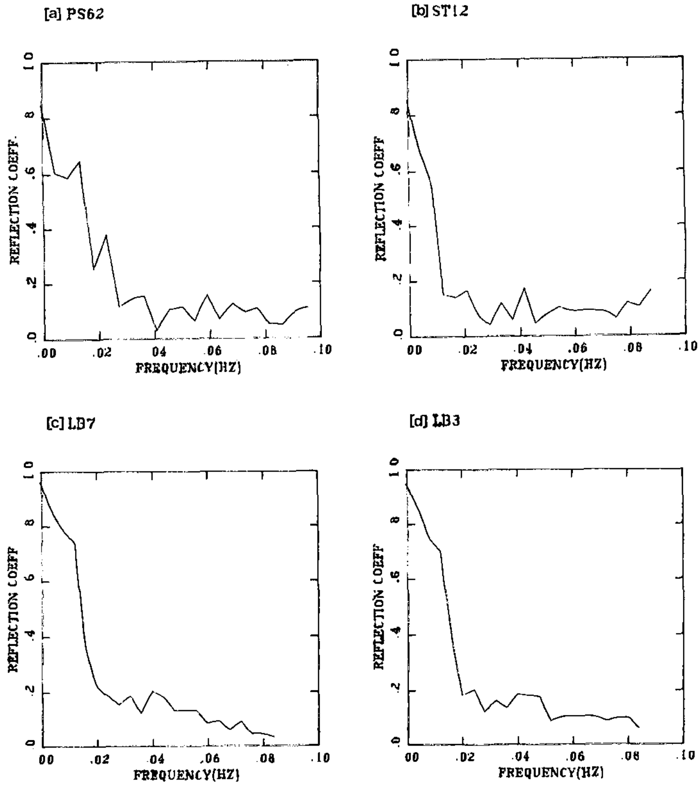

Figure 4. Frequency dependent reflection coefficient estimates obtained using equation (14) for (a) Pointe Sapin data set (PS62), (b) Stanhope lane data set (ST12), (c) Leadbetter data sets (LB7) and (d) LB3.

shows remarkably different characteristics from the planar beaches suggesting the influence of the beach slope on the reflection coefficient estimates. Figure 7b shows the reflection coefficients for data from different beaches and at different locations as a function of the Irribarren number ($\epsilon = \frac{2a_o\omega^2}{g\tan^2\beta}$). From Fig. 7b it is evident that inclusion of the beach slope did bring the reflection curve of ST12 data set towards the general trend shown by the other data sets. However, the inclusion of the record average wave amplitude for a_o, in the case of Leadbetter data setshas not reduced the scatter in the curves. It is not clear whether to use a frequency band averaged amplitude or a representative record averaged amplitude for a_o in the expression for ϵ. A frequency band averaged amplitude would be meaningless if the waves in that frequency band are breaking. This problem of extending the monochromatic wave results to a spectrum of incident waves naturally found on beaches is a general one for which no satisfactory solution exists.

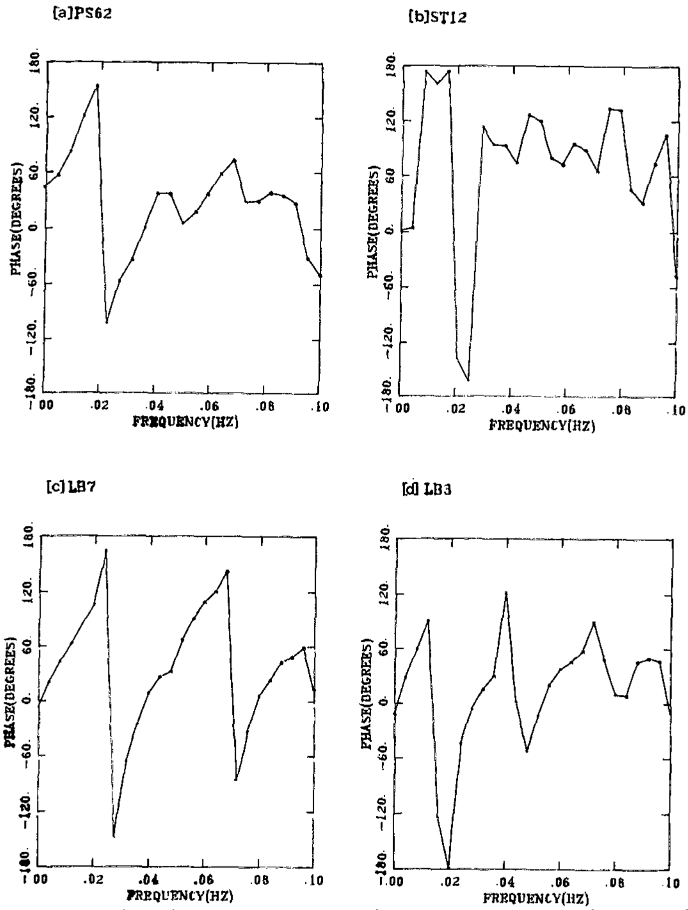

Figure 5. Relative phase between incoming and outgoing waves as a function of the frequency obtained using Equations (19) and (20) for (**a**) Pointe Sapin data set (PS62), (**b**) Stanhope Lane data set (ST12), (**c**) Leadbetter data sets (LB7) and (**d**) LB3.

Conclusions

i) Use of colocated velocity and elevation measurements avoids the resolution problem associated with the more common method of spatially separated sensors.

ii) Our technique minimizes the influence of noise, which tends to drive the reflection coefficient towards unity.

iii) The technique also gives the relative phase between incoming and outgoing waves and leads to decomposition of the original time series into time series of incoming and outgoing waves.

iv) Results from different beach sites and different wave conditions confirm that the re-

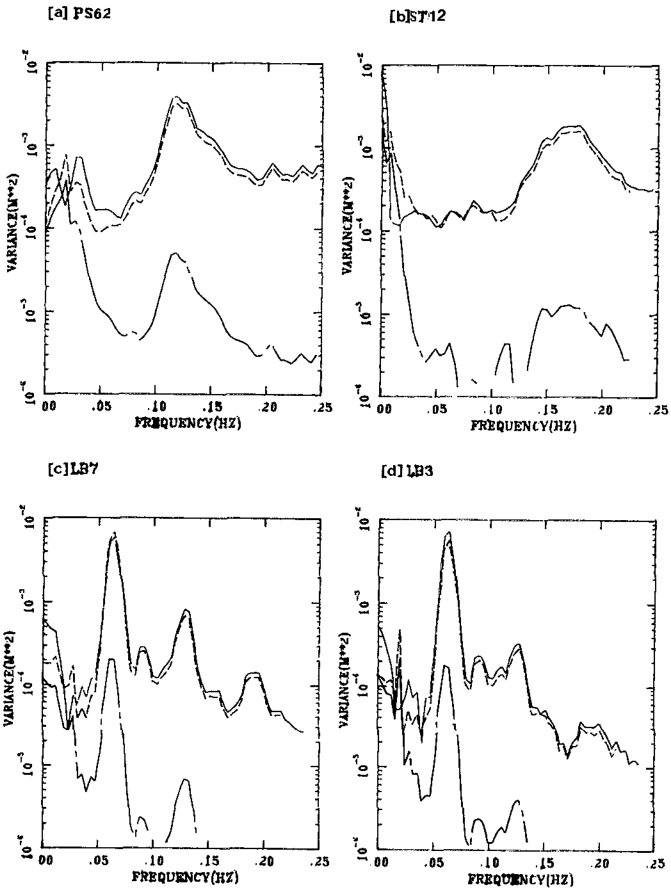

Figure 6. The measured elevation spectrum (solid line) and the decomposed incoming (short broken line) and outgoing (long broken line) for runs (a) PS62, (b) ST12, (c) LB7 and (d) LB3, d.o.f.=95.

flection coefficient is high at low frequencies but usually becomes very small at high frequencies.

v) The shape of the frequency dependent reflection coefficient appears to depend strongly on the shoreface beach slope, but the influence of wave amplitude is less clear.

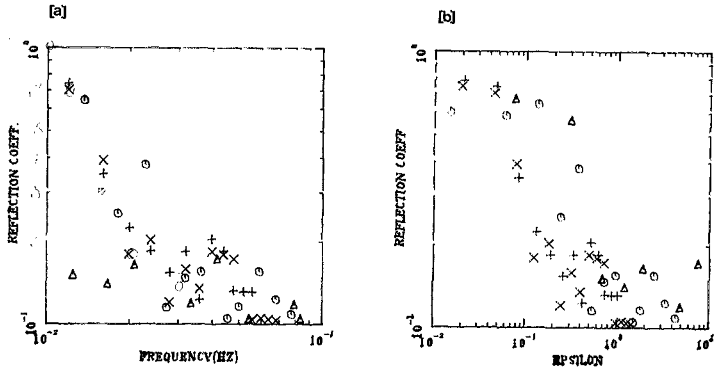

Figure 7. (a) Reflection coefficient as a function of the frequency for run PS62 (o), ST12 (△), LB7 (+), LB3 (×). (b) Reflection coefficient as a function of the non dimensional number ϵ for run PS62 (o), ST12 (△), LB7 (+), LB3 (×).

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