

CHAPTER 103

IMPROVED LONGSHORE SAND TRANSPORT EVALUATION

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Abstract

In this paper an improved bulk formulation for the longshore sediment transport rate is presented. It is based on a simplified hydrodynamic analysis of surf zone flow and supplemented by an exhaustive dimensional analysis. The proposed model includes the effect of the surf zone dynamic state (e.g. variation of longshore sand transport, ll , with breaker type) and it is now being used in the one- and two-line shoreline evolution models developed by the Maritime Engineering Laboratory in Barcelona.

1. Introduction

The interest of engineers in estimating longshore sand transport comes from the need to study coastal evolution in order to solve existing coastal problems.

General longshore sand transport models have been developed in recent years, all of them using stringent simplifying hypotheses, due to the poor present understanding of the physics of the problem. These models are usually based on extensive data measurement campaigns without providing the level of accuracy their cost would suggest.

Because of these drawbacks in the research stage, most of the models are still under development (analyses of basic hypotheses, comparison with other data sets, etc.). In this context, the bulk formulations, basically because of their simplicity, have been the most extensively calibrated and used for coastal engineering analyses.

2. Background

In nearly all approaches to longshore sand transport evaluation, no matter whether they are energetics based (Komar and Inman, 1970)(Bailard, 1981) or shear-stress based (Bijker, 1971)(Swart, 1976), the influence of the surf-zone dynamic state (e.g. breaker type) on the ll model or formulation is seldom considered. This influence is slight-

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ly better understood for surf zone hydrodynamics, which are the driving factor for the sediment transport. As it was proposed in earlier works (S.-Arcilla et al., 1986) the mean rate of wave energy dissipation per unit area, D (across-shore integrated value) can be related to the Iribarren parameter (Iribarren, 1949) (Battjes, 1974):

$$I_r = \frac{m}{(H_b/L_o)^{1/2}} \quad (2.1)$$

where:

m : bottom slope (average for the surf-zone).

H_b : breaker wave height.

L_o : deep water wave length.

This relationship can be derived from bore theory, energetic balance and dimensional analysis, and has been tested with an extensive data set, including field and laboratory measurements.

Similarly, and due to the relationship between the driving term for longshore currents and D (established from the alongshore momentum balance equation) the longshore current velocity, V_l , was also shown to be related to the surf zone dynamic state, via Iribarren's parameter (S.-Arcilla et al. 1986)

Both fits showed a bell-shaped trend for the relationship between the non-dimensional D and V_l and I_r , relating thus the breaker type with the amount of dissipated energy and the value of the longshore current. As an intermediate result of the research, the bottom friction coefficient, C_f , was also shown to be a function of the Iribarren parameter. The remaining step of this research is to analyse the influence of the surf zone dynamic state on the longshore transport rate, starting from the relationships obtained for D and V_l . This point is developed in the following sections.

3. Predictive Formulation

3.1. General

The sediment transport rate expressed in units of submerged weight per unit time, shall be denoted by I . This variable, as said before, has been evaluated in various ways during the last decades. One of the most physically-based approaches consists in assuming I to be proportional to a stirring term times a transporting term. This approach is shown in (Bagnold, 1966) (Bijker, 1968) and (Soulsby, 1986). The significant differences in the details illustrate the difficulties and uncertainties associated to this problem. In this paper, the alongshore component of I , across-shore integrated, is expressed by:

$$I_l = K_l V_l l_b (\bar{Z}_b - \bar{Z}_{bc}) \quad (3.1)$$

in which:

I_l : alongshore component of the sediment transport rate, expressed in Newtons/second and corresponding to the sub-

merged weight of sediment passing through a section (active profile) of the surf zone per unit time.

K1: dimensionless proportionality parameter.

V1: across-shore averaged longshore current velocity.

lb: some measure of the width of the surf zone (e.g. horizontal distance from breaker- to shore-line).

τ_b : bottom shear stress.

In equation (3.1) the transporting term is (V1.lb) and the stirring term is ($\tau_b - \tau_{bcr}$). This last term includes a threshold condition in the form of a critical value, τ_{bcr} , for the bottom shear stress. This means that for $\tau_b \leq \tau_{bcr}$, there is no Il. For $\tau_b > \tau_{bcr}$ the sediment transport rate is given by (3.1). The transporting term requires a longshore current and, thus, a non-zero value of D (mean rate of wave energy dissipation per unit time) to act. The stirring term, on the other hand, is associated to τ_b and depends on the dynamics of the wave/current boundary-layer.

The transporting term associated to V1, will be mainly due to the incident energy flux, Efi, because these incident waves (Hbi) will generate a significant dissipation, D, and, thus, V1 associated to the breaking process. The reflected waves (Hr), in an across-shore integrated sense, will generate smaller D and V1.

The stirring term, on the other hand, will be due to both wave trains (incident and reflected) because both of them generate a boundary layer and, thus, contribute to τ_b . In other words, the reflected waves will increase the roughness felt by the current and τ_b is, therefore, estimated based on a total wave-height, H_T :

$$H_T = H_{bi} + \beta H_r \quad (3.2)$$

in which $\beta = \beta(Ir)$ is a parameter between 0 and 1. We can now consider the effect of increased reflection (due, e.g., to some new structure built in the surf zone) on the across-shore integrated sediment transport rate, Il. Increased reflection means that H_T increases from which both τ_b and the stirring term increase. On the other hand, for a given incident energy flux (Efi) and width of the surf zone (lb), increased reflection (Efr) implies a smaller D, according to the equation (S.-Arcilla et al., 1986):

$$E_{fi} = E_{fr} + D lb \quad (3.3)$$

Therefore, the effect of increased reflection is to reduce D and V1 and, consequently, the transporting term. The global effect on Il, product of a stirring term that increases and a transporting term that decreases, is not easy to estimate a priori and must be obtained from the Il formulation.

3.2. Il Formulation

This formulation requires, according to equation (3.1), estimates for both V1 and τ_b . They will be derived from simplified hydrodynamic arguments. On the other hand, K1 and τ_{bcr} will be empirically calculated, by comparing this model with an alternative formulation derived from dimensional a-

analysis.

i) V1 ESTIMATION

The alongshore component of the vertically integrated momentum conservation law reads for stationary conditions (e.g. (Mei, 1983)):

$$\frac{\partial S'_{xy}}{\partial x} + \frac{\partial S''_{xy}}{\partial x} + \tau_{yb} = 0 \quad (3.6)$$

in which:

S'_{xy} : excess momentum flux tensor due to wave motion.

S''_{xy} : excess momentum flux tensor due to turbulent motion.

τ_{yb} : bottom shear-stress (y-component).

We can now evaluate the wave thrust in terms of D (Longuet-Higgins, 1970):

$$\frac{\partial S'_{xy}}{\partial x} = D \frac{\sin \alpha b}{Cb} \quad (3.5)$$

and D can be written (including reflection effects) as (S.-Arcilla et al., 1986):

$$D = \rho_w g \frac{Hb i^2}{T} \frac{\sqrt{2\pi}}{8} \gamma^{1/2} I_r \cos \alpha b (1 - Kr^2) \quad (3.6)$$

The bottom shear stress is usually expressed by the following linearised equation (obtained assuming small $V1$ and αb):

$$\tau_{yb} = \rho_w C_f U_{orb} V1 \quad (3.7)$$

Substituting all these equations into (3.4), we can obtain (after neglecting turbulent mixing):

$$V1 = F(I_r) m (gHbi)^{1/2} \sin 2\alpha b \quad (3.8)$$

in which $F(I_r)$ is a function of Iribarren's parameter.

ii) τ_b ESTIMATION

The τ_b estimation is based on the eddy-viscosity concept which allows us to write:

$$\tau_b = \rho_w A_H \frac{\partial U_c}{\partial z} \quad (3.9)$$

in which:

ρ_w : mass density of sea-water

A_H : eddy viscosity coefficient

U_c : characteristic velocity in the boundary layer

z : vertical coordinate, positive upwards

The eddy viscosity coefficient, A_H , is estimated using the model proposed in (Battjes, 1975).

$$A_H = M \cdot h_{Turb} \cdot U_{Turb} \quad (3.10)$$

in which M is a parameter of order $O(1)$ and h_{Turb} and U_{Turb} are respectively, the length and velocity scales of the turbulent motion. By writing $U_{Turb} = (D/\rho_w)^{1/3}$ it is easy to obtain:

$$U_{Turb} = f_1(Ir) U_{orb} \quad (3.11)$$

with U_{orb} being the wave orbital velocity and $f_1(Ir)$ a function of Iribarren's parameter.

The vertical velocity gradient is expressed by:

$$\frac{\partial U_c}{\partial z} \approx \frac{U_{orb}}{z_0} \quad (3.12)$$

in which z_0 is the thickness of the boundary layer and U_c has been approximated by U_{orb} because the wave boundary layer is expected to be much thinner than that of the current and, thus, more relevant for the calculation of Z_b . If we express z_0 by (e.g. (Nielsen, 1986))

$$z_0 = (A_H \cdot T)^{1/2} \quad (3.13)$$

and substitute all previous equations into (3.9) it is easy to obtain:

$$Z_b = \rho_w f(Ir) G(\theta) U_{orb} \quad (3.14)$$

with θ the mobility parameter.

iii) 11 FORMULATION

Substituting (3.8) and (3.14) into (3.1) we get:

$$I1 = \rho_w \hat{F}(Ir) (gHbi)^{3/2} Hbi \sin 2\alpha b (\hat{G}(\theta) - \hat{G}(\theta_{cr})) \quad (3.15)$$

The obtained formulation, apart from including a threshold level for incipient motion, presents an explicit variation of the transport rate with the breaker type or, more generally, with the surf zone (S.Z.) dynamic state (via Ir). This allows to consider (albeit in an across-shore integrated manner and somewhat simplistically) the effect of increased reflection or other changes in the S.Z. dynamic state on the transport rate.

The functions $F(Ir)$ and $G(\theta)$ will be calibrated in section 5, making use of some of the results obtained from the dimensional analysis presented in section 4.

4. Dimensional Analysis

4.1. General

The general expression of the dimensional formula for longshore sand transport (in a longshore uniform coast with a plane shore) following (Kamphuis et al., 1986) can be written as:

$$I_1 = f(\mu, \rho_w, g, \rho_s, D, \psi, F(D), x_c, y_c, h_b, H_b, T, \alpha_b) \quad (4.1)$$

where: μ (dynamic viscosity), ρ_w (water density) and g (gravity acceleration) are the fluid parameters; ρ_s (sediment density), D (characteristic grain diameter), ψ (grain shape factor) and $F(D)$ (granulometric distribution function) are the sediment parameters; x_c (across-shore characteristic scale) and h_b (water depth at breaking point) are the domain parameters; H_b (breaking wave height), T (wave period) and α_b (angle of wave incidence at breaker line) are the wave parameters.

Stationary conditions are assumed (time is not included) and only wave parameters are included since currents are considered to be wave induced. Other variables usually related to longshore transport conditions (e.g., wave orbital velocity, u_{orb} , or the sediment fall velocity, w), turn out to be, in this context, not-independent parameters and will, thus, appear only by exchange. Shape and granulometric distribution factors are not included (in general) in the data set used for calibration, and they will not be further considered.

For the development of the dimensional analysis four sets of repeaters have been employed: $\rho_w, g, H_b, \rho_w, h_b, T, \rho_s, g, D, \rho_w, g, H_b, T$. The main conclusion of these analyses was that the first set provided a better fit while giving the more physically meaningful parameters. This set was, therefore, chosen for the development of the model and is the only one here presented. Applying the Π theorem, and noticing that 8 non-dimensional parameters are expected to appear (13 independent variables, 2 or which are neglected and three of which are taken as repeaters, i.e. associated to the measurement system), the following expressions are obtained:

$$\Pi_{I1} = \frac{I_1}{\rho_w g H_b^2 (g H_b)^{1/2}} \quad (4.2)$$

$$\Pi_{II} = \emptyset \left(\frac{\mu}{\rho_w H_b (g H_b)^{1/2}}, \frac{\rho_s - \rho_w}{\rho_w}, \frac{\rho_w H_b}{\rho_s D}, \frac{x_c}{H_b}, \frac{y_c}{H_b}, \frac{h_b}{H_b}, \frac{g^{1/2} T}{H_b^{1/2}}, \alpha_b \right) \quad (4.3)$$

Combining these expressions Π_{I1} can be approximately written as:

$$\Pi_{I1} = \emptyset (Re, Ir, Id, \theta, m) \sin 2\alpha_b \quad (4.4)$$

in which

Re: Reynolds number (ratio between inertial and viscous forces).

θ : Mobility or shields Number (ratio between disturbing and resisting forces).

I_D : Dean number (ratio between fall velocity and a characteristic wave velocity)(Dean, 1973)(Dalrymple, 1976).

and assuming that the relationship between the angle of wave incidence (α b) and the longshore sand transport is (Komar and Inman, 1970)(Longuet-Higgins, 1972):

$$I_l \propto \sin 2\alpha b \quad (4.5)$$

The expression for the θ function has been thoroughly calibrated with the following field and laboratory data:

- field data : (Watts, 1953)(Komar, 1969)(Kana, 1977)(Kana, 1980)(Inman et al., 1980)(Gable, 1981)(Bruno et al., 1981)(Kraus et al., 1982).

- laboratory data:(Kamphuis et al., 1978)(Vitale, 1981)(Saiyao and Kamphuis, 1983).

The calibration has been based mainly on field values due to scale effects. Exhaustive analyses have been performed in order to select the most adequate data from each set, deleting those data not fulfilling the imposed quality checks. The breaking depth, h_b , was obtained via Goda's criterion when not measured. Curve adjustment and fitting was obtained both visually and by a least-squares technique.

4.2. Relationships among parameters

4.2.1. Iribarren parameter/Dean number

Both parameters refer to onshore/offshore transport and beach profile variations. It is easy to show, from physical arguments, that a relationship as the one shown in figure 4.1 (corresponding to field data) must exist between the two parameters, defined by (2.1) and $I_D = H/wT$.

The proposed relation can be derived from the influence of grain size, D_{50} , on both bottom slope, m and fall velocity, w . This results in a somewhat equivalent use of the two parameters for across-shore-integrated analyses (e.g. I_l). As a consequence, only I_r will be employed in what follows.

4.2.2. Reynolds number/Mobility number

Both parameters refer to the stage of flow development and, thus, to the stage of transport. They are related for initiation of motion conditions (Shields threshold curve). By combining their expressions, they can also be related for actual transport conditions:

$$Re = \theta^{2/3} f(k)$$

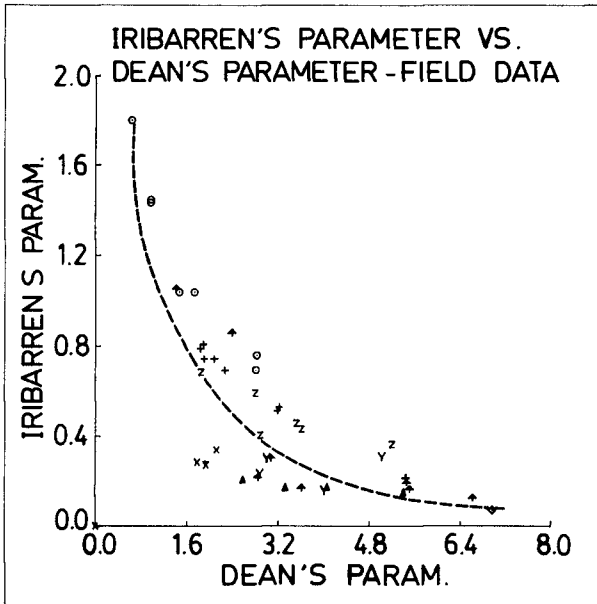


Figure 4.1: Iribarren parameter versus Dean's number (field data).

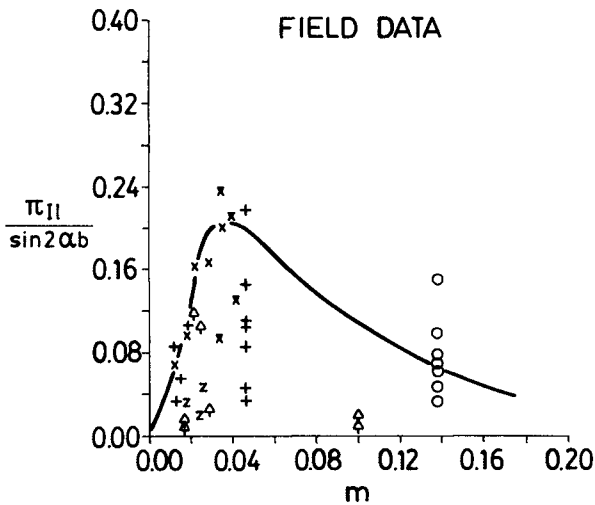


Figure 4.2: $\frac{\pi_{II}}{\sin 2\alpha b}$ versus m (beach slope) and the corresponding least-squares fit.

where k is a parameter depending on fluid and sediment characteristics. This relationship suggests the use of just one of the two parameters in this type of analysis. Only θ will be, thus, used in the subsequent development.

4.2.3. Bottom slope/Mobility number

A simple plot of these two variables provides an acceptable relationship for both field and laboratory data. These results are similar to those obtained by Kamphuis et al., 1982 and 1986) and suggest to discontinue the explicit use of m in equations such as (4.4). The influence of m on I_1 is, therefore, included via I_r .

4.2.4. Other relationships

Various other relationships among these parameters can be easily obtained. Since they are not required for the development of the I_1 formulation they will not be here included. Due care should be exercised when employing these relations due to the spurious effects inherent to this type of analysis.

4.3. I_1 formulation

The dimensional analysis formulation for I_1 is obtained by substituting the relationships derived in sections 4.2.1, 4.2.2 and 4.2.3 into equation (4.4):

$$I_1 = \sin 2\alpha b \cdot \emptyset(I_r, \theta) \quad (4.7)$$

This simplified expression is functionally identical to the one derived in section 3 (see equation 3.15).

The variation of I_1 with m , obtained by performing a least-squares fit to measured I_1 field data, turned out to be of the form (figure 4.2):

$$I_1 \propto m^{1/2} \cdot 10^{-8} m \quad (4.8)$$

The relationship $\pi_{I_1} \propto m^{1/2}$ was also proposed by (Bruun et al., 1986) and (Deigaard et al., 1986). The $10^{-8} m$ factor included here provides not only a better fit but also a bell shaped variation with breaker type coherent with the corresponding laws of evolution obtained for D and V_1 (see section 2).

To derive the form of the \emptyset function let us consider the variation of π_{I_1} with I_r . According to section 3, I_1 can be written as a stirring term times a transporting term. The first one can be shown to be proportional to D divided by some characteristic velocity while the second is mainly dependent on V_1 (see equation 3.1). Since both D and V_1 have been shown to vary in a bell-shaped manner with I_r , this same behaviour should be expected for π_{I_1} . When plotting π_{I_1} versus I_r (figure 4.3) a bell-shaped trend is in fact found. The mobility number θ appears to act as a parameter of the obtained family of curves, controlling the scale of the process. This type of variation with I_r is fully cohe-

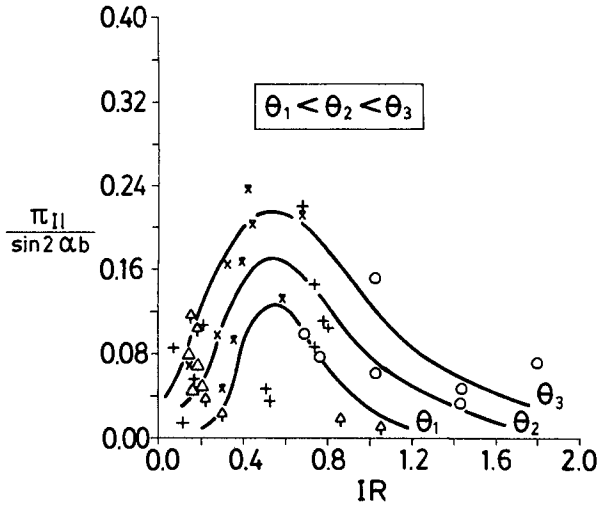


Figure 4.3: π_{I1} versus Iribarren parameter (Ir) with θ (mobility number) as family parameter.

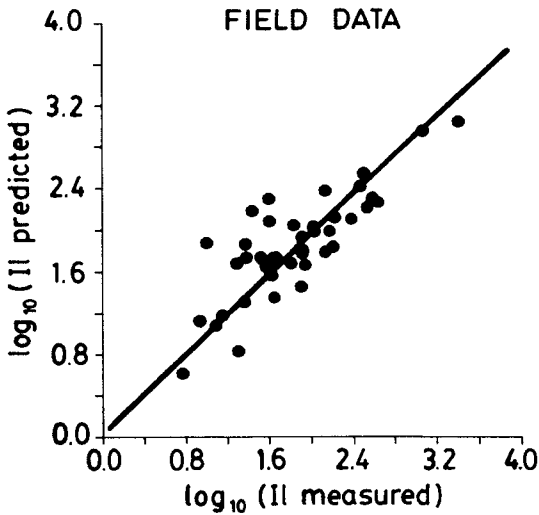


Figure 5.1: I1 predicted versus I1 measured (field Values).

rent with the variation of $\overline{I_1}$ with m given by equation (4.8) and it shall be further exploited in section 5.

5. Calibration of the Model

The variation of I_1 with m in equation (3.15) must be via the $F(Ir)$ function. Using the variation of $\overline{I_1}$ with m obtained in the previous section we may then write:

$$\hat{F}(Ir) = A Ir^{1/2} 10^{-8} Ir \quad (5.1)$$

This is the simplest possible function varying with m as prescribed by the dimensional analysis and possessing a bell-shaped behaviour with the breaker type.

Likewise, the simplest possible function of θ including this parameter as a scale factor is:

$$\hat{G}(\theta) = \left(\frac{\rho_s - \rho_w}{\rho_w} \right) \theta^c \quad (5.2)$$

in which the first term has been included to express I_1 as a function of $(\rho_s - \rho_w)$ instead of (ρ_w) .

The resulting final equation for I_1 is obtained by substituting (5.1) and (5.2) into (3.15):

$$I_1 = (\rho_s - \rho_w) A Ir^{1/2} 10^{-8.00} Ir^{3/2} Hbi^{5/2} \sin 2\alpha b. (\theta^c - \theta_{cr}^c) \quad (5.3)$$

By performing a least squares fit to measured I_1 values (field data only) it is easy to obtain:

$$A = 4.8 \times 10^{-3}$$

$$c = 0.44$$

$$\theta_{cr} = 12.77$$

in which the factor 10^{-3} in A can be obtained from dimensional analysis considerations. The quality of the obtained fit was not very sensitive to the precise θ_{cr} value, probably indicating that threshold conditions cannot be accurately established with the available data base.

To illustrate the structure of this formulation it is compared in Table I with three other well known models for I_1 . This comparison is by no means exhaustive since many other significant I_1 models have not been included (e.g. Dean, 1973)(Watanabe et al., 1988)). Its main purpose is to give a feeling of the functional structure obtained with various formulations. A quantitative measure of the corresponding goodness of fit appears in Table II. The agreement obtained with the proposed formulation is shown in figure 5.1. It should be, however, mentioned that the number of free parameters for the fit of the proposed model is higher

I1 model	PARAMETER DEPENDENCY						
	Hb	T	m	D ₅₀	K _R (via Ir)	αb	P _s
CERC, 1970	Hb ^{2.5}	-	-	-	-	sin2αb	-
Kamphuis et al. 1986	Hb ^{3.5}	-	m ¹	D ₅₀ ⁻¹	-	sin2αb	P _s - P _w
Kraus et al. 1986	Hb ^{2.5}	-	m ⁻¹	-	-	sin2αb	P _s - P _w
Arcilla et al. 1986	Hb ^{2.69}	T ^{1/2}	m ^{1/2}	D ₅₀ ^{-0.44}	10 ^{-0.201r(1-K_R)^{7/6}}	sin2αb	(P _s -P _w)(P _s /P _w) ^{0.66}

Table 1: Parameter dependency of various I1 models compared to the formulation proposed in this paper.

I1 model	S _{RSE}
CERC, 1970	0.6058
Kamphuis et al. 1986	0.3679
Kraus et al. 1988	3.7216
Arcilla et al. 1988	0.3308

Table II: Fit obtained with various I1 formulations using the following measure (standard error) of the scatter:

$$S_{RSE} = \sqrt{\frac{1}{N-1} \left[\sum (\log_{10}(I1)_{predicted} - \log_{10}(I1)_{measured})^2 \right]}^{1/2}$$

than for other formulations. This could be associated to the obtained reduction in dispersion. It should also be pointed out that the high scatter obtained for (Kraus et al., 1988) model is probably due to the fact that apparently there were some misprints in the coefficients reported in the original paper and used here for the fit (Kraus personal communication).

6. Conclusions

1. A bell-shaped variation with Ir (and thus, with the breaker type) has been obtained from physical arguments and field data for the following variables: D (mean rate of wave energy dissipation per unit area), V1 (mean longshore current velocity) and I1 (mean alongshore sand

transport rate).

2. A reasonable correlation has been found between the following pairs of dimensionless parameters:

- Iribarren's and Dean's parameters
- Reynolds and mobility parameters
- Beach slope and mobility parameter

This means that these parameters can be used somewhat equivalently in the context of across-shore integrated transport evaluation. It should also be remarked that this conclusion is limited to the employed data base and could also be affected by some spurious effects, always present when dealing with dimensionless parameters obtained from dimensional analysis. Because of this the predictive formulation proposed has been directly calibrated from equations (4.8) and (5.3), therefore avoiding the use of repeated variables in the two axes of the fitting diagram. The correlation among dimensionless parameters can illustrate the dangers of using models exclusively based on dimensional analysis.

3. The proposed I1 formulation calculates this variable as a transporting term (proportional to $V1$) times a stirring term (proportional to τb). The main advantages of this model are:

- It allows using different formulations for any of the two terms involved.
- It explicitly shows the variation of I1 with I_r and θ parameters and, in general, with the surf-zone dynamic state. In particular, this model includes the effect of varying reflection on I1.
- It provides a slightly better fit to the employed data base than other available formulations.

The proposed model represents, thus, an attempt to provide a more precise and physically based I1 formulation. This conclusion must be, however, interpreted in the context of the limitations (in quantity and quality) associated to the available I1 field data.

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