CHAPTER 80

Solitary Wave Transmission through Porous Breakwaters

by

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Abstract

A semi-empirical theory is formulated to predict wave reflection and transmission at a porous breakwater of rectangular cross section for normally incident solitary waves. The solution is based on the linearized form of the governing equations and on equivalent linearization of the friction loss in the porous structure.

Experimental results of transmission coefficients are presented for a large range of incident wave amplitudes, with several gravel sizes, water depths and breakwater geometries.

Experimental and theoretical results are compared and evaluated; the comparison shows satisfactory agreement for the transmission coefficient.

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Introduction

When a wave or surge reaches a rubble mound breakwater, it will be partly or totally reflected, depending on the permeability of the structure. Part of the flow may penetrate the porous barrier, and consequently the reflection is only partial. The incident wave energy is, in this way, split into reflected and transmitted. Reflection features determine the accessibility of the harbor trough its influence on the wave energy at the entrance, while transmission characteristics assess the effectiveness of the breakwater in protecting the harbor from the energy of incident waves.

An extensive literature has been reported to model the reflection-transmission features of a porous structure. The main approximations made in the literature are: the incident wave is harmonic and normal to the structure, the breakwater is a homogeneous isotropic medium governed by Darcy or Forchheimer type flow, the structure is of rectangular cross section. Sollit and Cross 1972, presented a summary of analytical approaches up to 1972. Madsen 1974 simplified the analysis presenting a linear long-wave model by assuming a Forchheimer type of flow and a linearized coefficient of friction. An extension of his model including trapezoidal cross section was given by Madsen and White, 1976. A statistical approach for random waves passing a perforated and porous breakwater was given by Massel and Mei, 1977. A generalization of Sollit and Cross work for multilayered structure with arbitrary cross section was presented by Sulisz, 1985. Damping of the incident and reflected waves due to bottom friction was considered in the long wave transmission through porous breakwater approach given by Scarlatos and Singh, 1987.

The long wave assumption made by several authors accounts for severe wave condition for most breakwaters. The governing equations for long waves in shallow water are the Boussinesq equations with the solitary wave as a typical solution. Theoretically, solitary waves have the advantage that, although non-linear, they can be described with two parameters: the wave height and the depth: experimentally they propagate with constant shape in constant depth and generally they can be separated from reflected waves. The objective of this study is to develop a semi-empirical model to the leading order for the motion of an incident solitary wave, the reflected wave and transmitted wave resulting from the presence of a normal rectangular cross section porous breakwater. This model will be checked with experimental tests. We assume first, that following ratios are small:

$$ak \ll 1 \quad kh \ll 1 \quad kb \ll 1 \quad O(b) \leq O(h)$$

in which a=wave amplitude, k=wave number, h=depth and b=breakwater width.

Following the procedure used by Mei, Liu and Ippen, 1974 for periodic long waves impinging on slotted or perforated breakwater and also used by Mei, 1983 for solitary waves climbing onto a shelf, three separate regions can be distinguished, within each of which different physical processes are dominant. In the evolution region, which is many wave lengths away from the breakwater, dispersion and nonlinearity are important and Boussinesq equations must be used. In the intermediate region roughly one wave length away from the breakwater, nonlinearity and dispersion are of second order in wave slope. In the neighborhood of the breakwater, where the typical distance is b, the loss friction effects are important but the apparent inertia due to local acceleration is not.

Theoretical formulation

Within the intermediate region, one-dimensional linearized equations are sufficient to the leading order:

$$\eta_t + h\, u_x = 0 \qquad (1)$$

$$u_t + g\, \eta_x = 0 \qquad (2)$$

the governing equations for the motion inside the porous rubble-mound are

$$n\, \eta_t + h\, u_t = 0 \qquad (3)$$

$$(1/n)\, u_t + g\, \eta_x + (C_1\, \mu/\rho d^2 n)\, u + (C_2/d\, n^2)\, u^2 = 0 \qquad (4)$$

where C_1 and C_2 account for the laminar and turbulent friction loss respectively and n is the porosity of the structure. The effect of friction loss is localized within O(h) or O(b) and can be represented as a boundary condition:

$$\eta_a - \eta_b = \frac{C_1\ \mu}{d^2\ n\ g\rho}\, b\, u_b + \frac{C_2}{d\ g\ n^2}\, b\, u_b^2 \qquad (5)$$

where subscripts $()_a$, $()_b$ indicate upstream and downstream respectively, and the apparent inertia (local acceleration) has been ignored in accordance with $kb \ll 1$.

As in the theory for perforated breakwater and for the scattering of a solitary wave at a sudden depth change and equivalent linear condition

$$\eta_a - \eta_b = c_a\, u_b \qquad (6)$$

such that the total square error

$$e = \frac{C_1 \ \mu}{d^2 \ n \ g \rho} \ b \ u_b + \frac{C_2}{d \ g \ n^2} \ b \ u_b^2 - c_a \ u_b \qquad (7)$$

is minimized over the wave period

$$\overline{e^2} = \int_{-\infty}^{\infty} e^2 \ dt$$

the minimum occurs when $\delta\overline{e^2}/\delta c_a = 0$. The optimum value of c_a is:

$$c_a = \frac{C_1 \ \mu}{d^2 \ n \ g \rho} \ b + \frac{C_2 \ b}{d \ g \ n^2} \ \frac{\overline{u_b^3}}{u_b^2} \qquad (9)$$

we assume the incident wave is given by the solitary wave

$$n_1 = a_1 \ \text{sech}^2 \ \lambda_1 \ (x - ct)$$

in which

$$\lambda_1 = (3 \ a_1 \ / \ 4 \ h^3)^{1/2}$$

$$c = (g \ h)^{1/2} \ (1 + a_1/2 \ h) \approx (g \ h)^{1/2}$$

Experimentally it has been observed that the transmitted and the reflected waves are similar in shape (sech^2 profiles) but not solitary waves, that is:

$$n_a = a_1 \ \text{sech}^2 \ \lambda_1 \ (x - c \ t) + a_1 \ R \ \text{sech}^2 \ \lambda_1 \ (x + c \ t) \qquad (13)$$

$$n_b = a_1 \ T \ \text{sech}^2 \ \lambda_1 \ (x - c \ t + \theta) \qquad (14)$$

where R, T represent the reflection and transmission coefficients and θ is a phase shift. To the leading order, this phase shift is negligible when kb << 1. The corresponding horizontal velocities are:

$$u_a = (g \ a_1/c) \left[\text{sech}^2 \ \lambda_1 \ (x - c \ t) - R \ \text{sech}^2 \ \lambda_1 \ (x + ct) \right] \qquad (15)$$

$$u_b = (g a_1 T \ / \ c) \ \text{sech}^2 \ \lambda_1 \ (x - c \ t) \qquad (16)$$

We remark that the experimental assessment of constant λ_1, can also be deduced from no phase shift and Eq. 1 in order to continuity to be satisfied for all t.

With these velocities, the coefficient c_w is:

$$c_w = \frac{C_1 \ \mu}{d^2 \ n \ g \rho} \ b + \frac{C_2 \ b}{d \ \ n^2} \ \frac{a_1 \ T}{c} \ \frac{4}{5} \tag{17}$$

Substituting Eq (13), (14), (16) into the boundary condition Eq (6), we obtain

$$1 + R = T \ (1 + \frac{c_w \ g}{c}) \tag{18}$$

Condition Eq 1 implies for the continuity of mass flux

$$1 - R = T \tag{19}$$

Combining Eq (18) with Eq (19), we obtain

$$T = \left[1 + \frac{c_w \ g}{2 \ c} \right]^{-1} \tag{20}$$

$$R = \left[\frac{c_w \ g}{2 \ c} \right] \left[1 + \frac{c_w \ g}{2 \ c} \right]^{-1} \tag{21}$$

the structure of Eq 20 also appears in the scattering of sinusoidal waves by perforated breakwater, in the scattering of solitary waves at a sudden depth change or in the scattering of sinusoidal waves by a porous breakwater.

Experimental Tests.

The tests were carried out in the 68 m long, 2 m wide, and 2 m deep wave channel of the water sciences Department of the University of Cantabria. In a length of 20 m, the flume has glass walls to allow visual observations. The incident solitary waves were generated with the procedure sketched by Goring, 1978. With this procedure, roughly one wave length away from the paddle the volume of water pushed by the paddle resembles a solitary wave.

As the solitary wave propagates on the horizontal smooth concrete bottom the frequency dispersion balances the non-linear effects and the waves remain stable without appreciable change. Only side wall and bottom friction cause a decrease of the wave amplitude. The measured profiles of the damped waves at the model test (47 m away from the paddle) are closely represented by the theoretical profile of Boussinesq, 1871. The agreement of both profiles is better on the upper 3/4 (Fig. 1).

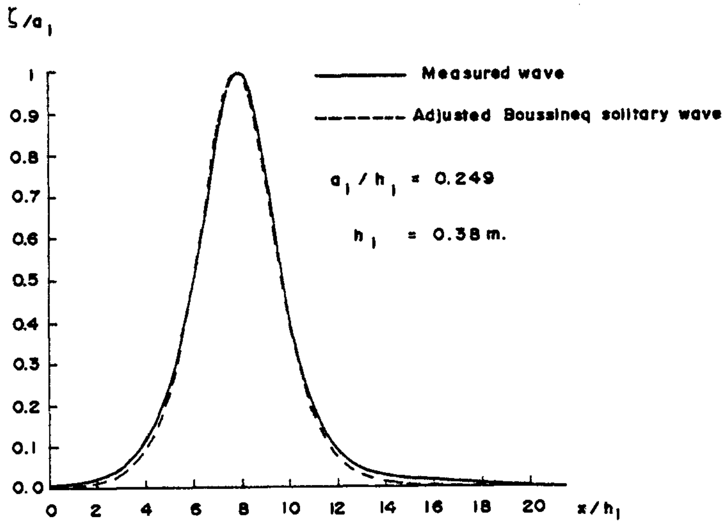

Fig. 1.- Recorded solitary wave and adjusted Boussinesq

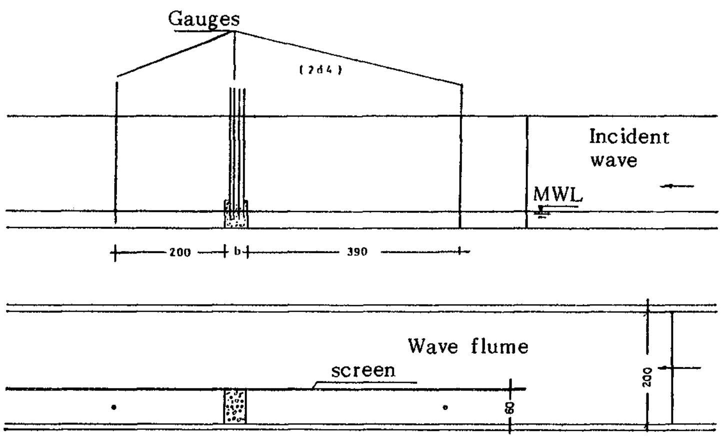

Fig. 2.- Experimental arrangement

TABLE I

TEST PARAMETERS AND EXPERIMENTAL RESULTS

$^{*}D_{50}$	$^{*}h$	$^{*}b$	$^{*}a_i$	T
1.43	30.0	20.0	1.92	0.45
1.43	30.0	20.0	4.28	0.38
1.43	30.0	20.0	7.91	0.31
1.43	30.0	40.0	1.97	0.34
1.43	30.0	40.0	4.41	0.29
1.43	30.0	40.0	7.97	0.21
1.43	24.6	40.0	1.91	0.30
1.43	24.6	40.0	4.29	0.24
1.43	24.6	40.0	7.64	0.19
2.43	30.2	20.0	1.87	0.57
2.43	30.2	20.0	4.27	0.45
2.43	30.2	20.0	7.07	0.38
2.43	30.2	20.0	9.39	0.34
2.43	25.8	20.0	1.90	0.55
2.43	25.8	20.0	4.40	0.43
2.43	25.8	20.0	7.05	0.34
2.43	25.8	20.0	9.49	0.32
2.43	25.8	20.0	11.46	0.29
2.43	31.7	40.0	1.95	0.50
2.43	31.7	40.0	4.63	0.34
2.43	31.7	40.0	7.64	0.28
2.43	31.7	40.0	10.21	0.25
2.43	24.9	40.0	2.19	0.33
2.43	24.9	40.0	4.74	0.29
2.43	24.9	40.0	7.65	0.23
2.43	24.9	40.0	10.31	0.21
2.43	24.9	40.0	12.37	0.19
3.15	30.1	40.0	2.01	0.46
3.15	30.1	40.0	4.38	0.34
3.15	30.1	40.0	7.22	0.28
3.15	30.1	40.0	9.65	0.25
3.15	25.0	40.0	1.84	0.43
3.15	25.0	40.0	4.23	0.33
3.15	25.0	40.0	6.87	0.26
3.15	25.0	40.0	9.02	0.25
3.15	25.0	40.0	10.78	0.21
3.15	30.1	20.0	1.99	0.57
3.15	30.1	20.0	4.53	0.46
3.15	30.1	20.0	7.38	0.39
3.15	30.1	20.0	9.96	0.37
3.15	25.1	20.0	1.86	0.56
3.15	25.1	20.0	4.36	0.46
3.15	25.1	20.0	6.83	0.38
3.15	25.1	20.0	9.30	0.35
3.15	25.1	20.0	11.18	0.34

* centimetres

The permeable structures tested had rectangular form and their characteristics are shown in table I. The tests were carried out with gravel of D_{50}=1.43, 2.43 cm and small cubic blocks of 3.15 cm side length. The measured porosities were n=0.44 for gravel and n=0.42 for cubes. The structures were 20 to 40 cm wide and water depth varied from 25 to 30 cm.

A wide range of incident wave amplitudes were tested so that the parameter a_i/h_o range was $0.05<a_i/h_o<0.5$. Note that this last parameter (a_i/h_o=0.5) violates the small wave amplitude hypothesis.

The experimental arrangement is shown in figure 2. As the flume width was excessive for the test requirements it was divided longitudinally by a screen so that the model covered a width of only 60 cm. Within this new narrow channel all the measuring equipment was placed as shown in figure 2. Upstream and downstream the model, two gauges were placed in order to measure the incident and the transmitted wave. Inside the permeable structure the pressure and water level were measured at several points depending on the structure width. A complete documentation of the experimental program is presented by Medina, 1988.

The experimental results of transmission coefficients for the different models are presented in table I. The transmission coefficient decreases when the structure width increases, the incident wave amplitude increases and the gravel size decreases. The influence of the water depth on the transmission coefficient is less important, and the transmitted wave amplitude decreases when the water depth decreases. Though the cubic block size is much larger then the gravel size, the transmission coefficient is slightly larger due to the lower porosity.

Comparison of theory and experiment.

For the experimental coefficients C_1, C_2,(Eq.17) we take the expressions given by Muskat,1946,also used by Engelund, 1953 and Madsen, 1974.

$$C_1 = \alpha \ (1-n)^3 \ / \ n$$

$$C_2 = \beta \ (1-n) \ / \ n$$

where $\alpha = 1092$, $\beta = 0.81$

The comparison of experimental transmission coefficient, T, and predicted transmission coefficient, T_t, for all the data given in table I is presented in fig. 3. The line T_t=T is also drawn. As is evident from the comparison in fig. 3, the difference is small and the predicted transmission gives a reasonable representation of experimental data. Note that the experiments correspond to values of the parameter $0.05<a_i/h<0.5$ thus the agreement is good even for not too small wave amplitude ratio.

The influence on the transmission coefficient due to the grain size, incident wave amplitude, water depth and breakwater width is represented in figures 4-5 in which theoretical (lines) and measured (symbols) coefficients are drawn.

Figure 4 shows the influence of the breakwater width on the transmission; the transmission decreases when the width increases. Figure 5 shows the influence of the incident wave amplitude; the transmission decreases when the a_i/h ratio increases, we can also observe the agreement between predicted and measured coefficients for a wide range of the ratio a_i/h ($0.05<a_i/h<0.5$).

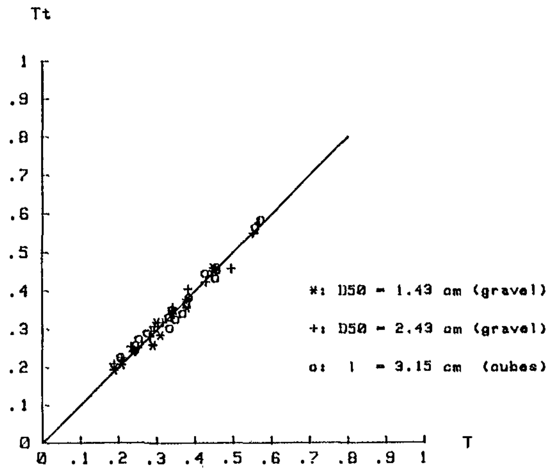

Fig. 3
Experimental versus Theoretical transmission

Summary and Conclusions

Based on a semi-empirical formulation, a simple solution for the reflection and transmission coefficients for a given incident solitary wave has been derived. This solution was based on the leading order of the governing equations and a linearized form of the friction loss in the porous structure. The linearization procedure was a slight modification of the Lorentz condition for sinusoidal waves. The coefficients depend on the geometry of the structure the incident wave amplitude, the water depth and on the properties of the porous medium (porosity and grain diameter).

Experimental results of transmission coefficients have been reported for a wide range of the involved parameters. These results have been compared with the predicted values showing a satisfactory agreement for the transmission coefficient.

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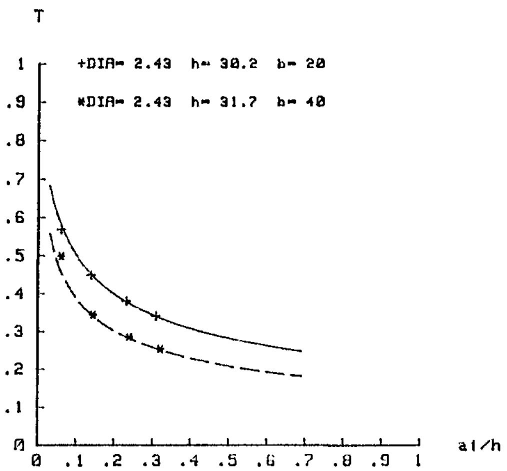

Fig. 4.- Influence of breakwater width on the transmission

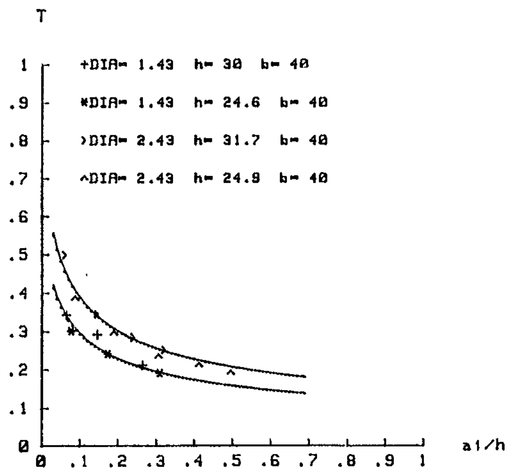

Fig. 5.- Influence of incident wave amplitude on the transmission

LIST OF REFERENCES.

Boussinesq, M.J. 1871; "Théorie de l'intumescence liquide appelée onde solitaire ou de translation se propagant dans un canal rectangulaire", C.R. Aca. Sci. 72:755.

Engelund, F. (1953); "On the laminar and turbullent flows of ground water through homogeneus sand", Trans. Danish Acad. Tech. Sci. 3(3).

Goring, D.G. (1978);"Tsunamis- the propagation of long waves onto a sehlf", Rep N. KH-R-38, W.M. Keck Laboratory of hydraulics and water Resources, California Institute of Technology.

Madsen, O.S. 1974; "Wave transmission through porous structures", ASCE, J.W! Harbors Coastal Eng. Div., 100 (WW3):169-188.

Madsen, O.S. and White, S.M. 1976; "Energy dissipation on a rough slope", ASCE, J.W. Harbors Coastal Eng. Div., 102 (WW1):31-48.

Massel, S.R. and Mei, C.C. 1977; "Transmission of random wind waves through perforated or porous breakwater", Coastal Eng., 1:63-78.

Medina, R. 1988; "Sobre la transformación de ondas solitarias. Propagación en un escalón y en medio poroso", Tesis doctoral, Departamento de Ciencias y Técnicas del Agua y del Medio Ambiente, Univ. de Cantabria.

Mei, C.C. Liu, P.L.F. Ippen, A.T. (1974); "Quadratic head loss and scattering of long waves", J. Waterway Harbors Coastal Eng. Div. Proc. ASCE 100, 217-239.

Mei C.C. (1983) "The applied dydamics of Ocean surface waves", J. Wiley and sons Inc. N.Y.

Muskat, M. (1946); "The flow of homogeneous fluids through porous media", J.W. Edwards, Inc. Ann Arbor, Michigan.

Scarlatos, P.D. and Singh, V.P. 1987; "Long wave through porous breakwaters", Coastal Eng. 11:141-157.

Sollit, C.K; Cross, R.H. (1972);"Wave transmission through permeable breakwater", Proc. XIII Coastal eng. Cong. ASCE, Vancouver, B.C. Canadá.

Sulisz, W.(1985); "Wave reflection and transmission at permeable breakwaters of arbitrary cross-section", Coastal Engineering, 9:pp 371-386.