CHAPTER 28

THEORETICAL MODEL FOR NEARSHORE CIRCULATIONS

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Abstract

The dynamics of nearshore circulations is investigated using mass, momentum and energy conservation equations with bottom friction, lateral mixing and wave-current interaction. By means of introducing a perturbation expansion for the mean variables, the first-order solutions are found in the surf and offshore zones according to the boundary conditions at the coast.

It is found that:(1) The rip velocity attains a maximum value in the offshore region near the break point as Yr* becomes larger. (2) The longshore velocities become higher in the surf zone and lower in the offshore region with increasing Yr*. (3) The rip and longshore velocities in the surf zone become relatively smaller due to the effects of the bottom friction, and that the rip and longshore velocities in the surf zone become smaller due to the wave-current interaction.

1. INTRODUCTION

Nearshore currents play an important role in the transportation of sediments and sea water in nearshore areas, and so several theoretical studies with respect to nearshore circulations have been presented. However many reseachers take only bottom friction into account to avoid the mathematical complexities resulting from the lateral mixing, then the results obtained theoretically suggests that the lateral mixing should be included in more detailed analysis. The turbulent viscosity as well as wave-current interaction is important in flow field of nearshore circulations. In order to consider nearshore circulations

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at the actual condition, it is necessary to have more realistic model for the dynamics of rip current circulation. In present paper, theoretical model including the bottom friction, the wave-current interaction and the lateral mixing is proposed.

According to Ozaki, Sasaki and Usui(1977), an important parameter on the dynamics of rip currents caused by normally incident waves in the symmetrical cellular(Harris,1967) is the nondimensional rip spacing Yr*, where $Yr^*=Yr/X_B$, Yr= the rip spacing, X_B =the width of the surf zone. When the value of Yr* is small, rip current circulation is confined to a very amall region near the shore, and as the value of Yr* becomes larger, rip currents flow rapidly from the surf zone to a large region where extends several surf zone widths offshore, and the rip currents grow intenser and narrower. The wave number and wave energy density are affected by the currents like the free jet. The wave-current interaction in nearshore circulations is important for the dynamics of rip currents.

2. BASIC FORMULATION

The beach considered here is of linear plane shape alongshore and of uniform slope offshore; the water depth, h, in the absence of waves is given by h=sx, where x is a distance in offshore direction normall to the shore line and s is the bottom slope. The waves are normally incident and break at a uniform distance from the shoreline at $x=x_B$ the breaker line. The fluid motion of currents is steady. The motion of water is described by Eqs.(1) and (2) in teams of the mean surface elevation ζ and the vertically averaged horizontal velocity components u and v in offshore and longshore directions,

$$ud=-\partial F/\partial y \qquad vd=\partial F/\partial x \qquad d=h+\zeta \tag{1}$$

$$u_j\frac{\partial u_i}{\partial x_j}+\frac{1}{\rho d}\frac{\partial S_{ij}}{\partial x_j}+g\frac{\partial \zeta}{\partial x_i}+\frac{B_i}{\rho d}-\frac{1}{\rho}\frac{\partial}{\partial x_j}(T_{ij})=0 \tag{2}$$

where d=the total mean depth, F=the transport stream function, Sij=the radiation stress, Tij=the effective stress, including effects of non-uniform velocity distribution, Bi=the bottom shear stress, and ρ is the fluid density. The steady state conservation of wave energy equation is

$$\frac{\partial}{\partial x_j}[(u_j+C_j)E]+S_{ij}\frac{\partial u_i}{\partial x_j}=-D \tag{3}$$

where $C_j=(-(gd)^{1/2}+u,v)$ is the velocity of the waves, E=the wave energy density and D=an energy dissipation function.

The radiation stress Sij, the bottom shear stress Bi and the lateral stress Tij can be written as follows:

$$S_{xx}=\frac{3}{2}E \qquad S_{yy}=\frac{1}{2}E \qquad S_{xy}=S_{yx}=0 \tag{4}$$

$$B_x=2f\bar{u} \qquad B_y=f\bar{v} \qquad f=\rho C_f|\bar{u}|/\pi \tag{5}$$

$$\bar{u} = \frac{1}{2}\frac{H}{d}\sqrt{gd}\,(x \leqq x_B),\ \frac{1}{2}\frac{H_B}{d_B}\sqrt{gd_B}\,(x \geqq x_B)$$

$$\frac{1}{\rho}T_{ij}=\nu_t\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)-\frac{2}{3}K\delta_{ij} \tag{6}$$

$$\nu_t=\mu\frac{ET}{\rho d}\,(x \leqq x_B) \ ,\ \mu\frac{E_B T}{\rho d_B}\,(x > x_B)$$

where H=the wave height, H_B=the breaking wave height, d_B =the breaking depth, ν_t =the eddy viscosity, T=the wave period, $E_B=\rho g H_B^2/8$, μ=a coefficient($\fallingdotseq$0.5) and K=the turbulent kinetic energy. We assume that K can be modelled as a function of the energy dissipation rate D/ρ using the breaking-waves approximation(Wind and Vreugdenhil(1986))

$$K=(D/\rho)^{2/3} \tag{7}$$

The dissipation term D plays an important role in the energy balance. We assume that

$$D \propto \sqrt{gd}\,u'u' \qquad |u'| \propto |u_w| \tag{8}$$

where u' and u_ω are the turbulence and the wave orbital velocity. Then, using the shallow water long wave approximation, the energy dissipation D is given by

$$D=q\sqrt{gd}\frac{E}{d} \tag{9}$$

where q=the dissipation factor. In the absence of the currents q is found by substituting Eq.(9) into the surf zone energy equation:

$$q=\frac{5}{2}m \qquad m=\partial d_0/\partial x \qquad d_0=h+\zeta_0 \tag{10}$$

where ζ_0 represents the wave set-up in an equilibrium state.

Small perturbations are imposed on the steady state, and the variables are expressed as

$$E=E_0(x)+\epsilon E_1(x,y)$$

$$\zeta=\zeta_0(x)+\epsilon\zeta_1(x,y)$$

$$F=\epsilon F_1(x,y) \tag{11}$$

$$D=D_0(x)+\varepsilon D_1(x,y)$$

where ε is a small oredering parameter.

3. FIRST-ORDER SOLUTIONS

The energy dissipation to the first order becomes, by using the relation shown in Eq.(9),

$$D_1=q_1\sqrt{gd}\,\frac{E_1}{d} \tag{12}$$

Then the dissipation factor q_1 can be written as

$$q_1=\frac{5}{2}m\left(\frac{d}{d_B}\right) \quad (x\leq x_B)\ ,\ \frac{5}{2}m\left(\frac{d_B}{d}\right) \quad (x\geq x_B) \tag{13}$$

where the first expression is modified partly from Eq.(10) due to the necessity of keeping the solution bounded at the shoreline, and the offshore expression corresponds to the expression in the surf zone.
In deriving Eqs.(12) and (13) we have assumed $\zeta_0 \ll d_0$.
The radiation stress Sij can be expressed as

$$S_{xx}=\frac{3}{2}(E_0+\varepsilon E_1) \quad S_{yy}=\frac{1}{2}(E_0+\varepsilon E_1) \tag{14}$$

Dimensionless variables are introduced as follows:

$$\xi=\frac{d}{d_B}=\frac{X}{X_B} \qquad X=x+x_S \qquad d=mX \qquad \eta=\frac{Y}{Y_r}$$

$$\zeta_1^*=\frac{\zeta_1}{d_B} \qquad E_0^*=\frac{E_0}{E_B} \qquad E_1^*=\frac{E_1}{E_B} \tag{15}$$

$$F^*=\frac{F}{(u_0 d_B X_B)} \qquad u_0=\sqrt{gd_B} \qquad u^*=\frac{u}{u_0} \qquad v^*=\frac{v}{v_0}$$

Making nondimensional form of Eqs.(1)-(3) and then eliminating the nondimensional perturbation energy E_1^* from a equation obtained by taking the curl of Eq.(2), finally, we find that, for the nondimensional stream function F* only,

$$L\{F^*\}=0 \tag{16}$$

where the deferential operator L is defined as

$$L=L_1\left(\frac{\xi^{3/2}}{q^*_1}L_2\right)-\frac{5}{2}\xi^2 L_2-C_4 L_1\left(\frac{L_3}{q^*_1}\right) \tag{17}$$

where

$$L_1=\xi\frac{\partial}{\partial\xi}+\frac{1}{2}$$

$$L_2=\left(\frac{\partial^2}{\partial\xi^2}-\frac{\partial^2}{\partial\eta^2}\right)\left\{\nu^*_t\left(\frac{1}{\xi}\frac{\partial^2}{\partial\xi^2}-\frac{1}{\xi}\frac{\partial}{\partial\xi}-\frac{1}{\xi}\frac{\partial^2}{\partial\eta^2}\right)\right\}$$

$$+\frac{\partial}{\partial\xi}\left\{2\nu^*_t\left(2\frac{1}{\xi}\frac{\partial}{\partial\xi}-\frac{1}{\xi^2}\right)\right\}\frac{\partial^2}{\partial\eta^2}$$

$$-C_5\left\{\frac{C^*}{\xi}\frac{\partial^2}{\partial\xi^2}+\frac{\partial}{\partial\xi}\left(\frac{C^*}{\xi}\right)\frac{\partial}{\partial\xi}+2\frac{C^*}{\xi}\frac{\partial^2}{\partial\eta^2}\right\}$$

$$L_3\left(2E^*_0\frac{1}{\xi}\frac{\partial}{\partial\xi}-\frac{7}{2}E^*_0\frac{1}{\xi^2}+2\frac{\partial E^*_0}{\partial\xi}\frac{1}{\xi}\right)\frac{\partial^2}{\partial\eta^2}$$

$$q^*_1=\xi^{1/2}\quad(\xi\leq 1)\ ,\ \xi^{-3/2}\quad(\xi\geq 1)$$

$$E^*_0=\xi^2\quad(\xi\leq 1)\ ,\ \xi^{-1/2}\quad(\xi\geq 1) \tag{18}$$

$$C^*=\xi^{-1/2}\quad(\xi\leq 1)\ ,\ \xi^{-1}\quad(\xi\geq 1)$$

$$\nu^*_t=\xi\quad(\xi\leq 1)\ ,\ 1\quad(\xi\geq 1)$$

$$C_4=(\mu T\sqrt{g/d_{13}})^{-1}$$

$$C_5=(4/\mu\pi\gamma^2)(C_f/m)(mT\sqrt{g/d_{13}})^{-1}$$

where C_4 and C_5 are the nondimensional parameter representing the relative importance of the interaction and the bottom friction, and γ is the ratio of the waveheight to the local mean depth in the surf zone.

Equation (16) is to be solved subject to the following boundary conditions:

$$u^*=v^*=F^*=0\quad\text{at}\quad\xi=0 \tag{19}$$

$$u^*=v^*=F^*=0\quad\text{at}\quad\xi\to\infty \tag{20}$$

$$(u^*_i)_{in}=(u^*_i)_{off}\quad\text{at}\quad\xi=1$$

$$\left(\frac{\partial u^*_i}{\partial\xi}\right)_{in}=\left(\frac{\partial u^*_i}{\partial\xi}\right)_{off}\quad\text{at}\quad\xi=1 \tag{21}$$

We introduce a function given by

$$F^*(\xi,\eta)=\phi(\xi)\sin k\eta \tag{22}$$

where $k=2\pi/Yr^*$, Yr^* is the nondimensionnal rip spacing. The solution in the surf zone to Eq.(16) is written as a series of power of :

$$\phi = \sum_{m=0}^{\infty} a_m \xi^{\rho + \frac{m}{2}} \tag{23}$$

The roots of the indicial equation are $\rho = 0, 2, 2, 5/2, 3$. The first, second and third roots are dorroped through Eq.(19). Hence the solution bounded at the shoreline in the surf zone is found as

$$\phi^*(\xi) = A_{S1} X_1(\xi) + A_{S2} X_2(\xi) \tag{24}$$

where

$$X_1 = \sum_{m=0}^{\infty} a_m \xi^{\rho_1 + \frac{m}{2}} \qquad (a_0 = 1, \rho_1 = 3)$$

$$X_2 = \sum_{m=0}^{\infty} \frac{\partial}{\partial \rho}\left(a_m \xi^{\rho_2 + \frac{m}{2}}\right)\Big|_{\rho=\rho_2} \quad (a_0 = \rho - 5/2, \rho_2 = 5/2)$$

$$\sum_{i=0}^{10} f_i\left(\rho + \frac{m-i}{2}\right) a_{m-i} = 0$$

$$f_0(a) = a(a-2)^2\left(a-\frac{5}{2}\right)(a-3)$$

$$f_1(a) = -C_5 a(a-2)\left(a-\frac{5}{2}\right)$$

$$f_2(a) = -\frac{5}{2} a(a-2)^2(a-3)$$

$$f_3(a) = \frac{5}{2} C_5 a\left(a-\frac{5}{2}\right) \tag{25}$$

$$f_4(a) = -2k^2\left\{a(a-1)^2 + \frac{3}{4}a - \frac{1}{2}\right\}$$

$$f_5(a) = k^2 a\left\{2C_4\left(a+\frac{1}{4}\right) + 2C_5\right\}$$

$$f_6(a) = \frac{5}{2} k^2 (2a^2 - 3a + 2)$$

$$f_7(a) = -5C_5 k^2$$

$$f_8(a) = k^4\left(a+\frac{3}{2}\right) \qquad f_9(a) = 0 \qquad f_{10}(a) = -\frac{5}{2} k^4$$

where A_{S1} and A_{S2} are a constsnts of integration.

In the deep-water region, we can consider the case of water of infinite depth, i.e. $\xi = \infty$. Hence the operator L given by Eq.(17) is rewritten as

$$L = \xi^4 \frac{\partial}{\partial \xi}\left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2}\right)\left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2}\right) \tag{26}$$

Using the general solution of Eq.(26), the approximate solution to Eq.(16) in the offshore region is found according to the boundary condition Eq.(20) as

$$\phi(\xi)=A_{01}e^{-k\xi}+A_{02}\xi e^{-k\xi} \tag{27}$$

where A_{01} and A_{02} are a constants of integration.

The fluid motion of the currents must be continuous at the breaker line as shown in Eq.(21). However three constants of the four constants, A_{S1} , A_{S2} , A_{01} and A_{02} , are determined through the matching condition, then, we introduce normalized velocities as follows:

$$U^*=\frac{u^*(\xi,\eta)}{u_1^*} \qquad V^*=\frac{v^*(\xi,\eta)}{u_1^*} \tag{28}$$

where

$$u_1^*=u^*(\xi,\eta)\Big|_{\xi=1,\eta=0} \tag{29}$$

4. DISCUSION

The normalized rip and longshore velocities, U* and V*, are shown in Figure 1. The Figure 1(a) suggests that the rip velocity attains a maximum value in the offshore region near the break point as Yr* becomes larger. This results are in good agreement with laboratory experiments of Ozaki et al(1977) and Sasaki(1985). Figure 1(b) shows that the longshore velocities become higher in the surf zone and lower in the offshore region with increasing Yr*. It is also found that the derivative of the longshore velocity is continuous at the breaker line. The solutions in the surf and offshore regions, Eqs.(24) and (27), are adequate in each zone including the neighbourhood of the break point.

Figure 2 shows the normalized rip and longshore velocities in case of Yr*=4 and C_4 =1 with respect to C_5=0 and 2. C_5 denotes the ratio of the bottom fiction term to the lateral mixing term, and it means the relative effects of the bottom friction. Then, as shown in Figure 2, the rip and longshore velocities in the surf zone become relatively smaller due to the effects of the bottom friction.

Figure 3 shows the profiles of the normalized rip and longshore velocities in case of Yr*=4 and C_5=1.0 with respect to C_4=0.5, 1.0 and 2.0. As above mentioned, C_4 means the effects of energy coupling between currents waves. Then, the figure demonstrates that the rip and longshore velocities in the surf zone become smaller due to the wave-current interaction.

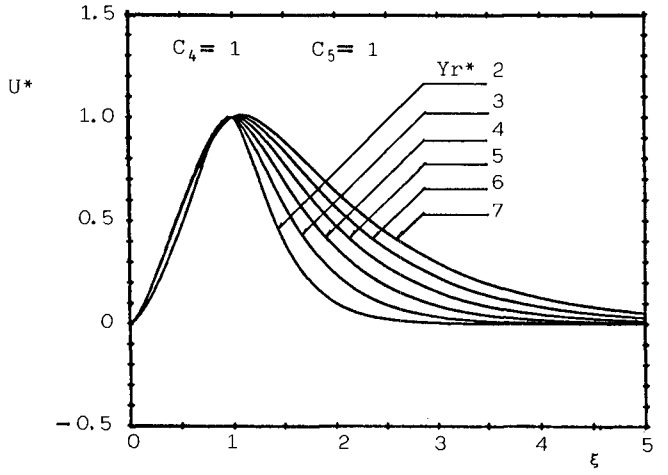

Fig.1(a) The normalized rip velocity U*.

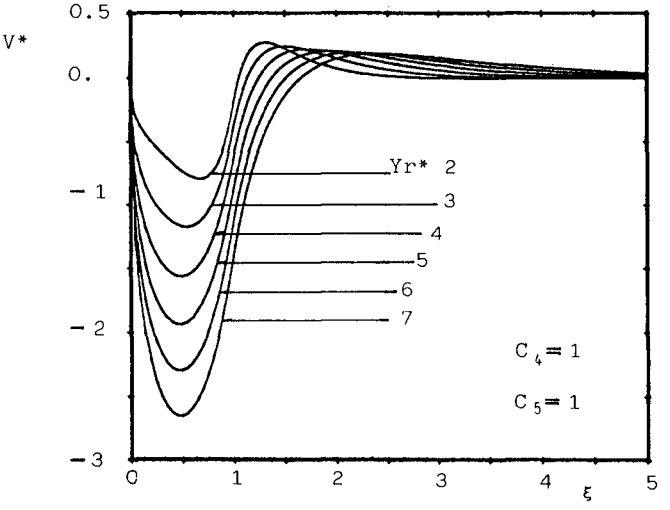

Fig.(b) The normalized longshore velocity V*.

Figure 1 Profiles of the normalized rip and longshore velocities, U* and V*.

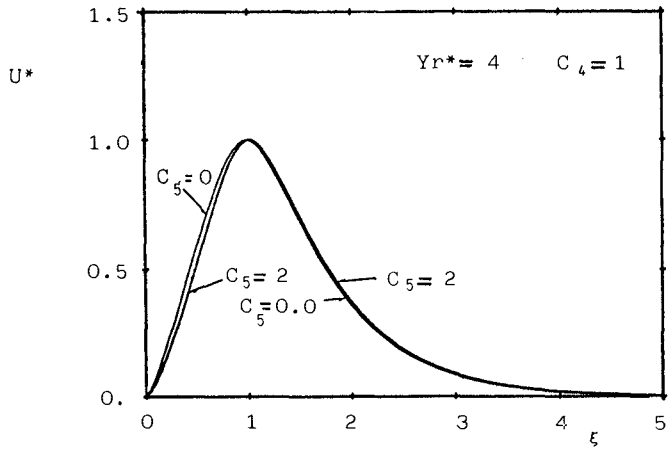

Fig.2(a) The normalized rip velocity with/without the effect of the bottom friction.

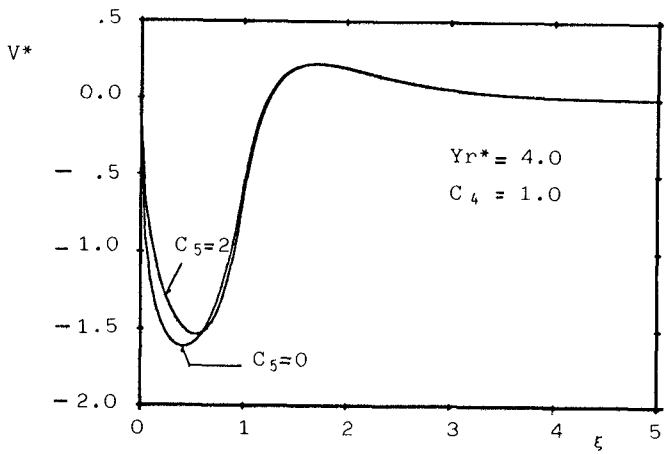

Fig.2(b) The normalized longshore velocity with/ without the bottom friction.

Figure 2 Profiles of the normalized velocity for typical two conditions of the bottom friction.

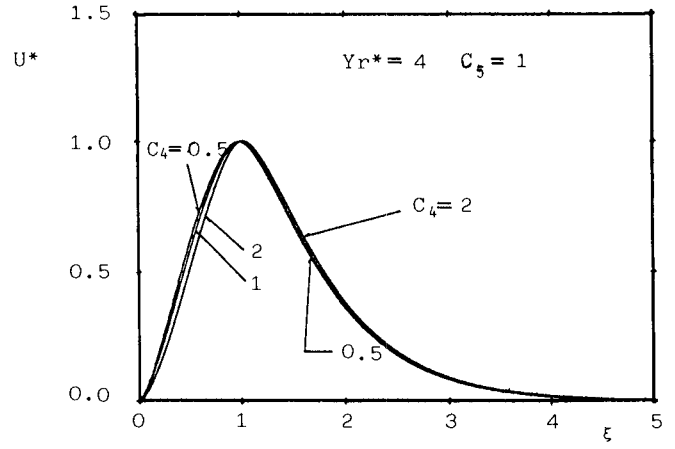

Fig.3(a) The normalized rip velocity.

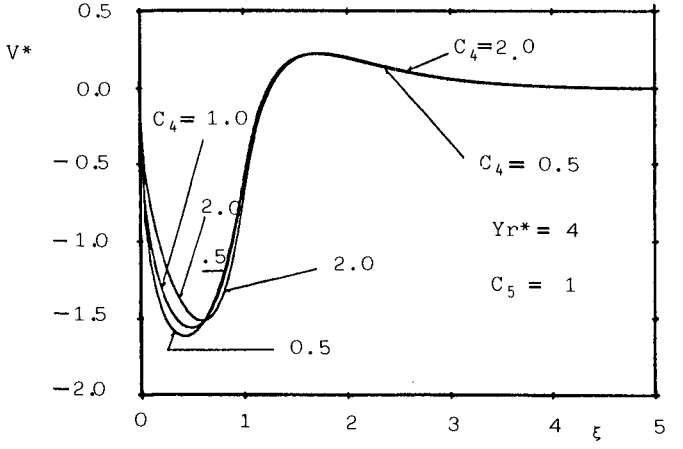

Fig.3(b) The normalized longshore velocity.

Figure 3 Profiles of the normalized velocity for various conditions of the wave-current interaction.

5. CONCLUSIONS

We have extended the theory of rip currents to include the bottom friction, the wave-current interaction and the lateral mixing. By means of introducing a perturbation expansion for the mean variables, the first-order solutions are found in the surf and offshore zones according to the boundary conditions at the coast. In present model the continuity at the breaker line in the derivative of the longshore velocity keeps for the first time. The several results in this investigation conform with results of observations reported by Ozaki et al(1977) and Sasaki(1985).

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