CHAPTER 52

THREE DIMENSIONAL FLOW PROFILES ON LITTORAL BEACHES

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ABSTRACT: The problem of combined cross-shore and longshore currents generated by waves in and around a surf zone is considered in its full three-dimensional formulation. The equations for the two current components are decoupled and it is found that for a cylindrical coast with no longshore variations the longshore current variation with depth and distance from the shoreline satisfies a Poisson equation. This equation is solved by a perturbation method and it is shown that the longshore velocities are always larger than the velocities found by classical theory. In the simple uncoupled case, the full 3-D current profile is constructed by combining the results with cross-shore velocities determined in previous publications. Also, the total velocities are larger than velocities found from simple depth averaged models.

1. INTRODUCTION

The present paper deals with the variation over depth of the magnitude and direction of the wave induced mean currents on a beach.

In the past, the dept variation of wave induced currents have been studied for the purely two-dimensional case of cross-shore flow with particular aim at the seaward going undertow (among the latest contributions may be mentioned Stive & Wind (1986) and Svendsen & Hansen (1988)).

Longshore currents have mostly been analyzed in the simplified case of a long beach with straight bottom contours and no flow variations in the longshore direction. The first contributions date back nearly 20 years (Bowen, 1969; Thornton, 1970; Longuet-Higgins, 1970), and numerous works have been published on the topic since then. They all, however, consider depth integrated equations only, and therefore can only determine a sort of depth averaged value of the longshore current velocity.

Some contributions have claimed three dimensionality for the current description without actually being so. There is also a quasi-3D model inspired by river hydraulics was recently suggested by deVriend & Stive (1987) (essentially the same as Stive & deVriend, 1987). They divide the current motion into a primary part which has the direction and depth averaged magnitude of the net discharge; and a secondary flow which has zero depth mean, goes in the direction of the wave energy flux, and is generated by the wave motion. Further, the secondary current is assumed small relative to the primary current. As an example, however, the undertow is generated by the waves and has

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zero depth mean and hence would be a secondary current according to their definition. On the other hand, it will usually be significant in comparison to net flux current and for obliquely incident waves it will not be in the direction of the wave energy flux. Another example, the longshore current does not seem to fit into the idea that the secondary currents generated by the waves have zero depth mean. Thus the assumptions of de Vriend and Stive (1987) do not seem to cover those two important examples of wave generated currents.

In the present paper we shall take a very much different approach to the problem of three dimensional nearshore circulation and particularly discuss two aspects. One is the variation over depth of the longshore (as well as the cross-shore) currents. The other is the variation of the combined cross-shore and longshore flow. The discussion will be centered around the basic assumptions and the physical aspects of the solution. Readers are referred to Svendsen and Lorenz (1988) for some of the detailed derivations omitted here.

The starting point is taken from the three dimensional Reynolds' equations derived in the above mentioned reference for a combination of waves and wave induced currents in an environment with turbulence produced by primarily wave breaking. Although those equations are fairly general we concentrate the discussion here on the situation of a long straight coastline since some of the essential features of the combination of cross and longshore currents also appear in that canonical situation.

2. THE BASIC ASSUMPTIONS AND THE EQUATIONS USED

The flow situation on a beach corresponds to a combination of a turbulent, oscillatory (wave) motion and a current. In particular inside the breaker point the turbulence generated by the wave breaking will be very intensive.

The equations governing the wave induced currents can be derived from the general Navier-Stokes equations by transformations that correspond to turbulent (or ensemble) averaging (leading to Reynolds type equations) and to averaging over a wave period (corresponding to the equations derived by Whitham (1962) and Longuet-Higgins and Steward (1960,61)).

In accordance with the above description, the total velocity $\vec{u}$ = (u, v, w) is divided into three components by the relations

$$\left.\begin{aligned} u &= U + u_w + u' \\ v &= V + v_w + v' \\ w &= \phantom{V +{}} w_w + w' \end{aligned}\right\} \tag{2.1}$$

Here index w indicates oscillatory (wave) components, ' turbulent quantities. We use $\tilde{}$ to indicate ensemble averaging, which means that $\tilde{u}' = \tilde{v}' = \tilde{w}' = 0$. Similarly, $\bar{}$ denotes averaging over a wave period so that $\overline{u_w} = \overline{v_w} = \overline{w_w} = 0$ below trough level.

By the above mentioned averaging processes and eliminating the pressure by integrating the vertical component of the momentum flux we arrive at the following two equations for U and V, the horizontal current components.

$$\frac{\partial U}{\partial t} + \frac{\partial U^2}{\partial x} + \frac{\partial UV}{\partial y} + \frac{\partial(\overline{u_w^2 - w_w^2})}{\partial x} + \frac{\partial \overline{u_w v_w}}{\partial y} + \frac{\partial \overline{u_w w_w}}{\partial z} = -g\frac{\partial b}{\partial x} - \frac{\partial(\overline{\widetilde{u'^2}} - \overline{\widetilde{w'^2}})}{\partial x} - \frac{\partial \overline{\widetilde{u'v'}}}{\partial y} - \frac{\partial \overline{\widetilde{u'w'}}}{\partial z} \tag{2.2}$$

and

$$\frac{\partial V}{\partial t} + \frac{\partial V^2}{\partial y} + \frac{\partial UV}{\partial x} + \frac{\partial(\overline{v_w^2} - \overline{w_w^2})}{\partial y} + \frac{\partial \overline{u_w v_w}}{\partial x} + \frac{\partial \overline{v_w w_w}}{\partial z}$$

$$= -g\frac{\partial b}{\partial y} - \frac{\partial \overline{\widetilde{v'^2} - \widetilde{w'^2}}}{\partial y} - \frac{\partial \overline{\widetilde{u'v'}}}{\partial x} - \frac{\partial \overline{\widetilde{v'w'}}}{\partial z} \tag{2.3}$$

Eqs. (2.2) and (2.3) are in principle the momentum equations governing the wave induced current motion and it is those equations that are examined further in the following. They differ from the usual depth averaged equations in that they are not integrated in the vertical direction. Thus U, V are in general functions of x, y and z. The equations are only valid below trough level.

In order to solve these equations for U, V it is necessary

a) to assume that the terms with index w representing the wave motion are determined by some wave theory or by measurements.

b) to model in terms of quantities determined otherwise the turbulent normal and tangential stresses.

In the present context, we do not loose any of the effects we want to examine by using a simple time-independent eddy viscosity model for the turbulence. Besides, this has been shown for the cross shore flow to yield very reliable results (see e.g. the earlier quoted references and Svendsen et al. (1987)). Hence we let

$$\left.\begin{aligned} \overline{\widetilde{u'v'}} \quad , \quad \overline{\widetilde{v'w'}} &= -\nu_t \left[\frac{\partial V}{\partial x} \; , \; \frac{\partial V}{\partial z}\right] \\ \overline{\widetilde{u'w'}} &= -\nu_t \frac{\partial U}{\partial z} \end{aligned}\right\} \tag{2.4}$$

in which we have already introduced the simplification of uniform flow conditions in the longshore direction $(\overline{\partial/\partial y} = 0)$. We also neglect the turbulent normal stresses.

3. THE FLOW PROFILES ON A CYLINDRICAL COAST WITH NO LONGSHORE VARIATIONS

This canonical case corresponds to letting $\overline{\partial/\partial y} = 0$ in both (2.2) and (2.3) which yields equations amenable to analytical solution without the essential features of the flow are lost. We also limit our considerations to time independent current patterns. Hence the equations we consider for further simplification are the following

$$\frac{\partial}{\partial x}\left(U^2 + gb + (\overline{u_w^2} - \overline{w_w^2})\right) + \frac{\partial \overline{u_w w_w}}{\partial z} = -\frac{\partial(\overline{\widetilde{u'^2}} - \overline{\widetilde{w'^2}})}{\partial x} + \frac{\partial}{\partial z}\left(\nu_t \frac{\partial U}{\partial z}\right) \tag{3.1}$$

and

$$\frac{\partial}{\partial x}\left(UV + \overline{u_w v_w}\right) + \frac{\partial \overline{v_w w_w}}{\partial z} = \frac{\partial}{\partial x}\left(\nu_t \frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial z}\left(\nu_t \frac{\partial V}{\partial z}\right) \tag{3.2}$$

where also (2.4) has been substituted.

The Cross-Shore Motion

With suitable additional simplification (3.1) becomes the equation solved in the literature for undertow. Several of those simplifications were analyzed by Stive & Wind (1982) and Svendsen et al. (1987), using measurements to calculate the magnitude of the $\overline{w_w^2}$, the $\overline{u'^2} - \overline{w'^2}$ and U^2 terms. Although the U^2 term is found not to be quite negligeable, the measurements used by Svendsen et al. (1987) show that the dominating driving term in (3.1) is the $\partial b/\partial x$ - term. $\partial \overline{u_w^2}/\partial x$ is only 10-20% of that and the other terms are even smaller.

The measurements also show that $\overline{u_w^2}$ is remarkably constant over depth, a fact that has been further confirmed by later, yet unpublished, laboratory results.

The $\partial \overline{u_w w_w}/\partial z$ term has also been neglected in all previous investigations. It represents the net horizontal shear stress on a fluid particle generated by the oscillatory wave component. For constant depth wave theories (such as the linear wave theory which is often used for the wave contributions) this term is identically zero (except in the boundary layer). On a sloping bottom we have no wave theory from which we can evaluate the term. Simple estimates suggest, however, that the term is substantially smaller than $\partial \overline{u_w^2}/\partial x$. Hence in spite of the principal importance of the $\partial \overline{u_w w_w}/\partial z$ term in deriving the depth integrated version of the equations, it is a good approximation locally to disregard this term in the equation for cross-shore currents.

With the assumed constant eddy viscosity and with the driving term

$$\alpha_1 = \frac{\partial}{\partial x}\left[gb + \overline{u_w^2} - \overline{w_w^2} + U^2\right] \tag{3.3}$$

uniform over depth the solution for the undertow may be written

$$U = \frac{1}{2}\frac{\alpha_1}{\nu_t}\zeta^2 + \frac{A_1}{\nu_t}\zeta + U_b \tag{3.4}$$

where A_1 is an integration parameter related to the mean bottom friction. U_b is the bottom value of the undertow profile if extended to the bottom without consideration of the bottom boundary layer. For a closer discussion of evaluation of A_1 and U_b reference is made to Svendsen & Hansen (1988).

The Longshore Motion

The longshore motion is given by (3.2). The additional simplification needed to make this equation solvable is related to the UV-term. This term represents the effect which the undertow has on the longshore current. Although this effect may locally be appreciable it can readily be shown that averaged over depth the contribution will be almost nil. In order to proceed we choose here to neglect the term. Then (3.2) reduces to

$$\frac{\partial}{\partial x}\left[\nu_t \frac{\partial V}{\partial x}\right] + \frac{\partial}{\partial z}\left[\nu_t \frac{\partial V}{\partial z}\right] = \frac{\partial \overline{u_w v_w}}{\partial x} + \frac{\partial \overline{v_w w_w}}{\partial z} \tag{3.5.}$$

Since the right hand side is considered known, this is a Poisson equation for the variation of the longshore current velocity V over depth and cross shore directions.

It may be noticed that the effect of neglecting the UV-term is that the equation for the longshore current becomes independent of the solution for the cross-shore current. Similarly, in the general case of arbitrary bottom topography this would have freed the cross shore current from dependence of the longshore current solution.

Eq. (3.5) may also be written

$$\nabla(\nu_t \nabla V) = g_{xy}$$

where

$$g_{xy} = \frac{\partial}{\partial x}\left(\overline{u_w v_w}\right) + \frac{\partial}{\partial z}\left(\overline{v_w w_w}\right)$$

4. SOLUTION FOR THE LONGSHORE CURRENT VARIATION WITH x AND z

It turns out that by physically realistic assumptions about magnitude of the terms in (3.5) and the boundary condition at the bottom (3.5) can be solved by a perturbation technique which as the first approximation has the classical solution for depth averaged longshore current velocity.

First we notice that since (3.5) is an elliptic equation it requires boundary conditions along the full boundary.

The forcing term g(x,y) will only be non-zero shoreward of the transition point, x_t (Svendsen, 1984). Hence, at x_t, g_{xy} is discontinuous and matching of solutions must be arranged as in traditional longshore current solutions with depth independent velocities.

Eq. (3.5) is only valid below wave trough level. Therefore another boundary condition is that at the trough level

$$\tau_{zy} = \rho\nu_t \frac{\partial V}{\partial z} = \frac{\partial}{\partial x} S_{xy,s} \qquad \text{at } x = \eta_t \tag{4.1}$$

where $S_{xy,s}$ is the radiation stress contribution above trough level, and η_t is the wave trough level. This boundary condition is introduced into the mathematical formulation by combining it with the requirement that (3.5) be satisfied at all z below trough level to yield the depth integrated momentum balance which becomes

$$\frac{\partial}{\partial x}\int_{-h_0}^{b} \rho\, \nu_t \frac{\partial V(x,z)}{\partial x}\, dz - \tau_{yb} = \frac{\partial S_{xy}}{\partial x} \tag{4.2}$$

For V independent of z this expression reduces to the well known

$$\frac{\partial}{\partial x}\left(\rho\nu_t h \frac{\partial V}{\partial x}\right) - \tau_{yb} = \frac{\partial S_{xy}}{\partial x} \tag{4.3}$$

which is the equation normally solved for longshore currents.

The most important boundary condition, however, is at the bottom. Here we have an oscillatory boundary layer in which the turbulent eddy viscosity can be assumed much smaller than above (Svendsen et al., 1987). In contrast to cross-shore flow, however, the bottom shear stress is the (exterior) force resisting the driving radiation stress gradient. In this context the horizontal turbulent mixing--represented by the $\partial/\partial x(\nu_t\ \partial V/\partial x)$ term--only redistribute the radiation stress contributions from one place to another, and the integral of all radiation stresses from sea to shore must equal S_{xy} at the breaker point.

The boundary layer at the bottom will yield a relationship between the bottom velocity V_b and the bottom shear stress $\tau_{y,b}$, and we assume that this relationship can be represented by a generalization of the expression derived by Svendsen & Hansen (1988)

$$\tau_{y,b} = \left(\rho\nu_t \frac{\partial V}{\partial z}\right) = \frac{1}{2} \rho \ f_w \ u_{wb} \ V_b \tag{4.4}$$

where f_w is the friction factor for waves only, u_{wb} the oscillatory particle velocity in the wave direction. It is emphasized that although this expression formally may resemble the frequently used relationships between bottom shear stress and depth mean velocity (4.4) is actually based on the elaborate wave current boundary layer theory by Christoffersen & Jonsson (1983).

The turbulent mixing term cannot be neglected even in a first approximation since that would change the degree of the differential operator in the x-direction and radically change the nature of the x variation of V (Longuet-Higgins, 1970).

With the turbulence mixing as a modifying term, however, the longshore flow is very much a balance between the driving radiation stress gradient and the resisting τ_{zy} shear stress. Thus, if we for a moment assumed the right hand side of (3.5) and the turbulent mixing were uniformly distributed over depth then τ_{zy} would vary linearly over depth, with the largest values at the bottom. The small viscosity inside the bottom boundary layer and much larger value above therefore means that by far the largest velocity gradients will occur within the bottom boundary layer. Or, in other words, the velocity outside the boundary layer will only show a moderate variation in comparison to the variation inside the boundary layer.

Mathematically this can be expressed by assuming that the first approximation to the longshore current velocity is independent of z. Thus we assume a perturbation expansion of the form

$$\left.\begin{aligned} V &= V_0(x) + \epsilon \ V_1(x,z) + \dots \\ \tau_{yb} &= \tau_0(x) + \epsilon\tau_1(x) + \dots \\ V_b &= V_{b0} + \epsilon \ V_{b1} + \dots \end{aligned}\right\} \tag{4.5}$$

In order to obtain a consistent modification of the equations (3.5), (4.2) and (4.4) a scaling is required which introduces the assumption that the width of the surf zone $x_s - x_t$ is $<< h_t$ where h_t is the depth at the transition point x_t. It is also utilized that

$$\frac{f_w}{\nu_t} h_t \sqrt{gh_t} = O(s_x^2) \tag{4.6}$$

where $s_x = \epsilon^{1/2}$ is a characteristic value of the bottom slope. With

$$X = -s_x x \quad ; \quad Z = z \quad ; \quad \epsilon = s_x^2 << 1$$

the modified equations become

$$\epsilon \frac{\partial}{\partial X} \left(\nu_t \frac{\partial V}{\partial X}\right) + \frac{\partial}{\partial Z} \left(\nu_t \frac{\partial V}{\partial Z}\right) = - \epsilon g_{xy}/s_x^2 \tag{3.5a}$$

$$\frac{\tau_0}{\rho\nu_t} = \left(\frac{\partial V}{\partial z}\right)_{-h_0} = \epsilon\,\frac{1}{2}\,\frac{f_w}{\nu_t s_x^2}\,u_{wb}\,V_b \tag{4.4a}$$

and

$$\epsilon\,\frac{\partial}{\partial X}\int_{-h_0}^{b} \rho\nu_t\,\frac{\partial V}{\partial X}\,dz - \tau_{yb} = \frac{\epsilon}{s_x^2}\,\frac{\partial S_{xy}}{\partial x} \tag{4.2a}$$

The first order approximation shows that since $V_0 = V_0(x)$ outside the boundary layer we get $\tau_0 = 0$.

The system also yields $V_{b0} = V_0$ and hence

$$\frac{\tau_1}{\rho\nu_t} = \frac{1}{2}\,\frac{f_w}{\nu_t S_x^2}\,u_{wb}\,V_0 = \left(\frac{\partial V_1}{\partial Z}\right)_{-h_0} \tag{4.7}$$

V_0 is then determined from (4.2), and since V_0 is independent of z (4.2) takes the form of (4.3)

$$\frac{\partial}{\partial X}\left(\rho\nu_t h\,\frac{\partial V_{b0}}{\partial X}\right) - \tau_1 = \frac{1}{s_x^2}\,\frac{\partial S_{xy}}{\partial x} \tag{4.8}$$

where τ_1 is given by (4.7).

Thus we have found that the first approximation V_0 to the longshore velocity V is determined exactly by the equation which has so far been used in the literature for depth averaged longshore current velocities. V_{b0}, however, is not the depth averaged value of V as we shall see shortly.

Carrying the computations through to second order we find that

$$\begin{aligned} V(x,z) &= V_0 + \epsilon V_1 \\ &= V_{b0} + \epsilon V_{b1} + \frac{1}{2}\left[g_{xy} - \frac{\partial}{\partial x}\left(\nu_t\,\frac{\partial V_0}{\partial x}\right)\right]\left(z+h_0\right)^2 \\ &\quad + \frac{f_w}{2\nu_t}\,u_{wb}\,V_{b0}(z+h_0) \end{aligned} \tag{4.9}$$

Here V_{b1} can be determined from the differential equation

$$\frac{\partial}{\partial X}\left(\nu_t h\,\frac{\partial V_{b1}}{\partial X}\right) - \frac{1}{2}\,\rho f_w\,u_{wb}\,V_{b1} = -\,F(x) \tag{4.10}$$

where F(x) is a known function of x determined from the first order solution. One important observation here is that (4.9) shows that the depth average of V is always larger than V_{b0}, $V_{b0} = V_0$ is in fact the smallest velocity in the profile.

The results also show that there is a correction V_{b1} to the bottom velocity V_{b0} found by the traditional longshore current equation (4.8). This correction is entirely due to the fact that in (4.8) we use V_0 to evaluate the turbulent mixing, not V. Since V_{b1} will normally be a small term, we may get a good approximation to V by neglecting V_{b1}. That yields

$$V(x,z) \simeq V_{bo} + \frac{1}{2}\frac{g'_{xy}}{\nu_t}(z+h_0)^2 + \frac{f_w}{2\nu_t} u_{wb} V_{bo}(z+h_0) \qquad (4.11)$$

where

$$g'_{xy} = g_{xy} - \frac{\partial}{\partial x}\left(\nu_t \frac{\partial V_0}{\partial x}\right) \qquad (4.12)$$

5. NUMERICAL RESULTS FOR THE LOCAL LONGSHORE CURRENT VELOCITY PROFILE

For both V_0 and V_1, a full solution for the local profile requires that we solve the differential equations (4.8) and (4.10) for V_{bo} and V_{b1}, respectively. Properly done this also implies determination of the wave height variation across the surf zone to establish the variation of S_{xy}.

To illustrate the nature of the results, however, let us neglect the effect of the turbulent mixing and concentrate on the local velocity profiles only. This implies that $g'_{xy} = g_{xy}$ in (4.11).

Using Svendsen's (1984) description of the surf zone waves with a shape factor $B_0 = \overline{\eta^2}/H^2$ to account for the breakers we get

$$\overline{u_w w_w} = g\left(\frac{H}{h}\right)^2 h B_0 \frac{c^2}{gh} \sin\alpha \cos\alpha$$

where α is the angle of incidence for the waves. Approximating the $\partial\overline{v_w w_w}/\partial z$ term in a way that satisfies the known requirements for that term (for details see Svendsen & Lorenz, 1988) then yields

$$g_{xy} = \frac{5}{2}\left(\frac{H}{h}\right)^2 B_0 \frac{c^2}{gh} \frac{\partial h}{\partial x} \sin\alpha \cos\alpha$$

and

$$\frac{\partial S_{xy}}{\partial x} = \rho g h \frac{\partial h}{\partial x}\left(\frac{H}{h}\right)^2 \left(\frac{5}{2} B_0 + 3\frac{A}{H^2}\frac{h}{L}\right) \sin\alpha \cos\alpha$$

where the second term originates from the shear stress created at wave trough level by the surface roller. A is the roller area and L the wave length.

Fig. 1 shows some examples of velocity profiles obtained by using the following values of the parameters.

Phase velocity	$c = \sqrt{gh}$
Bottom friction coefficient	$f_w = 2 \cdot 10^{-2}$
Breaker shape	$B_0 = \overline{\eta^2}/H^2 = 0.08$
Roller contribution	$\frac{Ah}{H^2 L} = 0.09$
Eddy viscosity	$\nu_t/h\sqrt{gh} = 0.015$

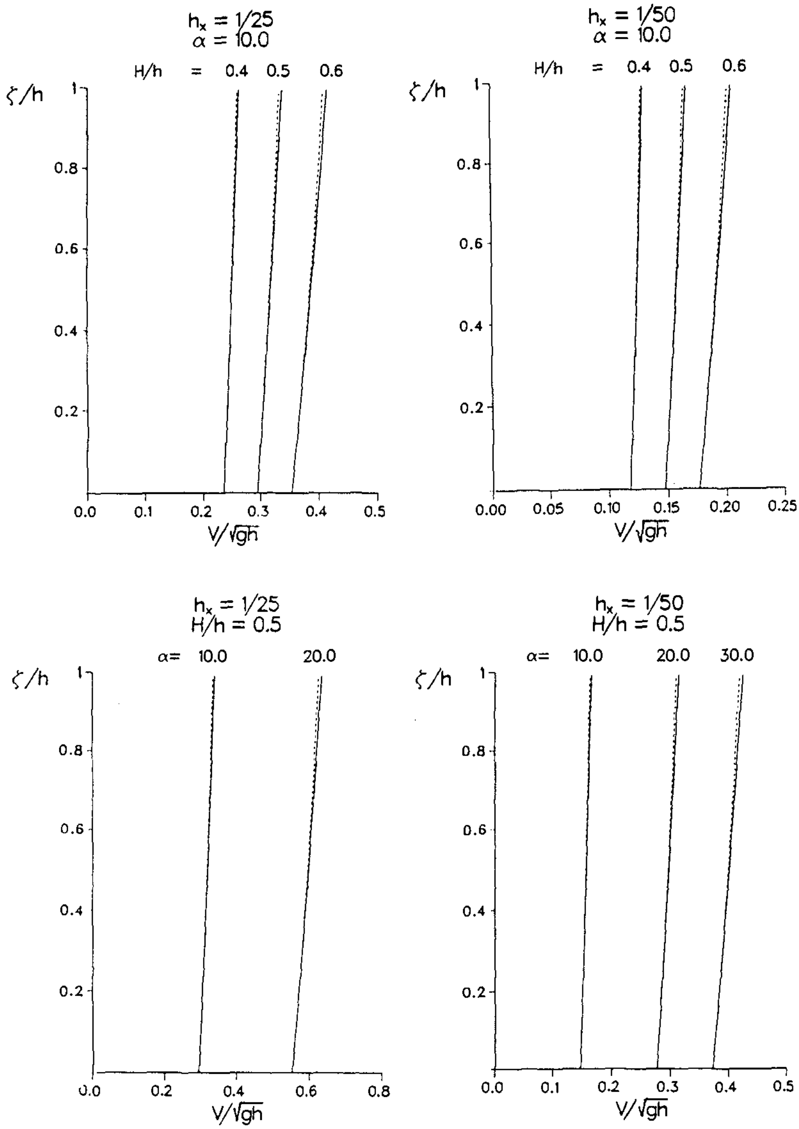

FIG. 1. Examples of longshore current proviles for different angles of incidence α, bottom slopes h_x and wave height to water depth H/h. No horizontal mixing.

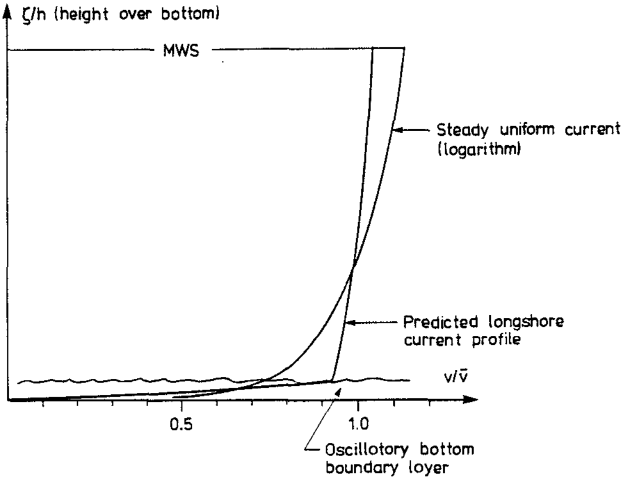

FIG. 2. A comparison between a longshore current profile and a logarithmic profile as in a river.

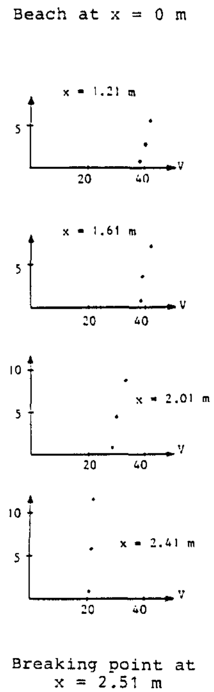

Fig. 3. Measurements of longshore velocities at three different elevations above the bed (from Visser, 1984).

The absence of turbulent mixing makes the calculated velocities larger than real. The important conclusion, however, is that the velocity profiles are very different from the logarithmic profile sometimes assumed in analogy with open channel flow. Fig. 2 shows a comparison between the two profiles. In fact the assumption of a uniform velocity over depth would be much closer to the real velocities than would a logarithmic profile. It also implies that the near-bottom velocities are significantly larger than for a logarithmic profile.

It may be emphasized that since the classical longshore current velocity found e.g. by Louguet-Higgins corresponds to $V = V_{bo}$, the real velocities will always be 10-20% larger than those found by classical theory.

Fig. 3 shows measurements of longshore currents in the surf zone at three different elevations above the bottom (from Visser, 1984). We see that the nature of the variation of depth is qualitatively in agreement with the theoretical profiles shown in Fig. 1.

6. COMBINED LONGSHORE AND CROSS-SHORE VELOCITY PROFILE

In the simplified case described here of no coupling between the cross-shore and the longshore current (the UV term in (3.2)) the total 3D velocity profile can be constructed by a simple vector addition of the cross-shore and the longshore velocity profiles determined independently. Such a 3D profile is shown in Fig. 4.

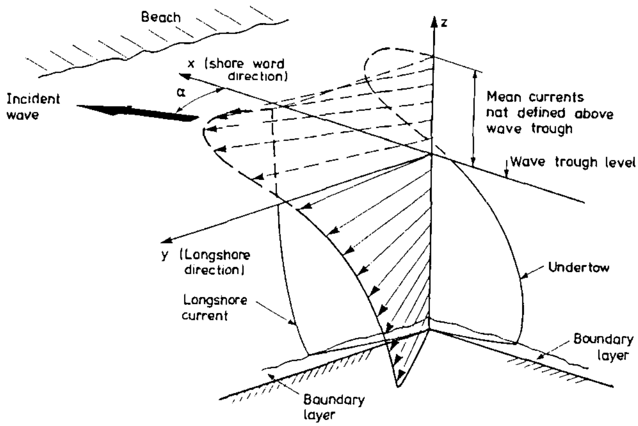

FIG. 4. The full 3-D current velocity profile as it occurs in a situation with both an undertow and a longshore current.

The result shows features that are of significant importance for the sediment transport on a coast.

From the knowledge of the fact that a strong undertow will be present in a surf zone even with no net cross shore current, we know that there is a tendency that sediments are being carried seaward by the current. The 3D profile, however, shows that close to the bottom the current direction may actually deviate 45° or more from the depth mean current vector which in this case is purely along-shore.

Moreover, if e.g. the longshore and crosshore bottom current velocities are of equal magnitude the total current velocity is 40% larger than given by a depth averaged model. Hence it can be quite misleading to use depth averaged models to predict bottom currents in the surf zone.

The predictions of the described 3D velocity profile are qualitatively confirmed by the tracer sediment observations by Ingle (1966). Fig. 5 shows a summary of his results.

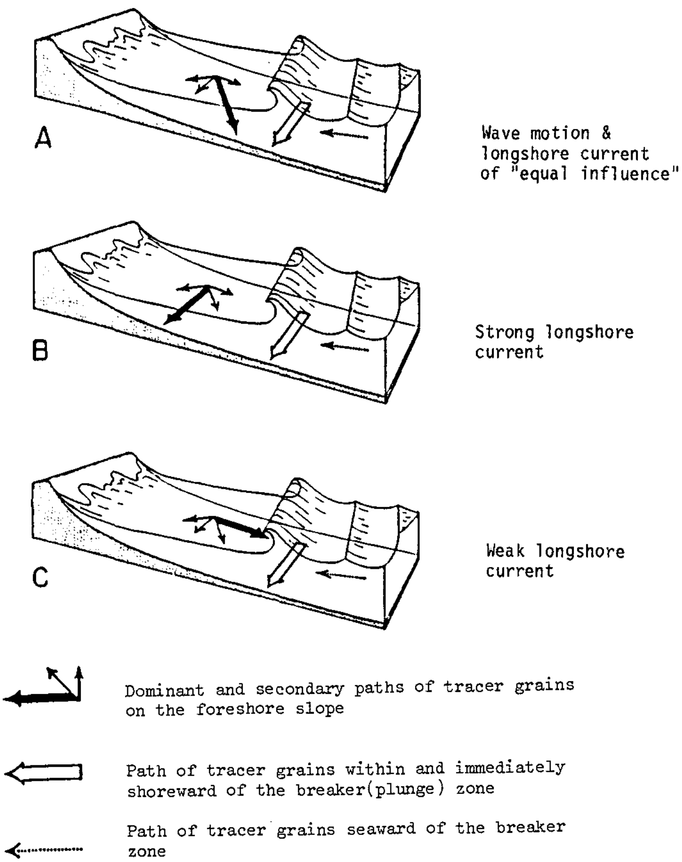

FIG. 5. Sediment movements in and near the surf zone. Summary of tracer observations by Ingle (1966).

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