CHAPTER 81

TIME-DEPENDENT WAVE SHEAR STRESS

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ABSTRACT

A knowledge of bed shear stress induced by waves is required to understand dynamic processes of nearshore morphologies as a results of sediment transport. However, the information on the stress is still incomplete due to lack of measured data. The study analyzes the unsteady horizontally averaged shear stresses measured over mobile beds in a water tunnel. It is found from the analysis that the presence of the third and fifth harmonics in the shear stress is in good agreement with the measured concentration of suspended sediments.

INTRODUCTION

Magnitude and phase of bed shear stress induced by waves over mobile bed depend on characteristics of waves and wave induced ripples on the bed. The magnitude of bed shear stress can be computed from known wave friction factor over mobile bed, Grant and Madsen (1982) and Vongvisessomjai (1987). Lofquist (1986) provided experimental results of the time-dependent drags on naturally rippled sand beds tested in a water tunnel which were obtained from the measured difference between pressure gradients in the test section split longitudinally into two channels, one containing the sand bed and the other having a smooth metallic bottom. Two sands were used, with diameters 0.55 and 0.18 mm. The bottom stress, including skin friction and profile drag components was found to be a fairly complicated function of the phase and not proportional to the square of the simultaneous sinusoidal velocity far above the bed. Average values of the bottom stress and coefficient of energy dissipation were both typically an order to magnitude greater than for a flat surface roughened with fixed grains. The wave friction factors on mobile beds f_{ws}, computed from the coefficient of energy dissipation $\bar{f}_1$ of Lofquist (1986) with the conversion $f_{ws} = (3\pi/4)\ \bar{f}_1$, were found to have the same order of magnitudes as those obtained by Vongvisessomjai (1987).

The time-dependent wave shear stresses obtained by Lofquist (1986) are analyzed and quantified for the phases, and the obtained friction factors together with the earlier results are used to develop a new expression.

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METHODOLOGY

The velocity field over the self-formed 2 dimensional ripple of $h = \eta \cos(2\pi x/\lambda)$ can be expressed as a sum of the fundamental component (n = 0) and its higher harmonics (n = 1, 2, 3...) caused by the presences of eddies or turbulences generated from the interaction of the fundamental component and the ripples as

$$u(x,z,t) = \sum_n \varepsilon^n u_n(x,z,t) \quad \text{.......(1)}$$

where ε = a small parameter.

The corresponding shear stresses along the ripple bed can be expressed as

$$\tau_o(x,t) = \sum_n \varepsilon^n \tau_{on}(x,t) \quad \text{.......(2)}$$

where subscript o denotes the bed, z = 0.

These velocity field, shear stresses and eddies will cause sediment suspension in the form

$$c(x,z,t) = \underline{c}(x,z) + \sum_n \varepsilon^n c_n(x,z,t) \quad \text{.......(3)}$$

These unsteady two dimensional velocity, the unsteady shear stress along the ripple and the suspended sediment concentration are too complicated to be measured and quantified. Only some measurements are presently available for horizontally averaged values of the above parameters which can be expressed respectively as

$$\bar{u}(z,t) = \frac{1}{\lambda}\int_o^\lambda u(x,z,t)dx = \sum_n \varepsilon^n \bar{u}_n(z,t) \quad \text{.......(4)}$$

$$\bar{\tau}_o(t) = \frac{1}{\lambda}\int_o^\lambda \tau_o(x,t)dx = \sum_n \varepsilon^n \bar{\tau}_{on}(t) \quad \text{.......(5)}$$

$$\bar{c}(z,t) = \bar{c}(z) + \sum_n \varepsilon^n \bar{c}_n(z,t) \quad \text{.......(6)}$$

The following analysis is based on the approach of Lofquist (1986) in analyzing his experimental results. The freestream velocity of flow above the bed and beyond its effects is

$$u_\alpha = \left(\frac{2\pi a}{T}\right) \sin\left(\frac{2\pi t}{T}\right) = U \sin\theta \quad \text{.......(7)}$$

where t is the time T is the period and 2a is the orbital diameter of the water motion. Equation (7) defines the maximum velocity, U, and the phase, θ. The bed surface profile, assumed two dimensional and periodic with length λ, has elevation $h(x,\theta)$, where x is the horizontal coordinate. The average bottom stress is then defined by

$$\bar{\tau}_o(\theta) = \frac{1}{\lambda}\int_o^\lambda [\tau_h + (p_h - p_\alpha)\frac{\partial h}{\partial x}]\, dx \quad \text{.......(8)}$$

where τ_{η} is the local tangential stress on the bed, and $(p_h - p_\alpha)$ is the difference between pressures at the bed surface and at any fixed elevation beyond the influence of the bed. Separate integrals for the two terms in the integrand would resolve $\tau_o(\theta)$ into components expressing the effects of tangential and normal stresses, the latter being a form drag. However, such a separation is generally impossible. A stress coefficient has been made dimensionless for bottom stress by dividing by ρU^2, rather than by $\rho u_\alpha^2(\theta)$, in order to keep it proportional to the bottom stress throughout the cycle where ρ is the density of water,

$$f(\theta) = \frac{2\bar{\tau}_o(\theta)}{\rho U^2} = f_s(\theta) + f_n(\theta) \qquad \text{.......(9)}$$

The dimensionless local tangential stress over the smooth boundary for laminar flow is

$$f_s(\theta) = \beta_4 \sin(\theta + \pi/4) \quad \text{where} \quad \beta_4 = \sqrt{\frac{\nu}{a^2\omega}} \qquad \text{.......(10)}$$

and the dimensionless normal stress is

$$f_n(\theta) = \beta_1 \bar{D}(\theta) \qquad \text{where} \quad \beta_1 = \frac{2H}{\rho L U^2} \qquad \text{.......(11)}$$

in which $\omega = 2\pi/T$; H = height of pressure taps from bed; and L = distance between pressure taps.

The measured differential pressure $\bar{D}(\theta)$ is found to be the summation of the fundamental harmonic $D_1(\theta)$ and its higher odd harmonics as

$$\bar{D}(\theta) = D_1(\theta) + D_3(\theta) + D_5(\theta) + \ldots \qquad \text{.......(12)}$$

It was found from the experiments that there existed a shift of the mean level of ripple bed from the initial bed of $\bar{h}$, a correction of $2\bar{h}/a = \beta_3$ is made to Eq. 9 as

$$f(\theta) = \beta_1\bar{D}(\theta) - \beta_3 \cos\theta + \beta_4 \sin(\theta + \pi/4) \qquad \text{.......(13)}$$

ANALYSIS AND RESULTS

Expressions of friction factor on fixed bed are first summarized for comparison with the obtained friction factor on mobile beds from this analysis. The friction factor can then be used to compute the magnitude of the maximum bed shear stress. The analysis of the phase of the shear stress is finally made.

Friction Factor of Fixed Bed.- Jonsson (1966) defined the wave friction factor $f_w = 2\,\bar{\tau}_{om}/(\rho U^2)$ where subscript m denotes the maximum value and expressed it in terms of the relative smoothness of the bed $k_* = a/k_s$ as

$$\frac{1}{4\sqrt{f_w}} + \log\frac{1}{4\sqrt{f_w}} = -0.08 + \log k_* \qquad \text{.......(14)}$$

The above expression was later confirmed by two additional sets of tests in a water tunnel by Jonsson and Carlsen (1976).

Kamphuis (1975) reanalyzed the measured values of the friction factors obtained by Riedel, Kamphuis and Brebner (1972) and proposed an expression for the friction factor also in terms of k_* as

$$\frac{1}{4\sqrt{f_w}} + \log \frac{1}{4\sqrt{f_w}} = -0.35 + \frac{4}{3} \log k_* \quad \text{.......(15)}$$

Grant and Madsen (1979) derived an expression of friction factor for combined waves and currents in fully rough turbulent flow. A relationship between f_w and k_* could be obtained for the limiting condition of a pure wave motion as

$$f_w = 0.08/[\mathrm{Ker}^2 2(\zeta_c)^{1/2} + \mathrm{Kei}^2 2(\zeta_o)^{1/2}] \quad \text{for } k_* > 1 \quad \text{.......(16)}$$

and $f_w = 0.23$ for $k_* < 1$; in which the dimensionless roughness length $\zeta_o = k_s/(30\ell)$; the characteristic length $\ell = 0.4\, u_*/\omega$; the shear velocity $u_* = \sqrt{f_w/2}U$; Ker and Kei = Kelvin functions of zero order.

Vongvisessomjai's (1984) expression of wave friction factor

$$f_w = 0.287\, k_*^{-2/3} \quad \text{.......(17)}$$

Friction Factor of Mobile Bed.- Grant and Madsen (1982) presented a model to predict the roughness in unsteady oscillatory flows over mobile beds using data of Carstens et al. (1969). The total roughness was the sum of the form drag component and the sediment transport component:

$$\frac{1}{k_*} = 28\,\frac{\eta}{a}\,\frac{\eta}{\lambda} + 160(s+K)\,\frac{D}{a}\left[\left(\frac{\psi}{\psi_c}\right)^{1/2} - 0.7\right]^2 \quad \text{.......(18)}$$

in which K = 1/2 for spherical sand; ψ = Shields parameters; and ψ_c = critical value of Shields parameter for initiation motion. Based on the law-of-the-wall, the expression of friction factor of the fixed bed, Eq. 16, was adopted for that of the mobile bed using k_* of Eq. 18.

Profiles of mean and unsteady concentration were derived by Vongvisessomjai (1986) from the mass conservation equation using the diffusion coefficient profile adapted from the eddy viscosity profile of Vongvisessomjai (1984) taking into account changes of the friction factor and boundary layer thickness in the presence of suspension. The theoretical profiles were expressed explicitly as functions of the friction factor, velocity and shear stress profile parameters, and the settling velocity of the sediment relative to the fluid velocity. The tabulated data available and the new experimental data of the mean concentration were used to fit the theoretical profile. The ratio of the friction factor f_{ws}/f_w obtained was then correlated to the densimetric sediment Froude number $Fd_* = U^2/[(s-1)gD]$ and the relative bed smoothness $a_* = a/D$ as

$$\frac{f_{ws}}{f_w} = 0.866\, Fd_*^{-0.50}\, a_*^{0.17} \qquad \text{for fine sand}$$

or

$$f_{ws} = 0.185\ Fd_*^{-0.50}\ a_*^{0.17} \qquad \text{for } f_w \doteq 0.214 \qquad \ldots\ldots\ldots\ldots(19)$$

Vongvisessomjai (1987) based on his friction factors obtained from suspended sediment profile data and those of Carstens et al. (1969) expressed f_{ws} as function of Fd_*, a_* and $D_* = (s-1)gD^3/\nu^2$.

$$f_{ws} = 0.049\ Fd_*^{-0.44}\ a_*^{0.21}\ D_*^{0.22} \qquad \ldots\ldots\ldots\ldots(20)$$

The data used for the present analysis are from Carstens et al. (1969), Lofquist (1986) and Vongvisessomjai (1986). Table 1 summarizes the sources and characteristics of the data. Note that Vongvisessomjai (1986) used various concentration profile data, D = 0.14 mm from Nakato et al. (1977) and Kennedy and Locher (1972), D = 0.18 mm from Hom-ma et al. (1965) and D = 0.21 mm for his own data.

TABLE 1.-Sources and Characteristics of Data of Wave Friction Factor

Source (1)	T (s) (2)	a (cm) (3)	D (mm) (4)	$k_* = a/(4\eta)$ (5)	f_{ws} (6)	Symbol (7)
Carstens	3.55	8-31	0.19	1.2-15	0.077-0.20	◐
	3.55	8-45	0.30	1.0-23	0.070-0.38	⊖
	3.55	10-42	0.59	1.0- 8	0.193-0.54	○
Lofquist	4-15	14-55	0.18	1.3-3.2	0.099-0.22	◫
	2-11	17-52	0.55	0.8-1.6	0.128-0.42	□
Author						
a) Author	2.00	9.55	0.21	1.77	0.083	▲
	1.00	3.18		1.14	0.132	
	1.00	6.37		1.59	0.059	
	1.00	9.55		1.59	0.053	
b) Nakato	2.40	10.20	0.14	1.41	0.137	▽
	1.80	7.50		1.56	0.107	
	1.20	5.10		1.42	0.079	
c) Kennedy	1.00	4.82	0.14	2.19	0.085	▽
d) Hom-ma	1.30	5.29	0.18	1.51	0.087	△
	1.60	5.27		1.41	0.125	
	1.60	7.10		1.73	0.080	
	1.60	9.12		2.05	0.071	
	1.72	6.75		1.63	0.104	
	1.72	9.25		2.02	0.086	

The friction factors of the mobile beds will first be compared with those expression for fixed beds, Eqs. 14-17. This depends on how to define the roughness of the bed, k_s, or the relative smoothness of the bed, $k_* = a/k_s$:

i) Madsen and Grant (1976) used $k_s = D$ and $k_* = a/D$;
ii) Kamphuis (1975) used $k_s = 2D$ and $k_* = a/(2D)$;
iii) Horikawa and Watanabe (1967) used $k_s = 4\eta$ and $k_* = a/(4\eta)$;
iv) Swart (1976) used $k_s = 25\eta(\eta/\lambda)$; and
v) Grant and Madsen (1982) used $k_s = 28\eta(\eta/\lambda)$.

Note that $k_s = 25\eta(\eta/\lambda) \doteqdot 4\eta$ when $\eta/\lambda = 1/6$ for growing ripple or equilibrium ripple.

Figs. 1a) and 1b) show the comparison plots of friction factors of the mobile beds with those expression for fixed beds, Eqs. 14-17, using the relative smoothness of the bed a) $k_* = a/[25\eta(\eta/\lambda)]$ and b) $k_* = a/D$ respectively. It can be seen in Fig. 1a) that the four expressions of friction factors for fixed beds have about the same magnitudes but mostly overestimate the measured values when assuming b) $k_* = a_* = a/D$.

The new expression of f_{ws} when data of Lofquist (1986) with the conversion $f_{ws} = (3\pi/4)\ \bar{f}_1$ are added to data of Carstens et al. (1969) and Vongvisessomjai (1986) is

$$f_{ws} = 0.043\ Fd_*^{-0.31}\ a_*^{0.16}\ D_*^{0.21} \qquad \text{......(21)}$$

From the above f_{ws}, the Shields parameter which is important to describe the sediment transport can be determined as

$$\psi = \frac{1}{2}\ f_{ws}\ Fd_* = 0.022\ Fd_*^{0.69}\ a_*^{0.16}\ D_*^{0.21} \qquad \text{......(22)}$$

Since the above friction factor of the mobile bed f_{ws} depends on three dimensionless parameters (D_* Fd_* and a_*) its dependence on these three parameters as well as $\psi_r = Fd_*/a_*^{1/2}$ and D_* or the diameter of the sediment D are shown in Fig. 2. The 3 diameters of fine, medium and coarse sands of Carstens et al. (1969) are used as the representatives in Fig. 2a) while in Fig. 2b) the 6 and 3 densimetric Froude numbers (Fd_*) respectively of the coarse sand and the fine sand of Lofquist (1986) are used as the representatives. Also included in Fig. 2b) are the 9 curves obtained by Lofquist (1986) with the conversion $f_{ws} = (3\pi/4)\ \bar{f}_1$ which show how his curves are smoothed to the present expression, Eq. 21. The additional plot of f_{ws} versus a_* of Carstens et al. (1969) and Vongvisessomjai (1986) has been presented in Fig. 1b).

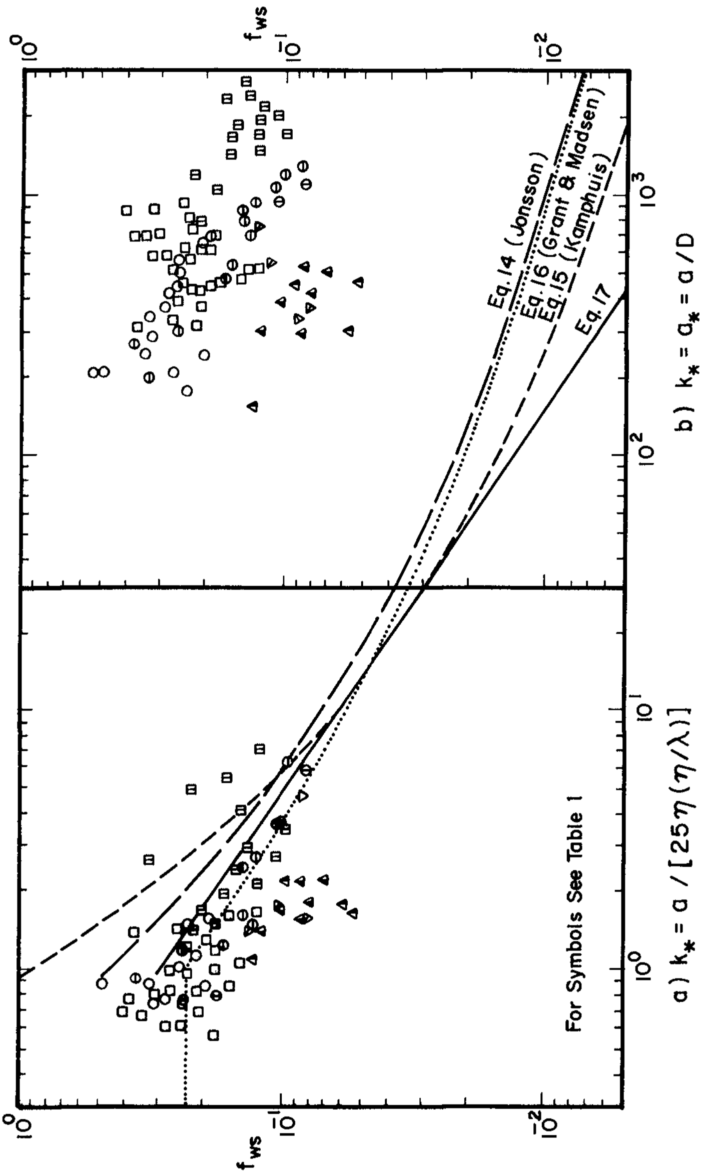

Fig. 1.- Comparison of Measured Friction Factors on Mobile Beds with Expressions of Friction Factors on Fixed Beds

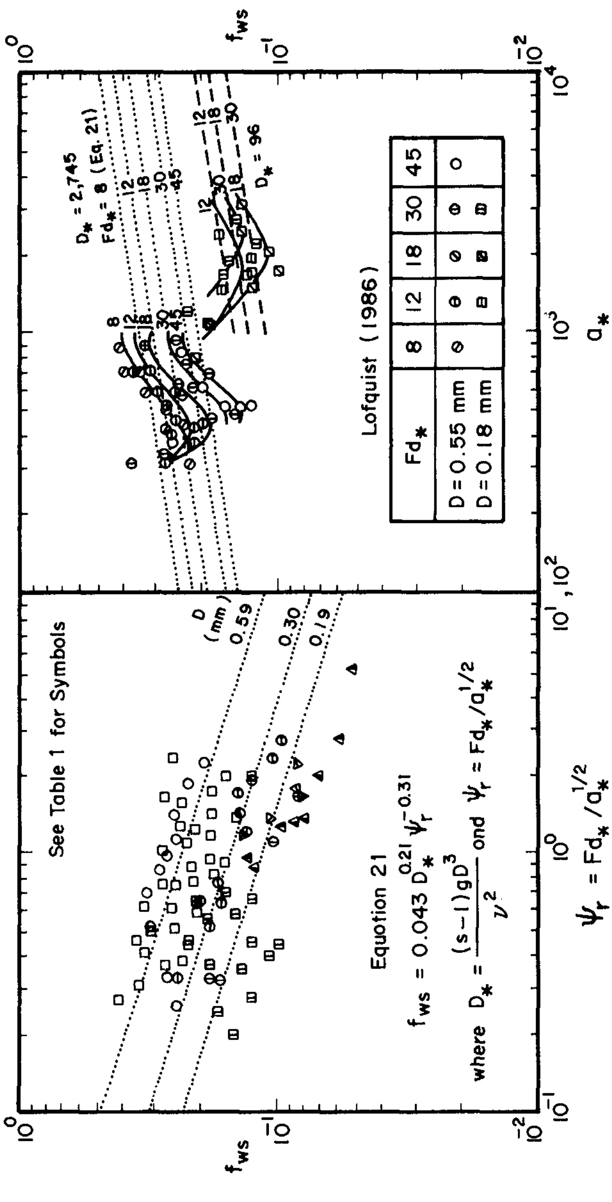

FIG.2 – Comparison of Measured Friction Factors on Mobile Beds with Equation 21

Phase of Bed Shear Stress.- Lofquist (1986) presented graphically the phase variations of dimensionless shear stress $f(\theta)$ of Eq. 13 for all experimental results for coarse sand and fine sand. Lofquist kindly provided all parameters used in plotting the graphs to the author for further quantification of the contributions by the fundamental harmonic and its higher harmonics. Tables 2 and 3 summarize results of coarse and fine sand experiments respectively. It can be seen from the tables that values of $-\beta_3$ (correction due to the presence of ripples on bed) and β_4 (tangential stress over smooth boundary) are small compared with $\beta_1\bar{D}(\theta)$ (normal stress).

The dimensionless normal stresses $\beta_1\bar{D}(\theta)$ of Eq. 13 for Run Nos. C28, C13 and C01 are plotted in **Fig.** 3. Run No. C28 is an example of high contributions of the third and the fifth harmonics where $D_{3m}/D_{1m} = 1.1733$ and $D_{5m}/D_{1m} = 0.5352$; Run No. C13 is an examole of high $D_{3m}/D_{1m} = 0.5421$ and low $D_{5m}/D_{1m} = 0.0746$ while Run No. C01 is an example of low $D_{3m}/D_{1m} = 0.0581$ and high $D_{5m}/D_{1m} = 0.2503$ where subscript m denotes the maximum value. It can be seen from these three examples that the resultant normal stresses have two peaks in half a cycle $\theta = \pi$ which imply that they have four peaks in a cycle $\theta = 2\pi$. The measured suspended sediment concentration by Hom-ma et al. (1965) and Nakato et al (1977) also exhibited four peaks in a cycle which could be quantified by the theory proposed by Vongvisessomjai (1986).

The densimetric sediment Froude number Fd_*, the relative bed smoothness a_* and the densimetric sediment Froude-Reynolds number D_* which have been used in correlating with f_{ws} are then used in correlating with D_{3m}/D_{1m} and D_{5m}/D_{1m} listed in Tables 2 and 3. The results obtained are

$$\frac{D_{3m}}{D_{1m}} = 2{,}670\ Fd_*^{1.50}\ a_*^{-1.68}\ D_*^{-0.44} \qquad \text{.............................(23)}$$

$$\frac{D_{5m}}{D_{1m}} = 1.49\ Fd_*^{0.60}\ a_*^{-0.64}\ D_*^{0.003} \qquad \text{.............................(24)}$$

The wave friction factor $f_{ws} = 2\ \tau_{om}/(\rho U^2)$ is the maximum of the dimensionless shear stress $f(\theta)$ which is mainly contributed by $D_1(\theta)$, $D_3(\theta)$ and $D_5(\theta)$. Using results of f_{ws} and $\beta_1\ D_{1m}$ listed in Tables 2 and 3 for coarse sand and fine sand respectively, a relationship is found

$$f_{ws} = 1.13\ (\beta_1\ D_{1m}) \qquad \text{..(25)}$$

The fundamental harmonic of the normal stress is the main contribution to the resultant stress, only 13 percent is contributed by higher harmonics and other factors.

The obtained results from this study could be used to further advance on the analytical descriptions of time-dependent sediment transport and suspension.

TABLE 2.-Results of Coarse Sand Experiments (D_*= 2,745)

Run	f_{ws}	$\beta_1 D_{1m}$	D_{3m}/D_{1m}	D_{5m}/D_{1m}	β_1x100	$-\beta_3$x100	β_4x100	Fd_*	a_*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C01	0.2853	0.2354	0.0581	0.2503	1.472	5.079	0.768	8.21	435
C02	0.2177	0.1794	0.3750	0.1058	1.245	5.079	0.642	18.50	435
C03	0.2483	0.2006	0.1271	0.2545	1.261	5.079	0.710	12.31	435
C04	0.2467	0.2080	0.1069	0.1399	1.354	4.761	0.686	12.35	464
C05	0.2580	0.1934	0.1680	0.1773	1.222	5.540	0.741	12.34	399
C06	0.2106	0.1940	0.2233	0.1713	1.233	5.917	0.766	12.14	374
C07	0.2641	0.2230	0.0514	0.1473	1.223	4.385	0.659	12.36	504
C10	0.2182	0.2292	0.2037	0.1638	1.121	1.475	0.924	8.19	316
C11	0.2691	0.2342	0.3945	0.1482	1.029	2.224	0.835	12.30	316
C12	0.2832	0.2421	0.5674	0.0730	1.367	2.961	0.755	18.35	316
C13	0.2773	0.2598	0.5421	0.0746	1.430	3.698	0.755	18.33	316
C15	0.2552	0.2144	0.0504	0.0950	1.242	0	0.615	12.31	580
C16	0.2891	0.2445	0.0245	0.1412	1.262	0.100	0.616	12.29	580
C17	0.3150	0.2660	0.0580	0.1371	1.323	0.194	0.680	8.25	580
C18	0.2825	0.2330	0.0627	0.1527	1.053	0.294	0.556	18.47	580
C19	0.2396	0.2068	0.5845	0.3035	1.728	0.388	0.491	30.46	580
C22	0.3477	0.2930	0.0861	0.1382	1.388	0	0.621	8.24	697
C23	0.3513	0.2962	0.0708	0.0903	1.449	0	0.733	12.37	697
C24	0.3141	0.2721	0.0224	0.1390	1.428	0	0.507	18.51	697
C25	0.1871	0.1766	0.2487	0.1985	1.580	0	0.507	30.79	697
C26	0.1845	0.1687	0.2672	0.1956	1.244	0	0.643	18.39	465
C27	0.1812	0.1672	0.5655	0.2200	1.053	0.779	0.544	30.70	472
C28	0.1628	0.1586	1.1733	0.5352	1.223	1.403	0.466	46.07	526
C33	0.1779	0.1555	0.2338	0.1995	0.839	0	0.639	18.22	443
C34	0.1977	0.1708	0.1251	0.1341	0.691	0	0.706	12.22	443
C35	0.2356	0.2038	0.0512	0.0921	0.800	0	0.779	8.25	443
C36	0.1496	0.1286	0.5233	0.3463	0.669	0	0.536	30.43	486
C38	0.2113	0.1806	0.2613	0.2012	0.873	0	0.643	17.73	443
C39	0.1911	0.1676	0.3520	0.2608	0.872	0	0.643	17.74	443
C40	0.2085	0.1802	0.3206	0.2077	0.874	0	0.643	17.75	443
C41	0.2160	0.1833	0.2173	0.1020	1.393	0	0.574	18.00	551
C42	0.2361	0.2035	0.4150	0.2355	1.231	0	0.496	30.68	566
C43	0.1986	0.1749	0.8459	0.8166	1.137	0	0.430	45.92	617
C44	0.2196	0.1948	0.1923	0.0167	0.902	0	0.591	18.42	515
C45	0.2771	0.2420	0.0823	0.1599	1.251	0	0.659	12.20	508
C46	0.2773	0.2364	0.0448	0.0267	1.357	0	0.734	8.16	501
C47	0.2146	0.1828	0.3230	0.1022	0.995	0	0.476	30.58	617
C48	0.2467	0.2122	0.3066	0.1553	0.995	0	0.467	30.72	639
C49	0.1835	0.1678	0.4950	0.5842	1.950	0	1.402	45.87	704
C50	0.2526	0.2152	0.0570	0.1200	0.961	0	0.553	18.42	588
C51	0.3169	0.2701	0.0100	0.1379	1.136	0	0.616	12.23	580
C52	0.3454	0.2908	0.0397	0.1237	1.543	0	0.682	8.17	580
C53	0.3091	0.2596	0.0518	0.1134	1.215	0	0.562	12.33	697
C54	0.3951	0.3331	0.0807	0.1250	1.602	0	0.622	8.19	697
C55	0.2828	0.2358	0.0264	0.1538	0.977	0	0.505	18.39	704
C56	0.2290	0.1958	0.2367	0.0769	1.411	0	0.428	30.68	762
C57	0.2375	0.2005	0.2489	0.2170	1.194	0.086	0.368	45.88	842
C58	0.3296	0.2759	0.1000	0.1287	1.239	0	0.503	12.30	871
C59	0.4154	0.3497	0.0592	0.0998	1.918	0	0.557	8.17	871
C60	0.3294	0.2763	0.0641	0.1150	1.628	0	0.453	18.40	878

TABLE 2 (Continued)

Run (1)	f_{ws} (2)	$\beta_1 D_{1m}$ (3)	D_{3m}/D_{1m} (4)	D_{5m}/D_{1m} (5)	β_1x100 (6)	$-\beta_3$x100 (7)	β_4x100 (8)	Fd_* (9)	a_* (10)
C61	0.2476	0.2096	0.0667	0.1745	1.572	0	0.384	30.77	943
C62	0.2120	0.1846	0.1307	0.1911	1.131	0.086	0.719	12.33	424
C63	0.2707	0.2375	0.1637	0.0720	1.456	0.137	0.797	8.19	424
C64	0.1831	0.1732	0.3277	0.0641	0.932	0.253	0.646	18.37	432
C65	0.1864	0.1726	0.5544	0.2273	1.002	0.423	0.548	30.58	464
C66	0.1421	0.1281	0.8019	0.2827	1.192	0.615	0.466	45.71	526
C67	0.2740	0.2562	0.2360	0.0392	1.374	0.210	0.840	12.28	312
C68	0.2241	0.3261	0.1760	0.1457	1.458	0.361	0.930	8.19	312
C69	0.2573	0.2396	0.6632	0.2377	1.260	0.661	0.757	18.47	319
C70	0.2745	0.2571	1.1181	0.4229	1.571	1.101	0.637	30.71	343
C71	0.2585	0.2409	1.5154	0.3462	2.004	1.636	0.542	45.76	385
C72	0.2429	0.2235	0.5336	0.1846	1.644	0.353	0.748	18.63	319
C73	0.2179	0.1859	0.1235	0.1336	0.679	-0.661	0.504	18.56	704
C74	0.1819	0.1655	0.3227	0.1392	0.890	0.149	0.640	18.42	439
C75	0.1751	0.1579	0.2335	0.2849	0.888	0.282	0.622	18.46	464
C76	0.1642	0.1693	0.3863	0.1459	0.849	0.492	0.672	18.35	399
C77	0.1845	0.1876	0.4784	0.1768	0.889	0.694	0.690	18.45	377
C78	0.1767	0.1557	0.2041	0.2302	0.889	0.646	0.599	18.45	501

TABLE 3.-Results of Fine Sand Experiments (D_*=96)

Run (1)	f_{ws} (2)	$\beta_1 D_{1m}$ (3)	D_{3m}/D_{1m} (4)	D_{5m}/D_{1m} (5)	β_1x100 (6)	$-\beta_3$x100 (7)	β_4x100 (8)	Fd_* (9)	a_* (10)
F01	0.1256	0.1086	0.2098	0.0283	1.043	0.386	0.801	18.40	1496
F02	0.1298	0.1094	0.3092	0.1130	1.025	0.423	0.750	18.42	1707
F03	0.1301	0.1138	0.0591	0.0794	1.251	0.715	0.841	12.26	1663
F04	0.1503	0.1286	0.0630	0.0513	1.396	0.927	0.940	8.17	1629
F05	0.1237	0.1042	0.1334	0.2653	1.037	1.302	0.750	18.41	1707
F06	0.1593	0.1308	0.1570	0.1538	1.326	1.838	0.623	30.61	1862
F07	0.1098	0.0933	0.1581	0.1351	1.420	0.087	0.686	18.38	2040
F08	0.1279	0.1075	0.0852	0.0653	0.864	0.150	0.782	12.19	1929
F10	0.1263	0.1056	0.2013	0.0440	1.588	0.270	0.583	30.71	2184
F11	0.1397	0.1194	0.0201	0.0655	1.059	0.041	0.626	18.28	2461
F12	0.1708	0.1441	0.0460	0.0535	0.885	0.070	0.705	12.22	2372
F13	0.1498	0.1294	0.1016	0.0498	1.613	0.124	0.528	30.39	2682
F14	0.1275	0.1184	0.0674	0.0814	1.051	0.171	0.801	18.41	1496
F15	0.1673	0.1434	0.0795	0.0568	1.271	0.301	0.903	12.27	1441
F16	0.1682	0.1389	0.0954	0.1768	1.900	0.522	0.674	30.60	1641
F17	0.1887	0.1674	0.1729	0.1813	0.903	0.123	0.941	18.34	1086
F18	0.1826	0.1672	0.1445	0.1317	0.013	0.211	1.057	12.26	1053
F20	0.2106	0.1989	0.2421	0.4439	1.084	0.084	1.098	18.35	799
F21	0.2233	0.2036	0.0346	0.1108	1.260	0.143	1.231	12.27	776
F23	0.1390	0.1149	0.1626	0.0322	0.903	0.025	0.559	18.42	3070
F28A	0.1199	0.1119	0.1579	0.0023	2.084	0	0.748	18.59	1707
B	0.1378	0.1179	0.1158	0.0196	2.084	0	0.748	18.59	1707
C	0.1053	0.0916	0.1795	0.0636	2.084	0	0.748	18.59	1707
D	0.0092	0.0792	0.2755	0.0656	1.041	0	0.748	18.59	1707

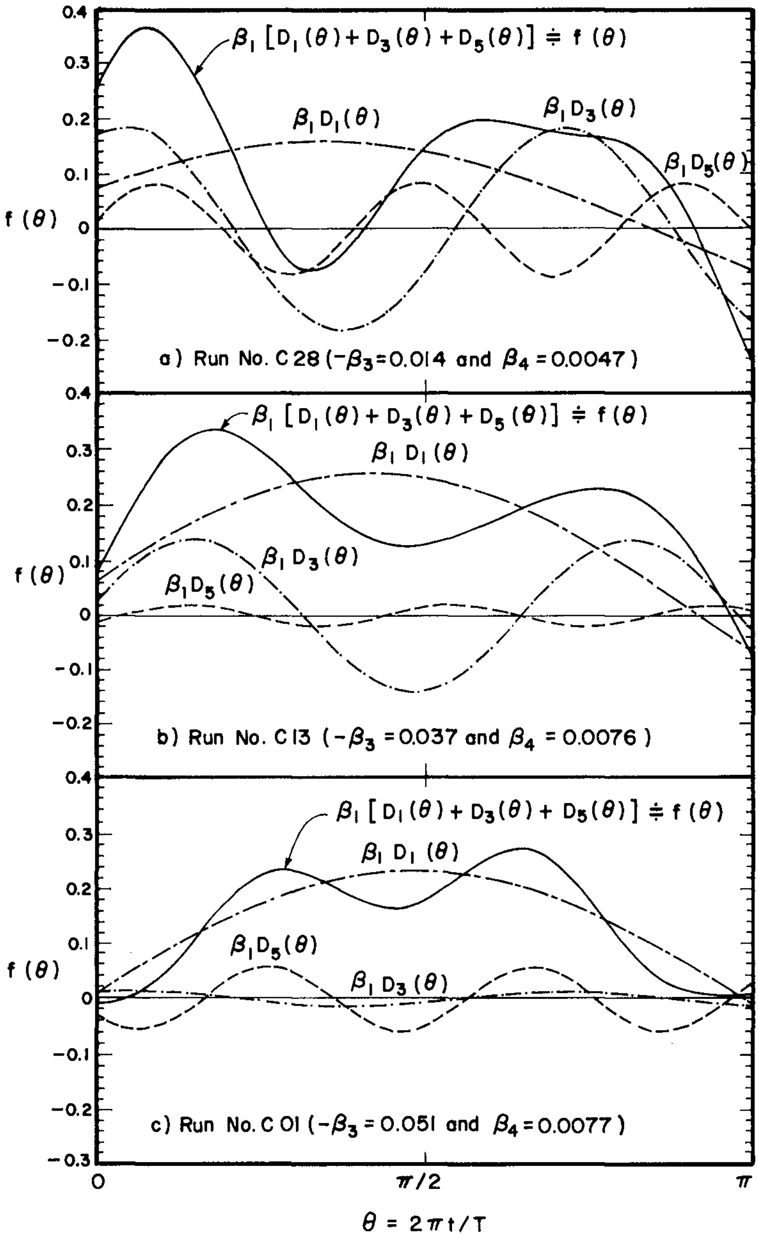

FIG. 3. – Phase of Bed Shear Stress, Eq. 13

CONCLUSIONS

The following conclusions can be drawn from this study:

(a) The wave friction factors on mobile beds were slightly smaller than those computed from expressions of friction factors for fixed beds with the use of ripple geometry as bed roughness [$k_s = 25\eta(\eta/\lambda)$] but they were one order of magnitude greater than the computed values with the use of sand grain as bed roughness ($k_s = 2D$).

(b) The wave friction factor on mobile beds f_{ws} was then correlated with the densimetric sediment Froude number Fd_*, the relative bed smoothness a_* and the densimetric sediment Froude-Reynolds number D_* (Eq. 21).

(c) The measured phases of the bed shear stresses over mobile beds of sands indicated the presence of eddies generated on the ripple bed which conformed to the measured profiles of suspended sediment concentration.

(d) The relative magnitudes of the third and the fifth harnomics over the fundamental harmonic were also correlated with Fd_*, a_* and D_* (Eqs. 23 and 24).

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REFERENCES

CARSTENS, M.R., NIELSON, R.M., and ALTINBILEK, H.D., "Bed forms generated in the laboratory under an oscillatory flow: analytical and experimental study," Technical Memo. No. 28, US Army Corps of Engineers, Coastal Engineering Research Center, 1969.

GRANT, W.D. and MADSEN, O.S., "Combined wave and current interaction with a rough bottom," Journal of Geophysical Research, Vol. 84, No. C4, 1979, pp. 1797-1808.

GRANT, W.D. and MADSEN, O.S., "Movable bed roughness in unsteady oscillatory flow", Journal of Geophysical Research, Vol. 87, No. C1, 1982.

HOM-MA, M. HORIKAWA. K., and KAJIMA, R., "Study on suspended sediment due to wave action, Coastal Engineering in Japan, Vol. 8, 1965, pp. 85-103.

HORIKAWA, K., and WATANABE, A., "Study on sand movement due to wave action," Coastal Engineering in Japan, Vol. 10, 1967, pp. 39-57.

JONSSON, I.G., "Wave boundary layers and friction factors," Proceedings of the Tenth International Conference on Coastal Engineering, Vol. 1, 1966, pp. 127-148.

JONSSON, I.G., and CARLSEN, N.A., "Experimental and theoretical investigations in an oscillatory turbulent boundary layer,"Journal of Hydraulic Research, Vol. 14, No. 1, 1976, pp. 45-60.

KAMPHUIS, J.W., "Friction factor under oscillatory waves," Journal of the Waterways, Harbors and Coastal Engineering Division, ASCE, Vol. 101, No. WW2, 1975, pp. 135-144.

KENNEDY, J.F. and LOCHER, F.A., "Sediment suspension by water waves," Waves on Beaches and Resulting Sediment Transport, R.E. Meyer, Ed., Academic Press, New York, NY, 1972, pp. 249-295.

LOFQUIST, K.E.B., "Drage on naturally rippled beds under oscillatory flows," Miscellaneous Paper CERC-86-13, US Army Corps of Engineers, Coastal Engineering Research Center, 1986.

MADSEN, O.S. and GRANT, W.D., "Sediment transport in the coastal environment", Report No. 209, Ralph M. Parsons Lab. for Water Resources and Hydrodynamics, Massachusetts Inst. of Tech., 1976.

NAKATO, T., LOCHER, F.A., GLOVER, J.R. and KENNEDY, J.F., "Wave entrainment of sediment from rippled beds," Journal of the Waterways, Harbours and Coastal Engineering Division, ASCE, No. WW1, Vol. 103, 1977, pp. 83-100.

RIEDEL, H.P., KAMPHUIS, J.W., and BREBNER, A., "Measurement of bed shear stresses under waves," Proceddings of the Thirteenth International Conference on Coastal Engineering, Vol. 1, 1972, pp. 587-603.

SWART, D.H., "Predictive equations regarding coastal transports," Proceedings of the Fifteenth International Conference on Coastal Engineering, Vol. 2, 1976, pp. 1113-1132.

VONGVISESSOMJAI, S., "Oscillatory boundary layer and eddy viscosity," Journal of Hydraulic Engrg., ASCE, Vol. 110, No,. 4, 1984.

VONGVISESSOMJAI, S., "Profile of suspended sediment due to wave action," Journal of Waterways, Port Coastal and Ocean Engrg., ASCE, Vol. 112, No. 1, 1986.

VONGVISESSOMJAI, S. "Wave friction factor on sand ripples". Proc. Specialty Conf. on Advances in Understanding of Coastal Sediment Processes, Vol. 1, 1987, pp. 393-408.