

CHAPTER 146

The theoretical temporal structure of the longshore currents

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1) INTRODUCTION

For a better knowledge of the causes about the littoral solid transport is essential to experience advances in the littoral hydrodynamic. In this field the longshore currents are may be who play the most important role due to the intensity, the distribution and the relative duration of them, causing the movement of the solid particles due to the suspension of them and the friction in bottom.

The first theoretical formulation about the longshore currents were established by Bowen (1969), Longuet-Higgins (1970) and Thornton (1970). Previously another works treated an approximation about the problem, obtaining a little precise results basing his works in simplest formulations, it shows in Sonu et al. (1966).

2) AN OBSERVATION ABOUT THE EXISTING SOLUTIONS.

The basic equations of motion are:

- The Conservation of mass equation
- The Conservation of energy equation
- The Conservation of momentum equation.

The conservation of energy equation is not used because two reasons:

1. It is a complex formulation
2. It utilize dissipation mechanism that they are not simples and precises for his formulation.

With the proposit of obtain a simple expression about the phenomenon there are realized series of simplifications, where the most important are:

- a) Vertical and temporal averages in all the equations.
- b) Elimination of the "Y" variable considering the case like a quasi-orthogonal problem; in this way X is the independent variable.

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- c) Utilization of Monochromatic waves.
This simplification together with the others represent waves like a curve $H(x)$.
- d) Utilization of ideal beach profiles, terms linealization and others.

Not all the models use the same simplifications, but the final results is always a distribution $V(x)$ (in some cases $V(x,y)$), that doesn't have in count the temporal nature of the phenomenon, eliminated by one of the simplifications before mentioned.

Battjes (1974) tried to improve this lack trough relations of the parallel velocity V with formulations of the radiation stress in truncated function of the Rayleigh distribution of the waves in a sea state.

3) A NEW ECUATION DEDUCTION.

The objective is to establish a new ecuation were we will find the local longshore current velocity value $V(x,t)$, like a consequence of waves gived by the displacement of the sea surface, like a time serie.

In this way, the longshore current will be a variable in time, which structure is possible to study.

For it we have to revert to the classic basic equations of Navier-Stokes, and make a parallel deduction to them (Phillips 1966, Mei 1983) but without the temporal average.

In conclusion the most important Hypothesis are:

- 1) Admiting the "Shallow Water wave" simplification we have

$$U = \frac{C}{\delta} \zeta \quad (U_x, U_y) = U (-\cos \alpha, \sin \alpha)$$

$$Pz = pg (-z+h+\zeta)$$

$$C = \sqrt{g\delta} \quad \delta = h + d \quad (\text{Figure 1})$$

- 2) Accepting the Snell's Law :

$$\frac{C}{\sin \alpha} = \text{cte}$$

$$\frac{\partial \alpha}{\partial x} \approx \frac{\sin 2\alpha}{2\delta} \quad \frac{\partial \delta}{\partial x} \quad \frac{\partial \alpha}{\partial y} = 0 \quad \frac{\partial \alpha}{\partial t} = 0$$

- 2) Admitting the hypothesis of "long crested waves" we have:

$$\frac{\partial \zeta}{\partial y} = -\operatorname{tg} \alpha \quad \frac{\partial \zeta}{\partial x} \approx -\sin \alpha \quad \frac{\partial \zeta}{\partial x}$$

- 3) For the internal stresses representation we can introduce it by the temporal adverage values:

$$\tau_{xy} \approx \rho \frac{gTH^2}{8\pi^2} \frac{\partial V}{\partial x}$$

Although the others hypothesis will guide us to a temporal equation, the inexistence of this equations take us to the mentioned expression. In this way we can interpret beginning with other previous models results, that the turbulent tensions have only a distribution effect on to the X axis; it has make that some works leaving aside this term presented models with anyone inclusion of this.

- 4) The Set-up will be the proposed by Bowen (1968)
 5) The section could be in any form $d = d(x)$ (Figure 1).
 6) The wave representation in breaking zone is in the following form (Figure 1):

$$\zeta_{X_i, Y_i, t} = \zeta_{X_o, Y_o, t} f_{fi} f_{ri}$$

... were

$\zeta_{X_i, Y_i, t}$ is the sea's displacement in (X_i, Y_i) point at instant t .

$\zeta_{X_o, Y_o, t - \Delta t}$ is the sea's displacement in a reference point (X_o, Y_o) .

Δt is the wave's time employed to go through the points (X_o, Y_o) and (X_i, Y_i)

$t \approx - \int_{X_o}^{X_i} \frac{dx}{C}$... in the hypothesis of "quasi-orthogonal" incidence

$f_{fi} = \frac{H_i}{H_o} \sqrt{\frac{\sin 2\alpha_o}{\sin 2\alpha_i}}$ is the refraction factor at the point i

$f_{ri} = \begin{cases} 1 & \text{if } H_o f_{fi} \leq \gamma_b \\ \frac{1}{H_o f_{fi}} & \text{if } H_o f_{fi} > \gamma_b \end{cases}$ is a breaking form factor at the (X_i, Y_i) point

With it we are looking for a temporal series representation "Sea's displacement at (X_i, Y_i) point" in relation with the correspondent at (X_o, Y_o)

With all is possible formulate an equation of the type:

$$\frac{dV}{dt} \approx f \left(V, \frac{\partial V}{\partial x}, \frac{\partial^2 V}{\partial x^2}, x, t, z \right)$$

This equation is :

$$\begin{aligned} & \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} \left[-c \frac{z}{\delta} - \left(\frac{gTH^2}{8\pi^2\delta D} \left(\frac{\partial z}{\partial x} + \frac{\partial \delta}{\partial x} \right) + \frac{\partial}{\partial x} \left(\frac{gTH^2}{8\pi^2\delta} \right) \right) \right] + \\ & + \frac{\partial V}{\partial y} \left[2 \frac{cz}{\delta} \sin \alpha + 2V \right] + \frac{\partial^2 V}{\partial x^2} \left[-\frac{gTH^2}{8\pi^2\delta} \right] + \\ & + V \left[\frac{1}{D} \frac{\partial z}{\partial t} + \frac{\partial \delta}{\partial x} \left(\frac{cz^2}{D\delta} - \frac{cz}{2\delta^2} \right) + \frac{\partial z}{\partial x} \left(-\frac{c(D+z)}{D\delta} \right) \right] \\ & + V^2 \left[-\frac{\partial z}{\partial x} \frac{\sin \alpha}{D} \right] + \frac{1}{\rho D} \tau^B = \\ & = \frac{\partial z}{\partial x} \left[\frac{c^2 z}{D\delta^2} (2D+z) \sin \alpha + g \sin \alpha \right] + \frac{\partial z}{\partial t} \left[-\frac{c(D+z)}{D\delta} \sin \alpha \right] + \\ & + \frac{\partial \delta}{\partial x} \sin \alpha \left[-\frac{c^2 \delta^2 (D+z)}{D\delta^3} + \frac{g z^2}{\delta^2} + \frac{c^2 z^2}{2\delta^3} \right] + \frac{1}{\rho D} \tau^S \end{aligned}$$

$$D = \delta + z$$

$$\tau^B = \rho C_F \sqrt{\left(\frac{cz}{\delta} \right)^2 + \left(\frac{cz \sin \alpha}{\delta} + V \right)^2} \left(V + \frac{cz}{\delta} \sin \alpha \right) \dots \text{is the}$$

stress in the bed

$$\tau^S = \rho k W^2 \cos \Theta \dots \text{or similar, is the stress due to}$$

$$k \begin{cases} 1.1 \cdot 10^{-6} & W \leq 7.2 \text{ m/s} \\ 1.1 \cdot 10^{-6} + 2.5 \cdot 10^{-6} \left(1 - \frac{7.2}{W} \right) & W > 7.2 \text{ m/s} \end{cases} \quad \text{the wind action}$$

For the explicit form resolution we must utilize a small Δt . If we admit that the time discretization error is the same order of magnitude then the spatial discretization, beginning with the wave phases term we can obtain:

$$\frac{2\pi x}{L} + \frac{2\pi t}{T} \quad \Delta x = C \Delta t$$

CONCLUSIONS

As a conclusion in accord with all before mentioned, we can conclude that we have basis for the study of the theoretical temporal structure of the longshore currents. This is based in anew deduction of the movement equation where we don't have to make a temporal averages in it development and let the inoident wave comes from a unic time serie $z(t)$ of the superficial sea displacement at (X_0, Y_0) point of reference in the contour, with discretization in Δt intervals.

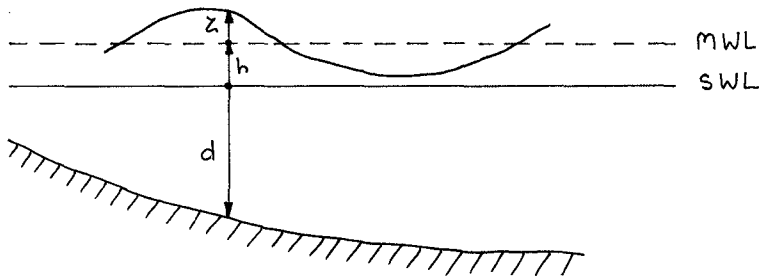


Figure 1: A section definition.

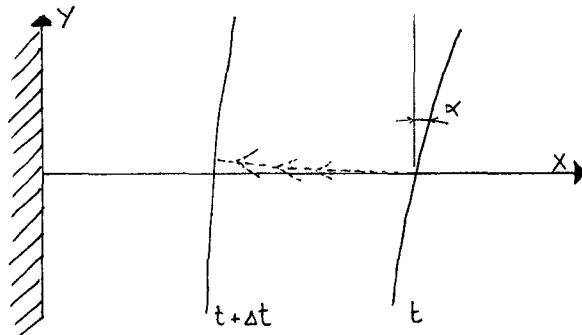


Figure 2: A Plant Definition.

REFERENCIAS

BATTJES, J.A. (1974). "Computation of Set-up, Longshore Currents, Run-up and Overtopping due to Wind-generated waves". Delft University

BOWEN, A. (1969). "The generation of Longshore Currents on a plane beach". J. of Marine Research Vol. 27, 2, pp. 206-215

LONGHUET-HIGGINS, M.S. (1970). "Longshore Current Generated by Obliquely incident Sea Waves. 2". J.of Geophysical Research. Vol. 75, n^o 33, pp. 6790-6801.

MEI C. (1983). "The Applied Dynamics of Ocean Surface Waves". John Wiley & Sons. N.Y.

PHILLIPS, O.M. (1966). "The Dynamics of the Upper Ocean". Cambridge University Press. London

SONU, C.; Mc. CLOY, J.M. y Mc. ARTHUR, D.S. (1966) "Longshore Currents and Nearshore Topographies" I.C.C.E., Chp. 32, pp. 325-349.

THORNTON, E.B. (1970). "Variation of Longshore Currents Across the Surf Zone". I.C.C.E., Chp. 18, pp. 291-308.

