CHAPTER 4

MODEL PREDICTIONS OF NONBREAKING SHOALING WAVES

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ABSTRACT

The predictions of linear and nonlinear (Boussinesq) shoaling wave models for nonbreaking unidirectional surface gravity waves are compared to field observations, with particular emphasis on quantities that may be important for cross-shore sediment transport. The extensive data sets were obtained on two natural beaches, span water depths between 1-10 m, and include incident wave power spectra with narrow, broad, and bimodal shapes. Significant wave heights varied between approximately 30 and 100 cm and peak periods between approximately 8 and 18 seconds. Only the nonlinear theory predicts the increasingly asymmetric sea-surface elevations and horizontal velocities (pitched-forward wave shape) and the weaker variation of skewness (difference between crest and trough profiles) which are observed to occur during shoaling. The nonlinear theory also models qualitatively well the large skewed accelerations which occur during the passage of asymmetric waves.

INTRODUCTION

Because nonbreaking shoaling waves are both weakly nonlinear and weakly dispersive they are frequently described by models based on the nonlinear Boussinesq equations (Peregrine 1967). In general, the Boussinesq equations include the effects of shoaling, refraction, reflection, and diffraction for arbitrary wave fields (i.e. directionally spread and broad banded in frequency). Most implementations of Boussinesq shoaling models include a subset of these phenomena, and have been successfully tested against a variety of laboratory data and analytical results. Freilich & Guza (1984, FG) and Liu et al. (1985) have respectively derived one- and two-dimensional nonlinear shoaling models based on perturbation solutions to the Fourier transformed Boussinesq equations. The models clearly identify nonlinear near-resonant triad interactions as the primary cause for evolution of third moments of the wave field. The one-dimensional, many mode (i.e. broad banded in frequency) Boussinesq model has been compared to a limited set of ocean field data (FG, Elgar and Guza 1985 (EG), 1986, Elgar at al. 1990).

The Boussinesq equations require both shallow water depths ($(kh)^2 << 1$, where k is the wavenumber and h is the water depth) and small wave amplitudes ($a/h << 1$, where a is the wave amplitude) such that the Ursell number, $U = (a/h)/(kh)^2$ is approximately unity. The one-dimensional shoaling model assumes that the waves are normally incident to a beach with plane-parallel contours, and neglects dissipation and reflection.

The one-dimensional model is cast in terms of coupled, nonlinear, ordinary differential equations with the (temporal) Fourier coefficients of the wave field as the dependent variables. Since the model describes the spatial evolution of the Fourier coefficients (i.e. both the amplitudes and phases), it contains information relating to wave shapes and instantaneous oscillatory velocities.

Freilich and Guza (1984) give details of numerically implementing the nonlinear model. Fourier coefficients used as initial conditions for nonlinear model predictions are provided by measurements at the seaward edge of the region of interest. The model equations are then integrated numerically, yielding predicted values of Fourier coefficients of sea-surface elevation in shallower water. The predicted and observed Fourier coefficients can then be manipulated and compared in various ways. Alternatively, after inverse Fourier transforming the predicted coefficients, comparisons can be made between predicted and observed time series.

The present study evaluates the performance of the 1-D shoaling model for nonbreaking waves in 18 data sets obtained from month-long field experiments at two beaches. A variety of incident wave conditions were observed, including swell from a distant storm, locally generated sea, and combinations of swell and sea. The model performance is good. The spatial evolution of sea surface elevation (SSE), velocity, and acceleration statistics are at least qualitatively well predicted for a wide range of ocean conditions.

FIELD EXPERIMENTS AND DATA REDUCTION

Two field experiments conducted in 1980 (Torrey Pines and Santa Barbara, California) provide the data used for model verification. The bottom contours were relatively straight and parallel at both experimental sites, and the mean beach slopes through the shoaling region were 0.022 and 0.050 at Torrey Pines and Santa Barbara, respectively. Data were obtained from wave staffs and bottom-mounted pressure and electromagnetic current meters. The field experiments, including representative beach profiles and descriptions of the sensors and data reduction are presented in FG, EG, and Thornton & Guza (1986). Measurements from cross-shore arrays extending for approximately 267 m (Torrey Pines) and 56 m (Santa Barbara) are used in the model-data comparisons presented below.

Initial conditions for the nonlinear Boussinesq shoaling model were generated with data from a bottom-mounted pressure sensor in 10 m depth at Torrey Pines and in 4 m depth at Santa Barabara. Short sections of data were Fourier transformed and converted to Fourier coefficients of sea-surface elevation using linear finite depth theory. Results of integrations of the Boussinesq shoaling wave model for consecutive short sections were averaged together for statistical comparisons. The maximum frequency considered is 0.234 and 0.4 Hz at Torrey Pines and Santa Barbara, respectively. The different cutoff frequencies reflect the requirement that the waves be relatively long compared to the depth, and the relatively deeper water at Torrey Pines.

All pressure and current meters were positioned within 80 cm of the sea bed, and the pressure data were converted to sea-surface elevation using linear theory. Because linear theory accurately relates local values of near-bottom pressure and elevation in nonbreaking waves (Guza & Thornton 1980 and references therein), hereafter no differentiation will be made between direct measurements of sea-surface elevation and sea-surface elevation inferred from pressure data. Comparisons between model predictions and current meter data are made at the known depth of the current meter sensing element (i.e. no theory is applied to the current meter data).

Energy dissipation was not important in the model-data comparisons discussed here because the evolution distances were relatively short, whitecapping was not pronounced, and the comparisons were terminated when measured energy losses owing to wave breaking were significant. The Torrey Pines experiment was designed to study nonbreaking waves, and thus all sensors were seawards of the breaking zone and dissipation was found to be negligible. Many of the Santa Barbara sensors were sometimes within the surf zone, and the estimated dissipation was sometimes significant in depths less than 1.6 m. Model-data comparisons are presented only for sensors where the total shoreward energy flux (integrated over all frequencies) was at least 85% of the value measured at the most seaward instrument.

The effects of directional spread and/or non-normal incidence in the incoming wave field on the nonlinear evolution of shoaling waves are not yet well understood. Boussinesq models appropriate for this case (e.g. Liu et al. 1985) have not been applied to random ocean waves. The data sets discussed here include locally generated wind-driven seas having broad directional spread, as well as wave fields composed of swell and sea arriving from different directional quadrants. Although the incident wave field was neither unidirectional nor normally incident, 1-D Boussinesq model predictions are possible because fundamentally nondirectional statistics are considered here. Moreover, as refracting surface waves propagate into shallower water they are strongly polarized in the cross-shore direction and thus the approximation of normal incidence often is not grossly violated. A longshore array of sensors in 10 m depth at Torrey Pines and a colocated pressure sensor-bidirectional current meter pair in 4 m depth at Santa Barbara showed that the principal wave directions at the offshore, initial conditions for the model predictions are less than 20° relative to normal incidence (FG, EG, Thornton & Guza 1986, Freilich et al. 1990)

MODEL-DATA COMPARISONS

As a primarily swell wave field (S11 and F2 in fig 1) shoals, the power spectrum undergoes significant evolution, with harmonics of the spectral peak increasing in power with decreasing water depth (fig 2). Linear finite depth theory (LFDT) does not predict the growth of harmonics, but the Boussinesq model accurately predicts the observed spectral evolution, except for high frequencies (e.g. $f \geq 0.3$ Hz for the data considered here). Along with cross-spectral transfers of energy (e.g. harmonic growth) owing to nonlinear interactions as the waves shoal, there is also substantial nonlinear phase evolution of the individual Fourier components (equivalent to a nonlinear effect on the phase speed). In shallow water where nonlinear effects are largest, phase differences between the nonlinear model and the data are small, while the phase differences between data and LFDT are large (fig 2). The coherence between the nonlinear model predictions and data is high (fig 2), except for a decreasing coherence with increasing frequency, which can be explained by directional spreading of the wave field (FG). Boussinesq model predictions for wave fields with broad band and bimodal spectra are also more accurate than LDFT predictions (not shown). LFDT predictions that include the effects of the directional distribution of energy (EG) are not substantially better than the unidirectional LFDT predictions shown in fig 2.

As waves shoal, their profiles evolve from nearly sinusoidal shapes in deep water to positively skewed (sharp peaks and broad troughs) shapes to vertically asymmetrical, sawtooth shapes just prior to breaking. The change in wave form during shoaling is statistically described by the skewness, S and

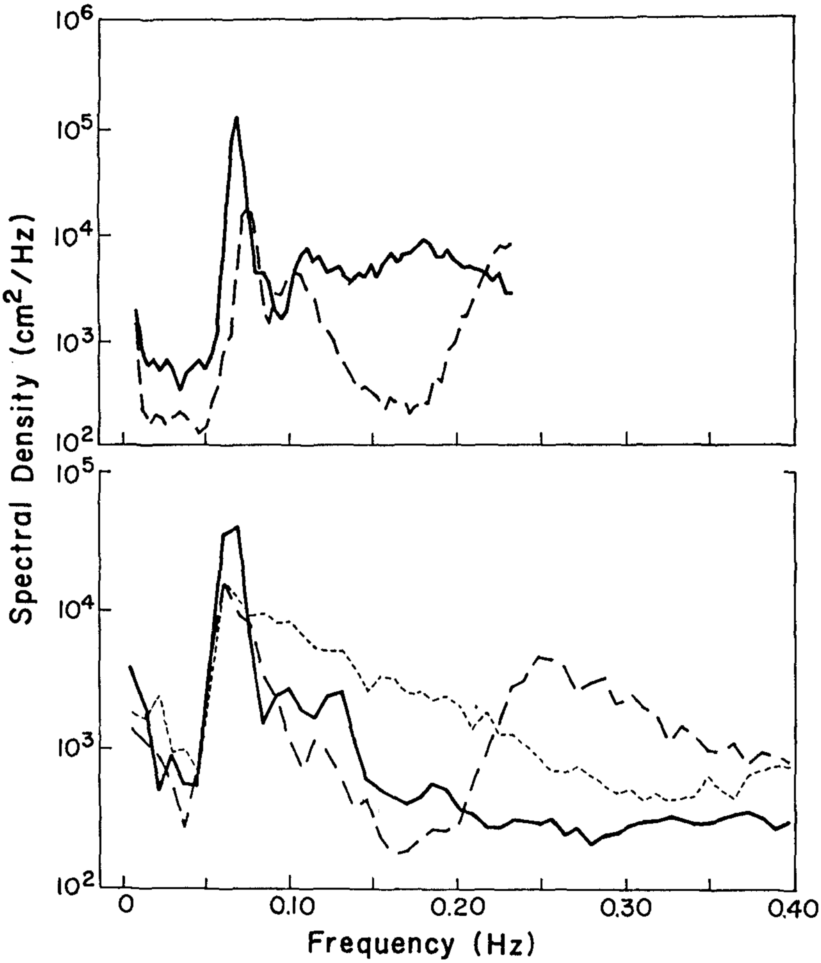

Figure 1. Initial power spectra of sea surface elevation for the model predictions. Top, Torrey Pines, $h \sim 10$ m (solid line, S11, $H_{sig} = 90cm$; dashed line, S16, $H_{sig} = 56cm$); bottom, Santa Barbara, $h \sim 4$ m (solid line, F2, $H_{sig} = 63cm$; dashed line, F12, $H_{sig} = 57cm$; dotted line, F15, $H_{sig} = 66cm$), where H_{sig} is the significant wave height at the depth indicated.

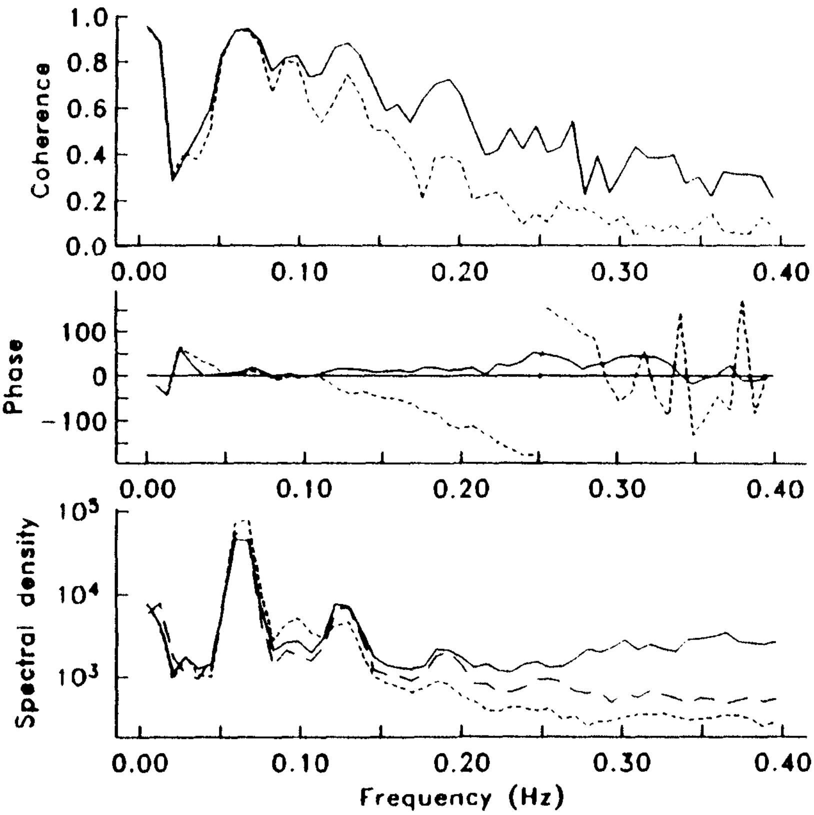

Figure 2. Comparison of model predictions (based on initial conditions in 4.5 m depth, F2 in fig 1) to swell data in 1 m depth. Coherence and phase differences between Boussinesq (solid line) and LFDT (dotted line) model predictions and observations are shown in the upper and center panels, respectively. The power spectra predicted by the Boussinesq (solid line) and LFDT (dotted line) models are compared to observed values (dashed line) in the lower panel.

asymmetry, A (third moments of the wave field, which measure deviations from symmetry about the horizontal and vertical axis, respectively (Masuda & Kuo 1981)). A sawtooth shape has $S = 0$ and $A \neq 0$, while a "Stokes wave" shape (broad, low troughs and narrow, tall crests) has $S \neq 0$ and $A = 0$.

The observed and predicted evolution of skewness and asymmetry for swell is displayed in fig 3. Third moments are small in deep water ($A = S = 0$ for linear waves), and increase owing to nonlinear interactions as the waves shoal. In both the observations and the model predictions, skewness of SSE and velocity attains a maximum and starts to decrease before the waves break (wave breaking is significant only for the shallowest sensor shown in fig 3). Asymmetry increases approximately monotonically, consistent with the steepening shape of shoaling waves. The random phase assumption underlying linear theory results in sinusoidal waves, and thus LFDT cannot predict the changes in wave shape as the wave field shoals. On the other hand, the Boussinesq model accurately predicts the observed evolution of third moments of the wave field, as shown in fig 3.

Observed and predicted third moments for a broad band wave field (i.e. locally generated sea, F15 in fig 1) are shown in fig 4. Although the shape of the power spectrum at the seaward edge of the shoaling region differs from the narrow band wave field discussed above, many aspects of the evolution during shoaling are similar. In particular, the wave shapes undergo similar shoaling evolution from sinusoidal to sawtooth profiles.

For broad band wave fields, the total skewness and asymmetry are not dominated by contributions from a few isolated harmonic triads, as is the case with narrow band wave fields. Rather, nonlinear interactions significantly couple many frequencies within the wind wave band, with each triad of coupled waves contributing to the overall third moments. The assumptions underlying the Boussinesq model become invalid at high frequencies where the lowest order Boussinesq dispersion relation deviates significantly from the exact finite-depth solution. Thus, it is not surprising that nonlinear model predictions of third moments for broad banded conditions (fig 4) are not as accurate as those for swell-dominated spectra (fig 3). This is especially true for acceleration statistics (fig 4c), where high frequency motions are even more important (Elgar et al. 1988). Nonetheless, the nonlinear model correctly predicts the depth-dependent trends in the third moments of sea-surface elevation, horizontal velocity, and acceleration.

Observed and predicted third moments for a wave field consisting of both sea and swell, S16 and F12 in fig 1) are shown in fig 5. The sea and swell arrived at the outer edge of the shoaling region from different directions, separated by about 45° at Santa Barbara and about 25° at Torrey Pines. As in the narrow- and broad-band cases discussed above, the steepening of the wave profile during shoaling is fairly well predicted by the Boussinesq model, as shown in fig 5a. The predictions of near-bottom velocity third moments (figs

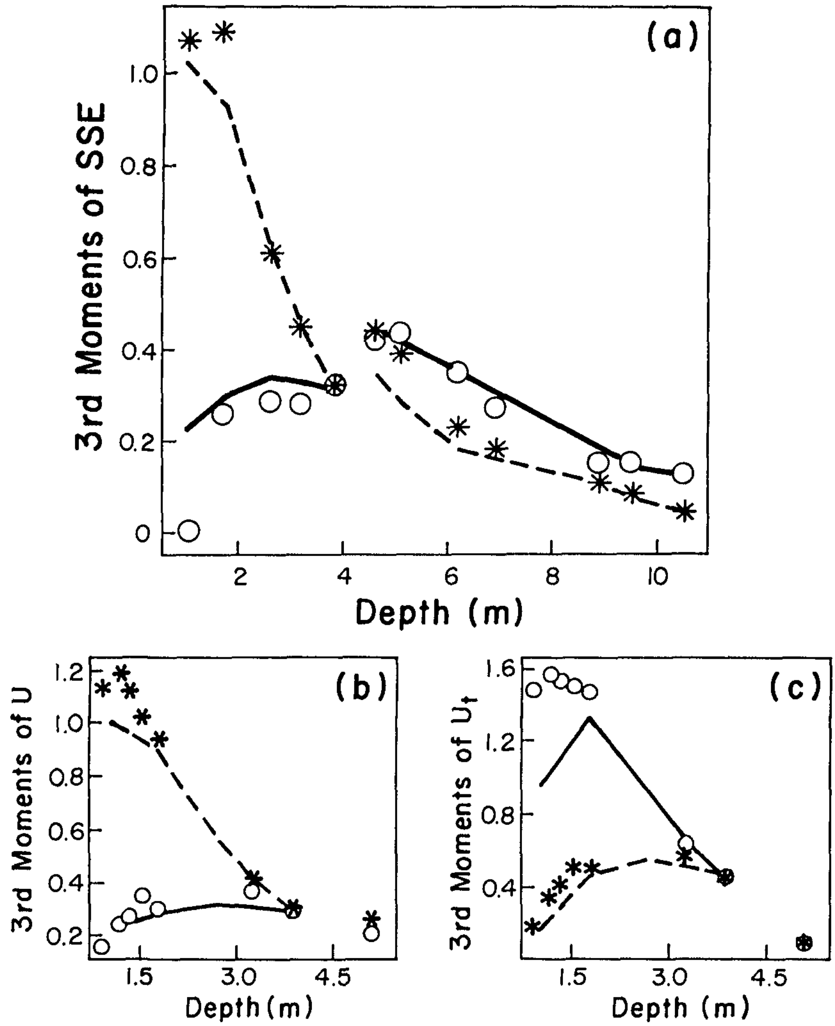

Figure 3. Predicted and observed normalized third moments versus depth for the narrow band swell data ($h > 4$ m, Torrey Pines (S11); $h \leq 4$ m, Santa Barbara (F2)). a) sea-surface elevation; b) near-bottom horizontal (e.g. cross-shore) velocity; c) near-bottom horizontal acceleration. Solid and dashed lines are model predictions of skewness and -asymmetry, respectively. Circles and asterisks are observed values of skewness and -asymmetry, respectively.

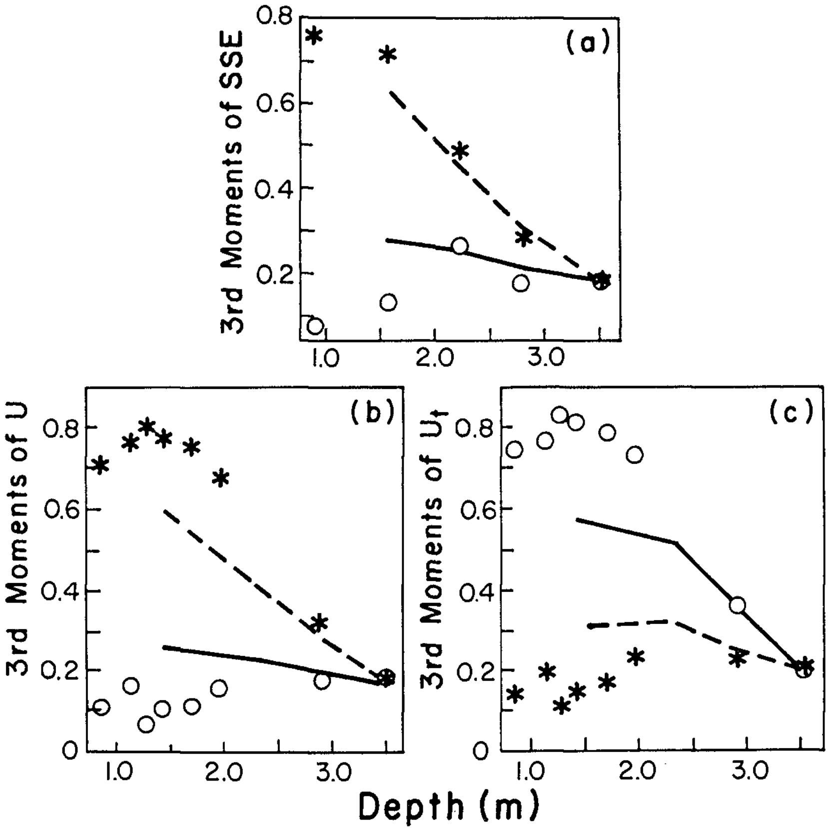

Figure 4. Predicted and observed normalized third moments versus depth for the broad band data, Santa Barbara (F15). a) sea-surface elevation; b) near-bottom horizontal velocity; c) near-bottom horizontal acceleration. Format is the same as Figure 3.

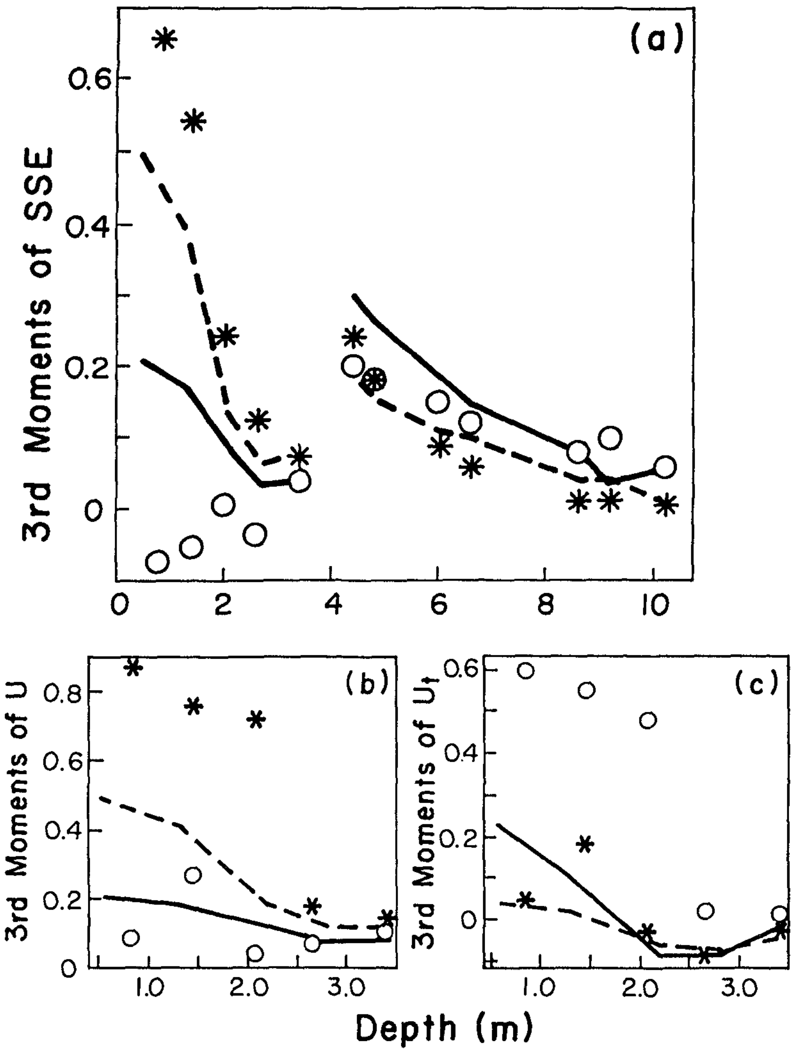

Figure 5. Predicted and observed normalized third moments versus depth for the bimodal data ($h > 4$ m, Torrey Pines (S11); $h \leq 4$ m, Santa Barbara (F2)). a) sea-surface elevation; b) near-bottom horizontal velocity; c) near-bottom horizontal acceleration. Format is the same as Figure 2.

5b,c) are considerably less accurate, perhaps owing to directional effects.

From the model-data comparisons discussed above it is clear that the one-dimensional Boussinesq model predicts the evolution of shoaling waves for the conditions considered at least qualitatively well. In addition, model-data comparisons for many more data sets are displayed in fig 6, where predictions of SSE and near-bottom horizontal velocity third moments are compared to observed values. The Boussinesq model predictions of SSE skewness are accurate for both field sites. The predictions of SSE asymmetry for the Torrey Pines data are qualitatively correct, but somewhat less than observed values, while the predictions of SSE asymmetry for the Santa Barbara data are accurate (fig 6a). Model predictions of third moments of near-bottom velocity in 4.5 m depth, 250 m from the initial conditions ($h \approx 10$ m) at Torrey Pines and for 12 and 56 m from the initial conditions at Santa Barbara are compared to observations in fig 6b. Overall the predictions are good. The Boussinesq model also provides accurate predictions of acceleration skewness and asymmetry (fig 6c).

CONCLUSIONS

Given measurements of the incident wave field, low-order statistics of the shoaling wave field seawards of the surf zone can be accurately predicted by the Boussinesq equations, as demonstrated by the model-data comparisons presented above. The Boussinesq model has no free or adjustable parameters, is not limited to any particular spectral shape, and accurately predicts the evolution of the wave field for swell, locally generated sea, combinations of swell and sea, and other typical field conditions. The nonlinear model also is not dependent on the particular field location, as long as dissipation outside the surf zone and reflection from the beach face are negligible. Although the two beaches discussed here were nearly planar, more complex bathymetry can, in principle, be incorporated into the nonlinear model.

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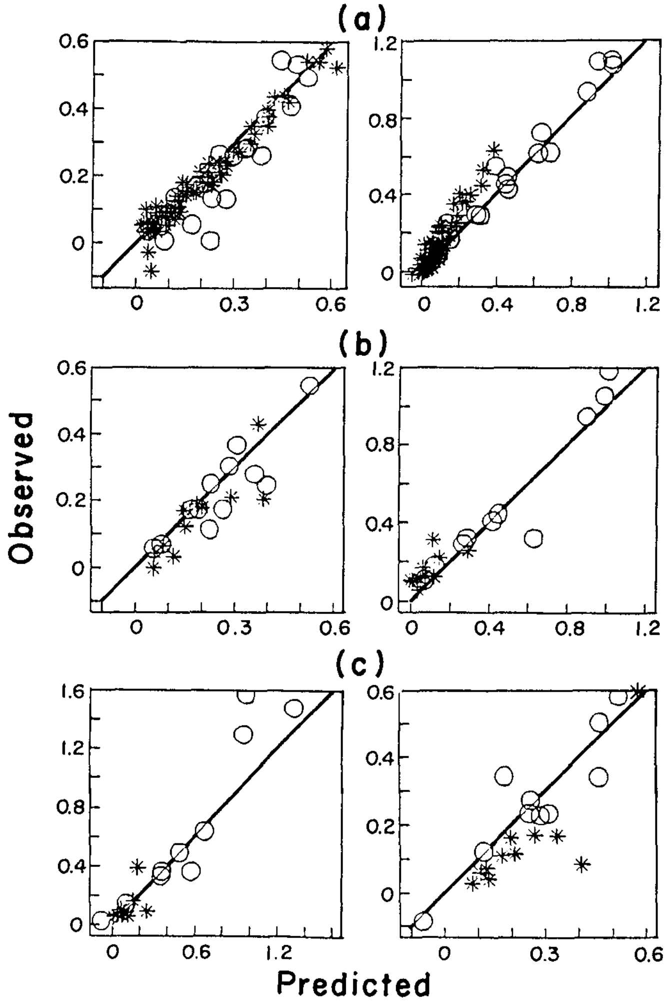

Figure 6. Observed third moments versus Boussinesq model predictions of third moments. Left hand panels, skewness; right hand panels, -asymmetry. a) SSE; b) near-bottom horizontal velocity; c) near-bottom horizontal acceleration. Asterisks are Torrey Pines beach and circles are Santa Barbara. Values falling on the 45° solid lines correspond to agreement between data and model predictions.

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