CHAPTER 134

Physical Modelling of Beach Erosion and Littoral Drift

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Abstract

A new procedure for physical modelling of beach sedimentary processes is presented. It is shown that the modelling requirements proposed by Dean (1985) are necessary but not sufficient for dynamic similarity. Quantification of scale effects due to slope and relative grain size scaling conditions enables extrapolation of the physical model results to prototype situations. Selected examples of the application of the proposed model design are also given.

1. Introduction

A model is dynamically similar to its prototype if all the dimensionless variables governing the model and prototype phenomena are identical. The dimensionless functional relationship of any mechanical quantity, particularly sediment transport, is given by,

$$\Pi_A = \varphi_A \; (x_1, \; x_2, \; \ldots \; x_n) \tag{1}$$

where Π_A is the dimensionless version of A (the mechanical quantity) and x_i are the dimensionless variables. For dynamic similarity between the model and prototype, the condition to be satisfied is:

$$(\Pi_A)_p = (\Pi_A)_m \tag{2}$$

where the subscript p refers to the prototype value of Π_A and the subscript m refers to the model value of Π_A.

In a conventional physical model of beach morphology with a rectangular co-ordinate system, the general model scale n is given by the vertical scale value n_z where n is defined as the ratio of the prototype value over the model value.

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Thus, Equation 2 may be written as:

$$n_{\Pi_A} = 1 \tag{3}$$

which is achieved if and only if,

$$(x_i)_p = (x_i)_m, \text{ or } n_{x_i} = 1 \tag{4}$$

is provided. If Equation 4 is satisfied then all x_i are identical in model and prototype and together with geometrical similarity, Π_A (the dimensionless version of a mechanical quantity of interest) is also identical in model and prototype.

2. Description of Littoral Processes

Sayao (1982) described sediment transport in the surf zone by the following dimensionless relationship:

$$\Pi_{Q_s} = \Phi(H_b/L_o, \ m_b, \ H_b/D, \ \rho_s/\rho) \tag{5}$$

where Π_{Qs} is the dimensionless littoral transport rate, H_b/L_o is the breaking wave steepness, m_b is the beach profile slope which characterises energy dissipation in the surf zone, H_b/D is the relative grain size parameter, and ρ_s/ρ is the density ratio (sediment to water) which reflects the influence of the density of the grains.

An additional dimensionless variable, especially important in situations where cross-shore transport is dominant, is the dimensionless fall time parameter, F_b or F_o,

$$F_b = \frac{H_b}{wT} \quad \text{or} \quad F_o = \frac{H_o}{wT} \tag{6}$$

where H_b is the breaker height, H_o is the deepwater wave height, T is the wave period and w is the fall velocity of the sand grains. According to Dean (1973), this parameter provides an indication of whether net onshore or offshore motion of sand will occur on a beach profile. A critical value of F_o is given in the Shore Protection Manual (US Army Corps of Engineers, 1977), for $F_o > 1$, recession of the shoreline will occur. The validity of this parameter in predicting the net cross-shore transport direction in situations where longshore transport is also appreciable can be confirmed by the laboratory data of Sayao (1982) and Readshaw (1979). Dean's fall time parameter is plotted versus deepwater wave steepness and the relative grain size parameter in Figures 1 and 2 for the results of these two three-dimensional model studies.

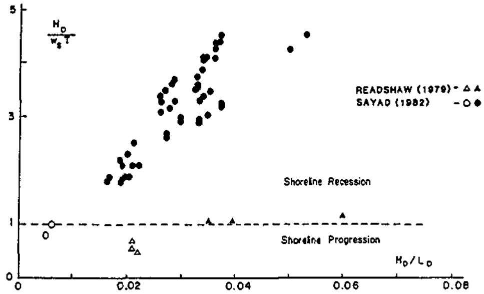

Figure 1. Dimensionless Fall Time vs. Wave Steepness

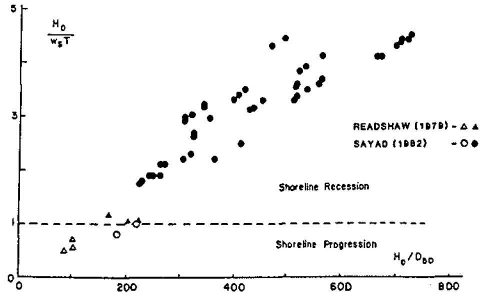

Figure 2. Dimensionless Fall Time vs. Relative Grain Size

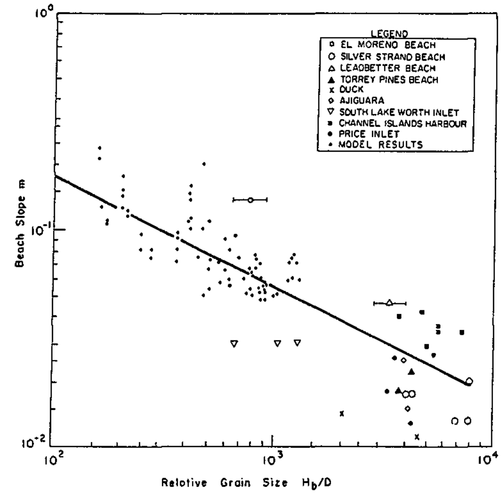

Figure 3. Beach Slope vs. Grain Size Relationship
(from, Kamphuis et al., 1986)

3. Dynamic Similarity

Dynamic similarity is only achieved if all the prototype and model values of the dimensionless parameters on the right hand side of the following equation (Equation 5 extended to include the fall time parameter),

$$\Pi_{Q_s} = \varphi(H_b/L_o,\ m_b,\ H_b/D,\ \rho_s/\rho,\ F_o) \tag{7}$$

are equal, i.e. if their scales are:

$$n_{H_b/L_o} = 1;\ n_{m_b} = 1;\ n_{H_b/D} = 1,\ n_{\rho_s/\rho} = 1;\ n_{F_o} = 1.$$

Therefore, for physical model studies of surfzone processes (including cross-shore and alongshore sediment transport) in a given geometrically similar model, if the set of scaling conditions derived from Equation 7 are satisfied, then the dimensionless version of any mechanical quantity (e.g. Π_{Qs} in Equation 7) is identical in model and prototype. The implications of each of these conditions are investigated.

(i) A Wave Steepness Condition : $n_{Hb/Lo} = 1$.

The provision of this condition yields,

$$n_{H_b} = n_{L_o} \tag{8}$$

which results in a Froudian scale for the wave period as follows,

$$n_T = \sqrt{n} \tag{9}$$

This is a conventional scale for most physical models.

(ii) A Beach Slope Condition : $n_{mb} = 1$.

The representative beach slope is assumed to be,

$$m_b = d_b/\lambda_b \tag{10}$$

where the depth of breaking defines the offshore limit of the horizontal breaking distance λ_b, measured from the still water line. To satisfy this condition, the following equality must be achieved,

$$n_{d_b} = n_{\lambda_b} \tag{11}$$

This will only be possible in geometrically undistorted models with model grain size reduced geometrically as well, i.e.,

$$n = n_X = n_Z = n_D \tag{12}$$

In an undistorted small scale hydraulic model all geometrical scales are the same. In a geometrically distorted model, the two horizontal scales n_X and n_y are identical but differ from the vertical scale n_Z. The geometrical model distortion N may thus be defined as:

$$N = \frac{n_X}{n_Z} = \frac{n_y}{n_Z} \tag{13}$$

An empirical relationship for beach slope was given in Sayao (1982) as follows (see also Sayao et al, 1985),

$$m_b = 1.5(H_b/D_{50})^{-\frac{1}{2}} \tag{14}$$

valid for medium and fine sands. Figure 3 shows a plot of beach profile slope versus the relative grain size parameter for the model data of Sayao (1982) and Readshaw (1979) and field data as compiled and analyzed by Davies (1984). Equation 14 shows good agreement with the trend of the data, albeit with large scatter. Sayao et al (1985) suggested that this relationship may be improved by introducing the wave steepness. However, comparison with available data did not improve scatter, indicating a need for the analysis of further field measurements where both wave characteristics and beach profiles are recorded simultaneously. Equation 14 can also be shown to fit the data of van Hijum and Pilarczyk (1982) for gravel beaches.

The relationship described by Equation 14 can be used to develop a scaling relation for the beach slope condition as follows,

$$n_{m_b} = (n_{H_b}/n_{D_{50}})^{-\frac{1}{2}} = (n_D/n)^{\frac{1}{2}} \tag{15}$$

If the model grain size is not reduced geometrically then

$$n_{m_b} = (n_D/n)^{\frac{1}{2}} = M_1 \tag{16}$$

where $M_1 \neq 1$ is the distortion due to the beach slope condition, which leads to a scale effect.

In an attempt to compensate for this distortion, Bijker (1967) suggested that for distorted models the geometric distortion should be equal to the ratio between the equilibrium beach slopes (which according to Dean, 1977 is a function of the grain diameter).

(iii) A Relative Grain Size Condition : $n_{Hb/D} = 1$.

The imposition of this condition yields

$$n_{H_b} = n_D \tag{17}$$

which will only be possible when the sand grain size is scaled down geometrically. In practice, this is only possible for shingle beaches or beaches with coarse sand grains. Thus, an imposed grain size distortion (M_2) may be present in beach models, when $M_2 \neq 1$, and the scale effect from this non-similarity is quantified as follows,

$$n_{H_b/D} = n/n_D = M_2 \tag{18}$$

(iv) A Density Ratio Condition : $n_{\rho s/\rho} = 1$.

To satisfy this condition the following must be provided.

$$n_{\rho_s} = n_\rho = 1 \tag{19}$$

In conventional physical models using water and sand this condition is fulfilled. Departure from this condition (for example, through the use of lightweight materials) leads to significant scale effects and dissimilarity between model and prototype morphological development in the surfzone (see Kamphuis, 1975).

(v) A Fall Velocity Condition : $n_{Fo} = 1$.

The adherence to this condition leads to the Froudian scaling of fall velocity as derived from the following relationship,

$$n_{H_b} = n_w . n_T \text{ or } n_w = \sqrt{n} \tag{20}$$

Based on the work of Dean (1973), fall velocity scaling has been suggested by many researchers (e.g. Dalrymple and Thompson, 1976; Kamphuis, 1982; Hallermeier, 1984; Ito and Tsuchiya, 1984; Dean, 1985; Kriebel et al, 1986 and Yalin et al, 1986).

Several scaling laws dealing with the distortion of the fall velocity condition have also been presented in the literature (Valembois, 1961; LeMehaute, 1970; Vellinga, 1982 and 1986; Sayao and Guimaraes, 1984). For example, using the Vellinga (1982) scaling law:

$$N = n_x/n_z = (n_z/n_w^2)^{\alpha} \tag{21}$$

where by definition $N = n_x/n_z = n_y/n_z$ and the fall velocity scaling criterion, if satisfied, gives $n_x/n_z = 1$ for any value of the exponent α, i.e. no geometrical distortion. The value of the exponent α in Equation 21 varies between 0.25 (Vellinga, 1982 and 1986) and 0.50 (Valembois, 1961; LeMehaute, 1970; Sayao and Guimaraes, 1984).

4. Sediment Transport Scale and Time Scale for Littoral Processes

The dimensionless littoral transport rate (Π_{Qs}) expression depends on the units of the sediment transport rate Q_s. Adopting Q_s as a volumetric rate (m^3/s), Sayao (1982) defined,

$$\Pi_{Q_s} = \frac{(1 - p)\, Q_s}{(\rho/\rho_s) H_b^2\, (H_b/T)} \tag{22}$$

where p is the beach sand porosity. Assuming $n_{(1 - p)} = 1$, the scale of dimensionless sediment transport in the littoral zone is,

$$n_{\Pi_{Q_s}} = \frac{n_{\rho_s} n_{Q_s} n_T}{n_\rho\, n_H^3} = \frac{n_{Q_s}}{n^{5/2}} \tag{23}$$

for Froudian models with sand as the model material.

The time scale for littoral processes (or morphological development) may be derived from Equation 23, using the definition of Q_s (in m^3/s) as,

$$n_{Q_s} = n_V/n_{t_s} \tag{24}$$

where n_V is the scale of volume of sand. Thus

$$n_{t_s} = n_V/n_{Q_s} = \frac{n_V}{n^{5/2} n_{\Pi_{Q_s}}} \tag{25}$$

Equation 25 shows that the time scale is influenced by the scale of dimensionless transport rate and by the volumetric scale. There is a possibility that the former will be different from 1 and the latter to be different than n^3 depending on the model design (i.e. whether or not there are distortions due to deviation from the conditions (i) to (v) and whether or not there is geometrical distortion).

5. Proposed Moveable Bed Model Design

The design of a moveable bed model study is not a simple task. It requires a clear understanding of the potential scale effects, as well as a thorough interpretation of model results in order that solutions for the prototype engineering problem may be proposed. Also, prototype data must be available so that prototype parameters related to the coastal phenomena at the study site may be defined.

Further, it is necessary to define the physical model scales. It is proposed here that a geometrically undistorted model is used if at all possible. Otherwise, the scaling law of Equation 21 can be used to determine an appropriate geometrical distortion and model grain size.

With reference to the model grain size it is proposed that the fall velocity criterion is used in the moveable bed model study if possible. This is preferred over other scaling criterions (such as geometrically reduced model material) because:

(a) Dean's parameter would be the same in model and prototype. This means that both model and prototype beaches will be either eroding or accreting, and dynamic similarity of erosional and accretional processes is ensured.

(b) Scale effects resulting from the non-similarity of m and H_b/D are less detrimental and more easily quantifiable than the effects resulting from the non-similarity of F_o.

If fall velocity scaling cannot be used for model design due to other constraints, it is recommended that the moveable bed model should have F_o-values in the same region as the prototype ones, i.e., that for both model and prototype F_o-values are simultaneously above or below the critical value $F_o \sim 1$.

Finally, with respect to grain size, it is strongly recommended that the model value of H_b/D exceeds 300 to ensure that suspended load is able to occur at the model scale (see Nairn, 1985).

6. Quantification of Scale Effect

The modelling design criteria are summarised in Table 1. For the quantification of scale effect, a new procedure is presented for geometrically undistorted, fall velocity scaling models. In this case, distortions are due to non-similarity of beach profile slope and relative grain size. Thus using Equations 16 and 18, the combined scale effect M due to both contributing parameters is,

$$M = M_1 . M_2 = (n_D/n)^{\frac{1}{2}} . \ (n/n_D) = (n/n_D)^{\frac{1}{2}} \tag{26}$$

Also, $n_{\Pi Qs}$ is not equal to unity but for these types of models,

$$n_{\Pi_{Q_s}} = M = (n/n_D)^{\frac{1}{2}} \tag{27}$$

Hence, the rate of morphological development (or the littoral process time scale) may be quantified using Equations 25 and 27 as,

$$n_{t_s} = n/(n_D)^{\frac{1}{2}} \tag{28}$$

and the littoral transport rate scale becomes (using Equations 24, 27 and 28),

$$n_{Q_s} = n^2\sqrt{n_D} \tag{29}$$

Both Equations 28 and 29 are valid for geometrically undistorted, fall velocity scaling models only. If other scaling criteria are used, the derivation of these equations would be different. The expressions apply to both alongshore and cross-shore sediment transport. However, caution is advised in applying the time scale expression (Equation 28) to cross-shore problems where either the model or prototype profiles are near a stable equilibrium form.

Table 1. Geometrically Undistorted Fall Velocity Scaling Model

Scale Condition	Similarity	Comment
(i) wave steepness	$n_{H/L} = 1$	Froudian model
(ii) beach slope	$n_m = M_1 \quad (\neq 1)$	scale effect
(iii) relative grain size	$n_{H/D} = M_2 \quad (\neq 1)$	scale effect
(iv) relative density	$n_{\rho s/\rho} = 1$	Sand in model
(v) Dean's parameter	$n_{Fo} = 1$	Fall velocity scale is Froudian
time for littoral processes	$n_{ts} = n/\sqrt{n_D}$	Equation 28
littoral transport rate	$n_{QS} = n^2\sqrt{n_D}$	Equation 29

7. Applications of the Modelling Procedure

The recommended modelling procedure could be tested if a field data set were available. Unfortunately, due to the difficulties of determining sediment transport rates on beaches during storm events an ideal prototype data set is not presently available. However, the recommended procedure can be evaluated by (1) comparing pairs of flume test results at different scales (and thus regarded as model and prototype) and (2) the use of numerical models to compare profile development at different scales subject to the proposed modelling procedure.

Kriebel et al (1986) have used the fall velocity modelling procedure (originally proposed by Dean, 1985) and compared small-scale flume results (model) with the large-flume experiments (prototype) of Saville (1957). For the Kriebel et al (1986) data, the time scale for littoral processes is calculated to be, $n_{ts} \sim 6$ and Figures 4 and 5 show profile comparisons according to Equation 28. The match of model and prototype profiles in these figures is fair, but so is the match shown in Kriebel et al (1986) based on a Froude time scale. Sediment transport in this example was predominantly directed offshore.

As an example of the sediment transport rate scale, experimental data presented by Mimura et al (1986) and shown in Figure 6, is used. In this case, using their coarse sand tests (Series B) as prototype and fine sand tests as model (Series A), $n \sim 2.1$ and $n_D = 4.2$ are found for two pairs of tests, the B8 and A9 tests on a $^1/20$ initial profile, and the B2 and A3 tests on a $^1/10$ initial profile. The calculated value for the sediment transport rate scale according to Equation 29 is $n_{Qs} = 9.3$, and the experimental measurements of cross-shore sediment transport rate are in agreement with the proposed modelling procedure. (See Table 2). Also, beach profiles shown in Figure 6 are representative and show similar features. It should be noted that the tests of Mimura et al (1986) which have been investigated were characterised by onshore sediment transport.

Table 2. Cross-shore Transport Rate of Mimura et al (1986)

Initial Slope	Q_{s_p} (m^3/hr.)	Q_{s_m} (m^3/hr)	n_{Q_s}	comments
$^1/20$	0.0113	0.001	11	compares
$^1/10$	0.01	0.001	10	well with
				Equation 29

The recommended modelling procedure is also applicable to situations where alongshore sediment transport is predominant. Kamphuis et al (1986) have proposed an equation for alongshore transport rate as follows,

$$\Pi_{Q_s} = 0.002\ (H_{bs}/D)(m/(H_{bs}/L_o)^{\frac{1}{2}}) \tag{30}$$

Moreover, the applicability of this equation to both field results (from which its empiricity was derived) <u>and</u> model scale results was demonstrated by Sayao et al (1985). Equation 30 was shown to predict sediment transport rates around model circular sand islands at various scales and with different grain diameters based on the work of Nairn (1985), see Figure 7 reproduced from Sayao et al (1985). Since the variables in Equation 30 directly correspond to those of Equation 7, it follows that the time scale and sediment transport scale (Equations 28 and 29), based on the distortions of m and H_b/D respectively, will also apply to alongshore sediment transport models.

The cross-shore transport time scale was also investigated using the results of a numerical model for cross-shore sediment transport under random waves. The numerical model is based on a depth-integrated energetics approach as described by Nairn (1988). The modelling procedure is similar to that proposed by Stive (1986) based on the work of Bailard (1981). Profile change is determined from the net sediment transport rates calculated at points in a finite difference scheme

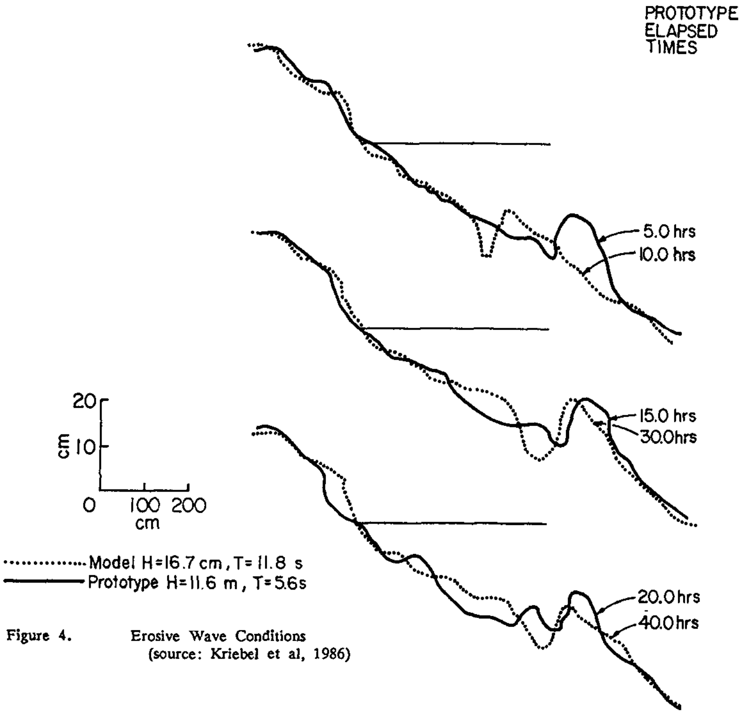

Figure 4. Erosive Wave Conditions (source: Kriebel et al, 1986)

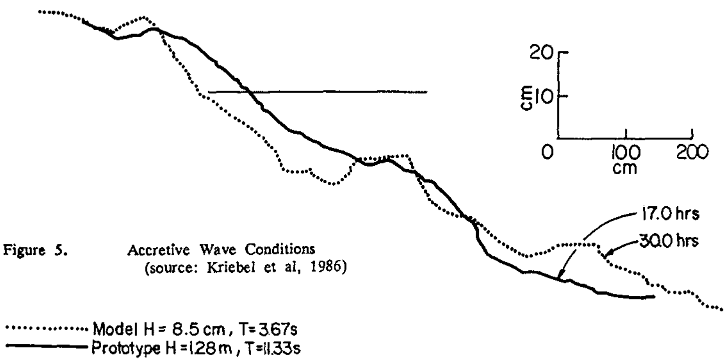

Figure 5. Accretive Wave Conditions (source: Kriebel et al, 1986)

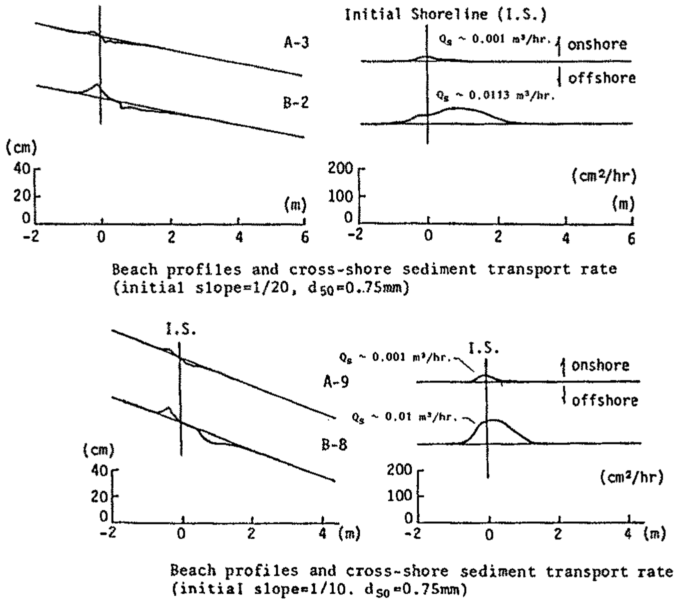

Figure 6. Profiles and Cross-shore Transport
(from, Mimura et al, 1986)

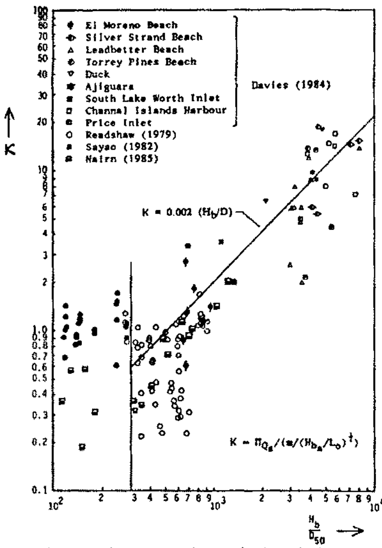

Figure 7. The Functional Dependency of Dimensionless Transport on H_b/D.
(from, Sayao et al, 1985)

across the profile. The sediment transport rates are taken to be proportional to local flow characteristics (including both mean return flow and orbital velocities) which are calculated from a statistical representation of the wave height at any point on the profile. The model has been shown to successfully predict both small and large scale flume laboratory erosion tests. Two pairs of model and prototype tests have been investigated using the numerical model. The model was assumed to be a $^1/10$ scale of a prototype which consisted of waves (H = 1.23 m, T = 6.33 s) incident on an initially plane $^1/40$ profile with a sand grain size of 0.2 mm. This corresponds to a model test described by Stive (1986) with H = 0.123 m, T = 2s and D = 0.09 mm. The results of two tests over different time periods are given in Table 3.

Table 3. Numerical Model Results.

PROTOTYPE RESULTS		MODEL RESULTS ($n = {}^1/10$)		
Prototype Period	Prototype Erosion Predicted (m^3/m)	Time Scales Used	Model Times	Predicted Prototype Erosion Volumes (m^3/m)
12 hrs.	15	$n_{ts} = 6.7$	1.8 hrs	7.0
		$\sqrt{n} = 3.2$	3.8 hrs	12.2
180 hrs.	86	$n_{ts} = 6.7$	27 hrs	49
		$\sqrt{n} = 3.2$	57 hrs	83

These results again demonstrate that the Froude time scale appears to be a better predictor of the offshore sediment transport time scale than that proposed by Equation 28.

8. Conclusions

The similarity of littoral processes including both cross-shore and alongshore sediment transport is due to 5 dimensionless parameters as follows (i) H_b/L_o, (ii) m_b, (iii) H_b/D, (iv) ρ_s/ρ and (v) F_o. In an ideal model the scales of each of these parameters should be equal to 1. However, this is generally not possible. The model design proposed is a geometrically undistorted Froude model with fall velocity scaling of the model sand. In this case scale effects are only due to

$$n_{m_b} \neq 1 \quad \text{and} \quad n_{H_b/D} \neq 1.$$

In the model proposed one can quantify the distortions due to the non-similarity of (ii) and (iii) by the following scales,

$$n_{t_s} = n/\sqrt{n_D} \qquad \text{(Equation 28)}$$

$$n_{Q_s} = n^2\sqrt{n_D} \qquad \text{(Equation 29)}$$

Comparisons to practical examples have successfully demonstrated the validity of these equations in situations of onshore transport in the cross-shore direction and for alongshore transport. The time scale for offshore sediment transport does however appear to be better represented by the Froude time scale than the scale proposed in Equation 28 above.

The conclusions noted above are based on comparisons to laboratory and numerical model results. Validation of the proposed model design and distortion expressions with a comprehensive field data set is still required pending the availability of an adequate data set.

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