CHAPTER 63

Spectral Wave Attenuation by Bottom Friction: Experiments

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ABSTRACT

This paper presents a summary of carefully conducted laboratory experiments on sediment response and resulting wave attenuation for waves, periodic as well as simulating a wave spectrum, propagating over an 18-m-long bottom section covered by a 10-cm-thick layer of 0.2-mm-diameter quartz sand. The results for wave-generated bedform geometry as well as the equivalent Nikuradse roughness are obtained. The results show that the sediment response to the agitation of a wave spectrum may be approximately described by the sediment response to an equivalent periodic wave having the same root-mean-square near-bottom orbital velocity and excursion amplitude as the spectral wave. This equivalent wave, defined by the near-bottom characteristics of the spectral wave, is identical to that derived by Madsen et al. (1988) in the context of dissipation within the wave boundary layer. Although limited by the range of experimental conditions achieved in this study, the results obtained here and in the companion paper suggest a predictive methodology for the evaluation of spectral wave attenuation from knowledge of sediment and wave characteristics.

INTRODUCTION

As waves propagate into waters of finite depth the waves respond to the presence of the bottom, e.g., through shoaling, depth refraction, and energy dissipation, while the bottom, consisting of movable sediment, responds to the presence of waves by exhibiting bedforms. In the present context of wave attenuation, the wave-sediment interaction is of crucial importance since the bottom bedforms and sediment transport caused by the wave motion itself, in turn, determine the bottom roughness and hence the rate of energy dissipation within the bottom boundary layer.

In a companion paper (Madsen et al., 1988) an approximate theory for the turbulent boundary layer flow over a rough bottom and the associated dissipation of energy has been

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developed for a wave motion specified by its directional frequency spectrum. While overcoming some of the problems of currently available theories for the evaluation of spectral dissipation by bottom friction, the theory by Madsen et al. (1988) assumes the equivalent Nikuradse roughness of the bottom, k_b, to be known a priori. From the preceding discussion of the response of a movable bed to the agitation by waves it is evident that the equivalent bottom roughness of a movable bed is a function of wave-sediment interaction, i.e., it is a dependent variable.

Based on the limited data of Carstens et al. (1969) and Stefanick's (1979) comprehensive review of experimental data on the geometry of wave-generated bedforms Grant and Madsen (1982) proposed a predictive expression for k_b in terms of sediment and wave characteristics. A major shortcoming of the Grant-Madsen relationship is that it, as others, e.g., Lofquist (1986), Vongvisessomjai (1987) is derived from experimental data obtained exclusively for monochromatic waves. To apply any of the existing relationships for movable bed roughness, in conjunction with a theory for spectral dissipation, to predict spectral wave attenuation by bottom friction therefore requires the determination of the characteristics of a periodic wave which is equivalent to the spectral wave in the context of wave-sediment interaction. To demonstrate that such an equivalent periodic wave does exist and to determine its characteristics are the objectives of the experimental study summarized in this paper.

The philosophy behind the experimental methodology and procedures used in the present study is to measure the attenuation of water waves (periodic as well as simulated spectral waves) propagating over a long section of movable bed and from the measured attenuation of each incident wave component backfigure the appropriate value of the wave friction factor for each incident wave component using the theoretical model for spectral wave attenuation developed by Madsen et al. (1988). From knowledge of the wave friction factor and the incident wave condition, which according to Madsen et al. (1988) is represented by a periodic wave having the same root-mean-square near-bottom orbital velocity and excursion amplitude as the simulated wave spectrum, the Grant and Madsen (1986) relationship between periodic wave characteristics, equivalent bottom roughness, and wave friction factor is used--in reverse--to back out a value of the equivalent bottom roughness, k_b. The geometry of bottom bedforms generated by the waves, whose attenuation was determined, is also determined and related to the wave and sediment characteristics as well as to the measured values of the equivalent bottom roughness, k_b, in order to arrive at a predictive relationship for the equivalent bottom roughness of a movable bed valid for spectral waves.

EXPERIMENTAL SET-UP

Laboratory experiments were performed in a 0.75-m-wide, 0.90-m-deep, 30-m-long wave flume in the R. M. Parsons Laboratory. This wave flume is equipped with a programmable piston-type wave maker and has a 1-on-10 sloping absorber beach, which in the present experiment was covered by 7.5-cm-thick horsehair mats to further decrease reflections. At the wave maker a trapezoidal wooden ramp provided a smooth transition to a level 10 cm above the flume bottom. The region between the ramp--approximately 3 m from the wave maker--to the start of the absorber beach--approximately 19 m from the wave maker--was covered by a 10-cm-thick layer of very uniform 0.2-mm-diameter Ottawa quartz sand.

The wave maker was programmed to simulate the generation of incident periodic waves, wave groups consisting of two frequency components, and wave spectra. Two different spectral simulation methods were used: one simulating the spectrum as equally spaced frequency components, the other representing the spectrum by different frequency components of equal energy. Although of no fundamental importance to the study it is mentioned for completeness that the spectral shape simulated in our experiments was that of a Neuman spectrum. In the majority of the experiments the water depth above the movable bed test section was 0.60 m.

In order to mobilize the bed material it was necessary to generate waves of large amplitudes and wave lengths. Our standard wave condition corresponded to a 6-cm-amplitude, 2.65-sec-period wave for which $h/L = 0.1$ in the 0.6-m-deep section. Wave group experiments were conducted with frequencies centered around the 2.65-sec-period while this period was taken as the peak period in spectral simulations. Due to limited wave-making capability it was only possible to generate multi-component wave conditions with an energy content equal to half of that of the standard periodic wave. The incident and reflected wave components were determined from surface profile measurements obtained from two three-gauge arrays (Goda and Suzuki, 1976) located 3 m and 18 m from the wave maker.

Bottom bedform geometry was determined photographically by taking pictures of 1-m sections of the bed profile at four stations along the test section. These pictures, including horizontal and vertical scales, were then projected onto a digitizing tablet and analyzed. Using a vertically mounted laser to trace the bottom profile at different distances from the flume sidewalls it was shown that the trace along the sidewalls accurately represented the bedform geometry.

EXPERIMENTAL PROCEDURE

Given the philosophy behind the experiments as outlined in the Introduction the procedure to be followed is in principle quite simple. From the analysis of Madsen et al. (1988) we may write the energy conservation principle for each wave component when only bottom dissipation is considered

$$c_{gn}\rho g a_n\left[\frac{\partial a_n}{\partial x}\right]_{bf} = -E_{d,n} = -\tfrac{1}{4}\rho f_{wr} u_{br} u_{bn}^2 \qquad (1)$$

where, for a finite number of wave frequency components

$$u_{br} = \sqrt{\sum_n u_{bn}^2}$$

$$A_{br} = \sqrt{\sum_n (u_{bn}/\omega_n)^2} = u_{br}/\omega_r \quad (2)$$

defines the representative periodic wave, and

$$u_{bn} = \frac{a_n \omega_n}{\sinh k_n h} \quad (3)$$

Thus, for a single wave component--whether it is alone or one of two or more--we have from Eqs. (1) through (3)

$$\left[\frac{\partial a_n}{\partial x}\right]_{bf} = - f_{wr}\frac{u_{br}}{4gc_{gn}}\left[\frac{\omega_n}{\sinh k_n h}\right]^2 a_n \quad (4)$$

In principle the experimental determination of a_n at two stations, Δx apart, suffices to determine a value of f_{wr} from Eq. (4). In practice, however, life is unfortunately not that simple.

First of all the experimental determination of Δa_n involves the "small" difference between two "large" quantities. As an example we may take the standard periodic wave (a_n = 6 cm, $T_n = 2\pi/\omega_n$ = 2.65 sec, h = 0.60 m) and a friction factor of f_{wr} = 0.2 to obtain a change in amplitude of $\Delta a \simeq$ 7 mm over the 15-m-long test section. While this value of Δa is well above the accuracy with which the individual amplitudes is determined experimentally--estimated to be of the order 0.5 mm--it should be recalled that component amplitudes of the order 1 to 2 cm are used in spectral simulations. Thus, we are is some cases attempting to measure differences of the order a few mm, i.e., approaching the magnitude of uncertainty of each measurement used to determine the difference. For this reason alone it is essential that the experiments be conducted with great thoroughness and repeated several times in order to minimize the effect of experimental errors.

However, other effects in addition to those directly related to measurement accuracy play a role in these experiments. Owing to the requirement of long waves of substantial amplitude--to set the bottom sediment in motion--it can be expected that nonlinear effects come into play. To illustrate this, assume the change of amplitude, Δa_n, to have been determined experimentally over a distance Δx. Formally, we may then write

$$\frac{\partial a_n}{\partial x} \simeq \frac{\Delta a_n}{\Delta x} = m = m_{nl} + m_{sw} + \left[\frac{\partial a_n}{\partial x}\right]_{bf} \quad (5)$$

in which m_{nl} denotes amplitude changes associated with nonlinearity, m_{sw} the changes associated with dissipation along the side walls, and $(\partial a_n/\partial x)_{bf}$ is the quantity expressing the contribution of bottom friction, i.e., the quantity we are looking for. A priori we cannot neglect the unwanted contributions, m_{nl} and m_{sw}, in particular when the overall requirement of accuracy is kept in mind.

To evaluate the magnitude of the terms $m_{nl} + m_{sw}$ in Eq. (5) preliminary experiments were conducted for conditions corresponding to a flat bed. The measured amplitude change in these preliminary experiments (denoted by subscript p)

$$\left[\frac{\Delta a_n}{\Delta x}\right]_p = m_p \simeq m_{nl} + m_{sw} + m_{fb} \tag{6}$$

were taken to represent nonlinear and sidewall effects also in the movable bed runs. The bottom friction effect, m_{fb} in Eq. (6), was calculated theoretically using the sediment grain size as the bottom roughness.

For experiments with simple periodic waves the nonlinear effects were eliminated by correcting the wave maker motion as suggested by Madsen (1971). For all other experiments a series of about five preliminary runs were performed to determine the nonlinear and sidewall contributions using Eq. (6). Following the preliminary runs the wave condition was maintained in the flume for sufficiently long time to ensure the bedforms on the bottom to be fully developed (in some cases this took several hours). Several wave measurements were then performed corresponding to fully developed bed conditions with bottom bedform geometry determined between individual wave measurements (to ensure that the bed indeed was fully developed). Combining the preliminary and the fully developed wave measurements, as outlined above, produced experimental results for the isolated effect of wave attenuation caused by bottom friction, which in turn were used in Eq. (5) to obtain friction factors.

SUMMARY OF RESULTS

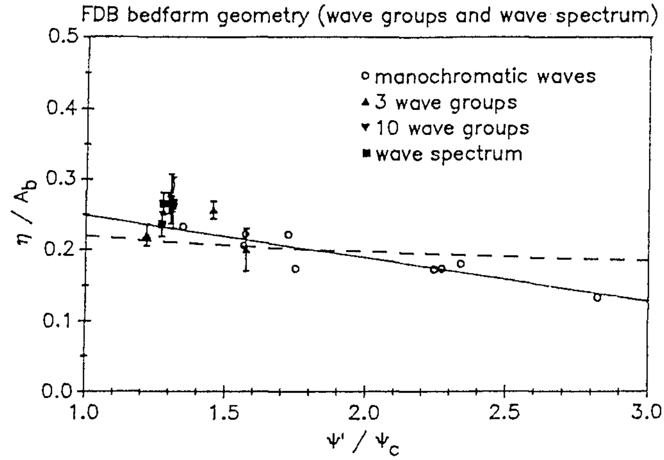
Since the present results are preliminary in nature due to the limited range of experimental parameters covered, only a summary of the conclusions most significant to application will be presented here. Further details may be obtained from Rosengaus (1987) and in subsequent publications. By comparison of bedform geometry generated by simple periodic and multi-component (spectral) waves it is found that the representative wave defined by Eq. (2) may be used to describe the response of a movable bed to wave agitation.

This conclusion is supported by the results presented in Figure 1a and b which shows measured ripple height, η, normalized by the excursion amplitude of the representative wave, defined by Eq. (2), and ripple steepness (η/λ), respectively, as functions of the sediment mobility parameter ψ'/ψ_c where

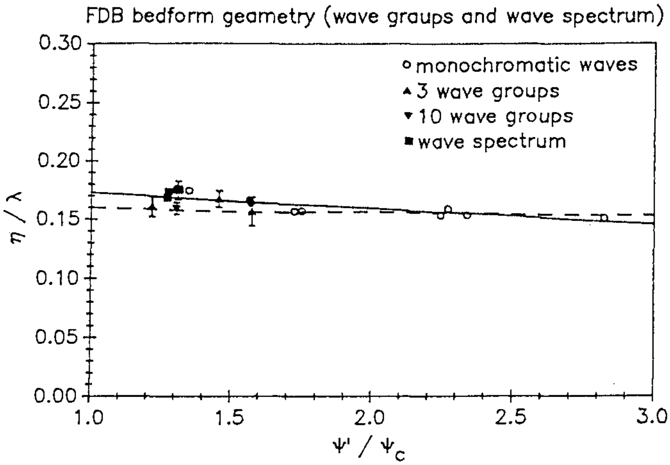
$$\psi' = \frac{\tau_b'}{(s-1)\rho g d} \tag{7}$$

is the Shields parameter based on grain size roughness evaluated for the representative wave and ψ_c is the critical value for initiation of sediment motion (see Madsen and Grant, 1976, for details).

For comparison the empirical relationships derived by Stefanick (1979) and used by Grant and Madsen (1982) are



a: Fully developed bedform height for non-monochromatic waves



b: Fully developed bedform steepness for non-monochromatic waves

Figure 1: Comparison of Bedform Geometry.

shown as dashed lines while new relationships derived from this investigation

$$\frac{\eta}{A_{br}} = 0.31 - 0.06(\psi'/\psi_c)$$

$$\frac{\eta}{\lambda} = 0.19 - 0.014(\psi'/\psi_c) \qquad (8)$$

are shown as solid lines. As is evident from the results presented in figures 1a and b the agreement between bedform geometry generated by monochromatic and multi-component waves (groups and spectra) is excellent, but tested only within a range of (ψ'/ψ_c) slightly above unity. Use of Eqs. (8) should therefore be limited to values of (ψ'/ψ_c) less than about 3 or, in the terminology of Grant and Madsen (1982), for conditions corresponding to the "equilibrium range." The relatively small difference between the present results for ripple geometry and the predictive relationships by Grant and Madsen (1982) tentatively suggests the use of their relationship for conditions beyond the equilibrium range.

A slightly modified version of the Grant and Madsen (1982) relationship for the equivalent bottom roughness of a movable bed, k_b, as a function of bedform geometry can be used also for multi-component (spectral) waves when these are represented by the periodic wave defined by Eq. (2), i.e.

$$\frac{k_b}{A_{br}} = K \frac{\eta}{A_{br}} \frac{\eta}{\lambda} \qquad (9)$$

is found to apply within the equilibrium range with the constant K, originally given as 28 by Grant and Madsen (1982), being replaced by K = 20, when using their empirical relationships for ripple geometry, and by K = 23, when the relationships given by Eq. (8) are used.

Table 1
Spectral Wave Friction Factors

Frequency (rad/sec)	Amplitude (cm)	Near-bottom velocity (cm/sec)	Friction factor
1.634	1.63	6.22	0.199
2.136	1.65	6.04	0.140
2.388	1.49	5.31	0.231
2.702	1.27	4.36	0.142
2.953	1.26	4.18	0.040
3.393	1.30	3.99	0.222
3.958	1.35	3.68	0.267
6.346	1.13	1.19	0.159

In support of this conclusion results from an "equal component amplitude" spectral simulation experiment are presented in Table 1. With a single exception the friction factors obtained for the different frequency components are remarkably similar (mean value of 0.194 and coefficient of variation 25% when the single exception is excluded). A weighted average of the friction factors listed in Table 1 which preserves total measured energy dissipation (weighting factor u_{bn}^2) gives a value of 0.192 (0.176 if the single exception is not excluded). Corresponding to this wave condition the representative wave, defined by Eq. (2), is characterized by

$$u_{br} = 13.06 \text{ cm/s}, \quad a_{br} = 5.86 \text{ cm}, \quad T_r = 2\pi/\omega_r = 2.23 \text{ sec}$$

and use of the Grant and Madsen (1982) empirical relationships for bedform geometry with K = 20 in Eq. (9) gives a value of $k_b/A_{br} = 0.68$. Similarly, use of Eq. (8) with K = 23 in Eq. (9) results is a value of $k_b/A_{br} = 0.94$. It is important to note here that when the information on relative bottom roughness is used to predict the representative value of the friction factor this must be done using the generalized friction factor relationship given by Grant and Madsen (1982, Figure 3) since this relationship was used (in reverse) when establishing Eq. (9) from experimental values of the friction factor. Thus, with these values of relative roughness use of Figure 3 in Grant and Madsen (1982) yields predicted values of the friction factor of 0.18 and 0.21, respectively, in excellent agreement with the experimentally obtained values. Again, this conclusion is limited by the range of experimental parameters covered, i.e., within the "equilibrium range" of bottom bedforms. Tentatively, however, one may adopt the Grant and Madsen (1982) formulation, with the modification of the form drag term, K = 20 in Eq. (9), as indicated by the present results for conditions beyond the equilibrium range.

SUMMARY AND CONCLUSIONS

The results of carefully conducted experiments have been used to show that a representative periodic wave defined by Eq. (2) as the wave having the same root-mean-square near-bottom orbital velocity and excursion amplitude as a wave motion specified by its frequency spectrum may be used in conjunction with empirically derived relationships for movable bed roughness, to predict the equivalent bottom roughness, k_b, from knowledge of wave and sediment characteristics. With knowledge of k_b for a movable bed, the theory developed in the companion paper (Madsen et al., 1988) may be used to predict the spectral wave attenuation due to bottom friction.

It is, however, emphasized that the above conclusion should be regarded as preliminary until further experiments, covering a wider range of experimental parameters, can support its general validity. Thus, the present experiments were limited to one single sand size and the wave conditions were such that only bottom bedforms within the equilibrium range (as defined by Grant and Madsen, 1982) were generated. Presently the experimental investigation is being continued and extended to a finer sand (0.1-mm diameter) with an improved wave generation capability (larger waves). Both of these modifications and extensions should contribute to the removal of the limitations of the present results.

ACKNOWLEDGMENTS

The research presented here was supported by the U.S. Department of Commerce National Atmospheric and Oceanic Adminstration's Office of Sea Grant under Grant NA86AA-D-SG089 and by the Office of Naval Research under Grant N00014-86-K0325. The expert typing skills of Read Schusky are also acknowledged.

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