CHAPTER 6

THE GROUP CHARACTERISTICS OF SEA WAVES

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Abstract -- A review concerning the methods of studying and describing wave groups is presented in this paper. After analysing 73 field records oollected in Shijiu Port, China, the measured parameters of wave groups and some factors describing wave groupiness and their variations are given. Moreover, these results are compared with that of theory.

1. INTRODUCTION

It is found that the higher waves often appear in group in a continuous record. The existence of wave groups has been known to sailors for a long time, but the phenomena had not been studied systematically until the first year of 1970's. Many experiments have shown that the wave groups are not only more disastrous to the rubble mound breakwaters, the moored floating bodies and capsizing of ships, but also important for harbour resonance. Therefore wave groups received more attentions and they are studied by field experiments, simulation (physioal and numerioal) tests and theoretical analyses. In this paper, the existing methods for studying wave groups and some factors describing wave groupiness are summarized. Then, the 73 field records collected in Shijiu Port are analysed. Based on these analyses, the wave group parameters and the factors describing wave groupiness and their variations are investigated, and their results are used to examine the existing theoretical methods. It may be a good reference for theoretical study and engineering practice.

2. METHODS OF STUDYING WAVE GROUPS

2.1 Theory of wave envelope

The ooncept of wave group has been proposed in classical hydrodynamics. Wave groups can be formed by two trains of waves of the same height propagating in the

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same direction with slightly different frequencies. But the sea waves are considerably more complicated. Nolte (1972) and Ewing (1973) used the envelope theory proposed by Rice and Longuet-Higgins to study the sea wave groups and obtained many results. In Fig.1, the crest and trough envelope can be obtained by linking the top and bottom points respectively. Making a horizontal line at the specified level to cross the crest envelope, t_1, t_2 etc. are obtained which are

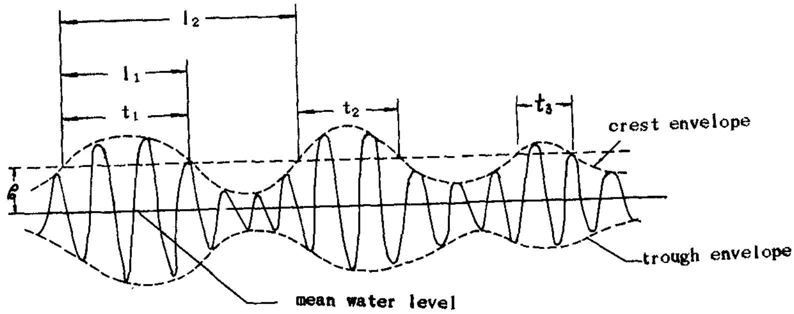

Fig.1 Sketch of wave envelope

called the time durations of wave groups at ρ_0 level. One can also use the wave number l_1 included in a wave group to describe the time duration. In the case of narrow-band spectrum, the mean wave number of wave group expressed by the spectral moment can be derived using the envelope theory, i.e.,

$$\bar{l}_1=[m_2/(2\pi\mu_2)]^{1/2}m_0^{1/2}/\rho_0 \qquad (1)$$

where m_r is the rth moment of wave spectrum and μ_r is the rth central spectral moment.

$$\mu_r=\int_0^{\infty}(\omega-\bar{\omega})^r\, S(\omega)d\omega \qquad (2)$$

where $\bar{\omega}=m_1/m_0$

The mean wave number between two successive wave groups is defined as

$$\bar{l}_2=[m_2/(2\pi\mu_2)]^{1/2}m_0^{1/2}/\rho_0\cdot\exp[\rho_0^2/(2m_0)] \qquad (3)$$

2.2 Statistical theory of runs

Goda (1976) determined the wave heights by means of the zero-up-crossing method and defined successive waves whose heights exceed a specified threshold Hc as a run. As shown in Fig.2, the threshold $H_c=H_{1/3}$ and the first run contains 3 waves in a group ($j_1=3$). He also defined the wave number between two successive wave groups exceeding Hc as the total run length j_2. For example, $j_2=8$ in Fig.2. Assuming the change of successive wave height is stochastic and independent, and letting the occurrence probability of $H > H_c$ be denoted as p_0

and non-occurrence probability by $q_o = 1-p_o$, the mean run length and its standard

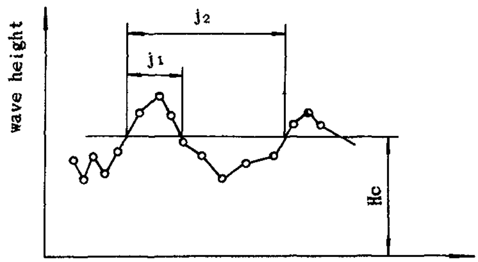

Fig.2 Sketch of run

deviation can be obtained by the probability distribution method as

$$\bar{j}_1 = 1/q_o \qquad \sigma(j_1) = \sqrt{p_o}/q_o \tag{4}$$

$$\bar{j}_2 = 1/p_o + 1/q_o \qquad \sigma(j_2) = \sqrt{p_o/q_o^2 + q_o/p_o^2} \tag{5}$$

In the case of $H_c = H_{1/3}$, according to the Rayleigh distribution $p_o = 0.1348$, $q_o = 0.8652$, one can obtain that $\bar{j}_1 = 1.16$, $\sigma(j_1) = 0.42$ and $\bar{j}_2 = 8.57$, $\sigma(j_2) = 6.91$.

Goda pointed out that in the case of wide-band spectrum, the run length got by Eq.(1) is probably less than 1, which is irrationable. He tried to modify it and derived the relationship between the average number of wave in a wave group $\bar{j}_1$ and the mean length of the group $\bar{l}_1$ as follows

$$\bar{j}_1 = 1/\{1-\exp[-1/\bar{l}_1]\} \tag{6}$$

Goda used the spectral peakedness parameter Qp to describe the spectral width, i.e.,

$$Qp = 2/m_o^2 \int_0^\infty f \cdot s^2(f) \cdot df \tag{7}$$

He obtained that $\bar{j}_1$ increases with increasing Qp from field data and numerical simulation. But the values of $\bar{j}_1$ from field data are very scattered and they are greater than that from Eq.(6) generally. It is because the successive wave heights in field are not independent. The average value of the correlation coefficients between successive waves heights is 0.324 (see Table 2) from our data.

Kimura(1980) gave the probability theory of wave run length by considering the correlation of wave heights. Assuming the distribution of wave height obey Rayleigh distribution, he derived the joint probability function, $p(H_1,H_2)$ of two successive wave heights (H_1 and H_2) as

$$p(H_1,H_2) = 4H_1H_2/[(1-4\rho^2)Hrms^4] \exp\{-(H_1^2+H_2^2)/[(1-4\rho^2)Hrms^2]\} \\ I_o\{4H_1H_2/[(1-4\rho^2)Hrms^2]\} \tag{8}$$

where Hrms is the root-mean-square value of wave heights, I_o denotes the modified Bessel function of zeroth order and ρ is a correlation parameter and can be defined as a function of the correlation coefficient $\gamma_{HH}(1)$ between H_1 and H_2, i.e.,

$$\gamma_{HH}(1)=[E(2\rho)-(1-4\rho^2)K(2\rho)/2-\pi/4]/(1-\pi/4) \tag{9}$$

where K and E are the complete elliptic intergrals of first and second kinds respectively. $\gamma_{HH}(1)$ will be defined by the following Eq.(21).

Let the probability that neither H_1 nor H_2 exceeds the threshold H^* be denoted as p_{11} and that of H_1 and H_2 simultaneously exceed as p_{22}, i.e.,

$$\left.\begin{aligned} p_{11}&=\int_0^{H^*}\int_0^{H^*} p(H_1,H_2)\, dH_1\, dH_2 \Big/ \int_0^{H^*} q(H_1)\, dH_1 \\ p_{22}&=\int_{H^*}^{\infty}\int_{H^*}^{\infty} p(H_1,H_2)\, dH_1\, dH_2 \Big/ \int_{H^*}^{\infty} q(H_1)\, dH_1 \end{aligned}\right\} \tag{10}$$

where $q(H_1)$ is the wave height distribution which Rayleigh distribution can be used, i.e,

$$q(H_1)=2*H_1/Hrms^2*\exp(-H_1^2/Hrms^2) \tag{11}$$

Then the probability of the run with the length of j_1 can be given by

$$p(j_1)=p_{22}^{j_1-1}(1-p_{22}) \tag{12}$$

The mean and the standard deviation of run length are

$$\bar{j}_1=1/(1-p_{22}) \qquad \sigma(j_1)=\sqrt{p_{22}}/(1-p_{22}) \tag{13}$$

The probability of the total run length is

$$p(j_2)=\frac{(1-p_{11})(1-p_{22})}{(p_{11}-p_{22})}\left(p_{11}^{j_2-1}-p_{22}^{j_2-1}\right) \tag{14}$$

The mean and the standard deviation of total run length are

$$\left.\begin{aligned} \bar{j}_2&=1/(1-p_{11})+1/(1-p_{22}) \\ \sigma(j_2)&=[1/(1-p_{11})^2+1/(1-p_{22})^2+1/(1-p_{11})+1/(1-p_{22})]^{1/2} \end{aligned}\right\} \tag{15}$$

The agreement between Kimura's theoretical results and $\bar{j}_1, \bar{j}_2$ obtained from field swell record ($\gamma_{HH}(1)=0.681$) is quite good.

2.3 Wave energy history method

Because the wave energy is directly proportional to the variance of the wave record, the energy at the crest of a crest envelope is greater than that at its trough (Fig.1). The wave groups can be described effectively by the wave energy history. Funke and Mansard (1979) and Nelson (1980) proposed some similar methods. Here, Funke's method is illustrated.

Calculate the mean square of the wave surface elevation within a time duration (time window) Tp and define it as the instantaneous wave energy E(t) at the middle time t of Tp.

$$E(t)= 1/Tp\int_{\tau=-Tp/2}^{Tp/2} \eta^2(t+\tau)d\tau$$

It can be rewritten as follows after smoothing above E(t).

$$E(t)= 1/Tp\int_{\tau=-Tp}^{Tp} \eta^2(t+\tau)Q_1(\tau)d\tau \qquad (16)$$

$$Tp \leqslant t \leqslant Tn-Tp$$

where Tp is the wave period at the spectral peak, Tn the total length of wave record and $Q_1(\tau)$ the smoothing function. A superior smoothing function is the Bartlett window which is given as

$$Q_1(\tau)=1-|\tau|/T_p \qquad -T_p \leqslant \tau \leqslant T_p \qquad (17)$$
$$=0 \qquad \text{others}$$

E(t) defined by Eq.(16) is called smoothed instantaneous wave energy history (SIWEH) and can describe the extent of wave groupiness (Funke and Mansard, 1980, Yu, 1986) effectively (Fig.3). Let $\mathcal{E}(f)$ denotes the spectral density of SIWEH, i.e.,

$$\mathcal{E}(f)=2/Tn\left|\int_0^{Tn} [E(t)-\bar{E}] \exp(i\omega t)dt\right|^2 \qquad (18)$$

$$\bar{E}=1/Tn\int_0^{Tn} E(t)dt=\int_0^{\infty} S_{\eta\eta}(\omega)d\omega=m_0 \qquad (19)$$

Funke and Mansard used the groupiness factor GF to describe the extent of wave groupiness, i.e.,

$$GF=\sqrt{1/Tn\int_0^{Tn} [E(t)-\bar{E}]^2dt} \;/\; \bar{E}=\sqrt{m_{\varepsilon,0}}/ m_0 \qquad (20)$$

where $m_{\varepsilon,0}$ and m_0 are the zeroth order moments of SIWEH spectrum and the wave spectrum respectively. The field data of wind waves from the Sea of Japan during a period of six months gave the value of GF in the range of 0.46 and 0.94. From our data, GF changes from 0.36 to 0.93.

Rye (1982) also studied the wave groups by the correlation function.

3. WAVE GROUOPINESS FACTORS

The parameters mentioned above such as the number of wave l_1 in a wave group whose heights exceed the specified threshold Hc (or run length j_1) and the number of waves l_2 between two wave groups in succession (or total run length j_2) and so on., are insufficient to represent the wave groupiness. So some factors have been proposed to describe the wave groupiness. In this paper, the following five factors will be studied.

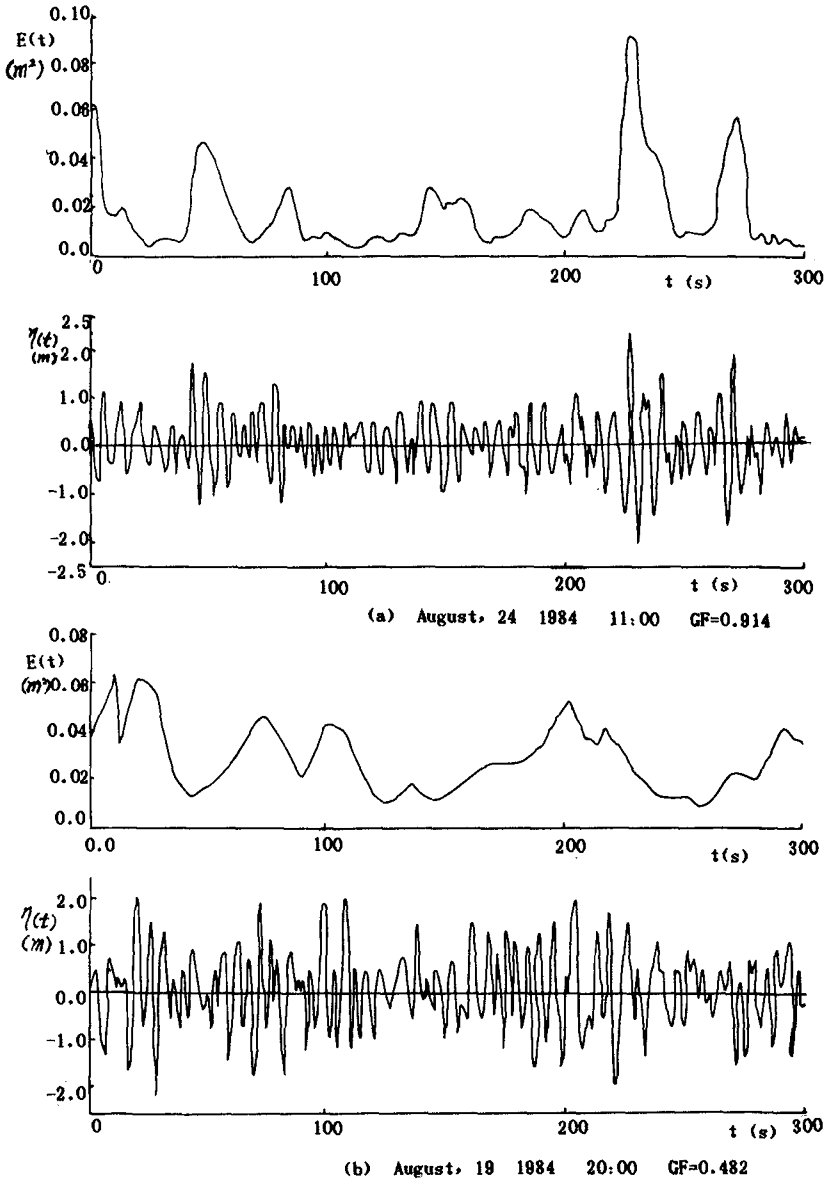

Fig.3 Examples of measured SIWEH

(1). Correlation coefficient between succeeding wave heights $\gamma_{HH}(1)$

$$\gamma_{HH}(1)=\frac{1}{\gamma_{HH}(0)(N-1)}\sum_{i=1}^{N-1}(H_i-\bar{H})(H_{i+1}-\bar{H}) \tag{21}$$

$$\gamma_{HH}(0)=\frac{1}{N}\sum_{i=1}^{N}(H_i-\bar{H})^2$$

where $\bar{H}$ is the average wave height determined by zero-up-crossing method. Larger $\gamma_{HH}(1)$ means that high wave follows high wave. Rye computed $\gamma_{HH}(1)$ for 60 wave records from the North Sea and the average value of $\gamma_{HH}(1)$ was 0.24. Su reported that it was 0.329 for 6 records of wind waves from the Mexico Bay. For swell, $\gamma_{HH}(1)$ is up to 0.68.

(2). Correlation coefficient between succeeding wave periods $\gamma_{TT}(1)$.

$\gamma_{TT}(1)$ is defined in the same way as $\gamma_{HH}(1)$ by replacing H_i and $\bar{H}$ with T_i and $\bar{T}$ respectively. Existing observed results show that $\gamma_{TT}(1)$ varied between 0 and 0.5.

(3). Correlation coefficient γ_{HT} between the wave height and the period.

$$\gamma_{HT}=\frac{1}{N\sigma_H\sigma_T}\sum_{i=1}^{N}(H_i-\bar{H})(T_i-\bar{T}) \tag{22}$$

where σ_H and σ_T are the standard deviations of wave height and wave periods respectively. γ_{HT} is generally greater than zero.

(4). Wave groupiness factor GF

It has been defined by equatin (20).

(5). Another parameter representing wave groupiness $\bar{T}_{SIVEH}$

$$\bar{T}_{SIVEH}=\frac{1}{T_p N_s}\sum_{j=1}^{N_s}(T_{SIVEH})_j \tag{23}$$

where T_{SIVEH} is the zero-up-crossing period of the time trace of $[E(t)-\bar{E}]$. $\bar{T}_{SIVEH}$ is the average of T_{SIVEH} normalized to peak period of wave spectrum and can represent the wave number in one group approximatelly.

4. THE CHARACTERISTICS OF THE WAVE GROUPS IN SHIJIU PORT

4.1 General description of field data

During September to November, 1984, a ENDECO 949 recording buoy made in U.S.A. was installed in Shijiu Port, Shandong Province, China, at 14.8m of water depth (d) by the First Design Institute of Navigation Engineering, Ministry of Communications. Wave data were collected for about 17 minutes every three hours. The sampling of record was done at the interval of 0.5 sencond and the data length was 2048 points in every record. In this paper, only 73 records whose significant wave heights are greater than 0.6m are analysed. The average wave heights are in the range of 0.42 and 1.34m, the maximum wave heights of 1.10 and 3.99m, and the average wave periods of 3.444 and 6.04 second. The wave number in one train varies from 167 to 296. $\bar{H}/d$ is equal to 0.03~0.09. So the field situation is basically close to deep water. The wave spectra showed that some records are of mixed wave with wind wave and swell, but most are wind waves only.

4.2 Parameters of sea wave groups

Letting the threshold height Hc equal to $H_{1/3}$, the statistical analyses are conducted for every record and the average number of waves $\bar{l}_1$ in a group (i.e., the mean run length $\bar{j}_1$) and the average number of waves $\bar{l}_2$ (i.e., the mean total run length $\bar{j}_2$) between two successive wave groups are listed in Table.1. The results obtained from the theoretical methods are also given in the Table. It is shown by comparing these results that the values from the wave envelope theory, Eq.(1) and Eq.(3), are evidently smaller than measured results, the values calculated by the statistical theory, Eq.(4) and Eq.(5), are also on the small side and $\bar{j}_1$ obtained from Eq.(6) is smaller yet. The average values of the Kimura's results $\bar{j}_1$, $\bar{j}_2$ are quite close to the observed values, but the calculated distribution are more concentrated.

Table.1 Observed and theoretical parameters of wave group

Theory	Parameter	the ranges	means	deviations
observed	$\bar{j}_1$	1.15~2.07	1.47	0.18
	$\bar{j}_2$	7.25~14.90	10.77	1.75
wave envelope theory	$\bar{l}_1$	0.40~0.52	0.45	0.03
	$\bar{l}_2$	1.44~3.05	2.33	0.37
theory of runs	$\bar{j}_1$	-------	1.16	0.42
	$\bar{j}_2$	-------	8.57	6.91
Eq.(6)	$\bar{j}_1$	1.09~1.17	1.12	0.02
Kimura's theory	$\bar{j}_1$	1.26~1.73	1.51	0.12
	$\bar{j}_2$	8.47~12.69	10.22	0.92

The relationships between $\bar{j}_1$, $\bar{j}_2$ and Qp are more scattered than that between $\bar{j}_1$, $\bar{j}_2$ and GF (Fig.4), $\bar{j}_1$ and $\bar{j}_2$ increase with the increase of GF generally.

4.3 Factors of wave groupiness

The factors of wave groupiness are analysed by above equations for 73 records, and their results are listed in Table.2. The spectral peakednesses Qp determined by equation (7) are also given in Table.2. The Table shows that some values of $\gamma_{TT}(1)$ are negative, but are positive mostly. It means that there are "memory" effect between successive wave heights and periods, which causes the wave grouping phenomena.

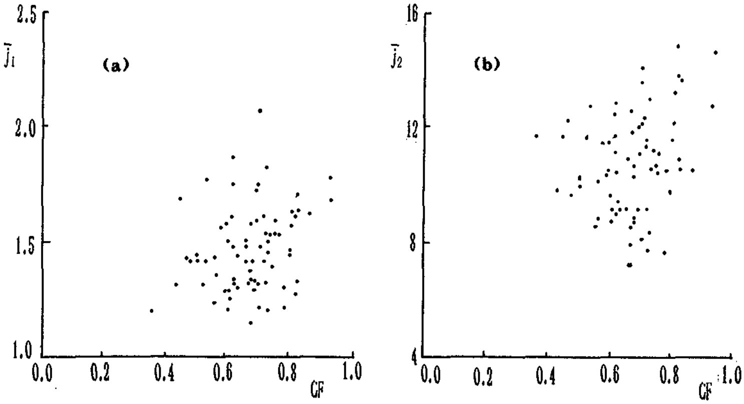

Fig.4 Relationships between $\bar{j}_1$, $\bar{j}_2$ and GF

Tbale.2 Observed wave groupiness factors and the spectral peakedness

factors	$\gamma_{HH}(1)$	$\gamma_{TT}(1)$	γ_{HT}	GF	$\bar{T}_{SIVEH}$	Qp
ranges	0.103~ 0.472	-0.149~ 0.346	0.147~ 0.652	0.360~ 0.932	3.661~ 7.778	0.704~ 2.821
average	0.324	0.085	0.413	0.673	4.953	1.597
deviations	0.093	0.100	0.110	0.113	0.801	0.393

4.4 Relationships between the wave groupiness factors and the spectral width parameter

Many scientists pointed out that the wave groupiness is directly correlated to the width of its spectrum, i.e., the narrower the spectrum is, the more strong of groupiness the wave is. Scientists used to judge the width of the spectrum by the width paramter ε . But Rye's work showed that the parameter ε is not able to express the spectral width definitely and it also varies significantly with the choice of the high cut-off frequency. However, Qp defined by Eq.(7) is more stable and reliable than ε . So the relationships between the five previously defined wave groupiness factors and the spectral peakedness parameter Qp are given in Fig.5 according to the field data. In general, $\gamma_{HH}(1)$, GF and $\bar{T}_{SIVEH}$ increase while γ_{HT} decreases as Qp increases although the points are scattering. But $\gamma_{TT}(1)$ is not evidently correlated to Qp. It proves that the wave groupiness is strengthened as the spectrum becomes narrower. But the effect of Qp on the wave groupiness gradually becomes weaker as Qp increases.

However, Van Vledder and Battjes (1990) pointed out that Qp can be affected by the smoothing method and the number of smoothing when estimating the spectrum and suggested a parameter K to express the speotral width, i.e,

$$K= \left| \int_0^{\infty} s(f)\exp(2\pi i f T_{02})df \right| / m_0 \qquad (24)$$

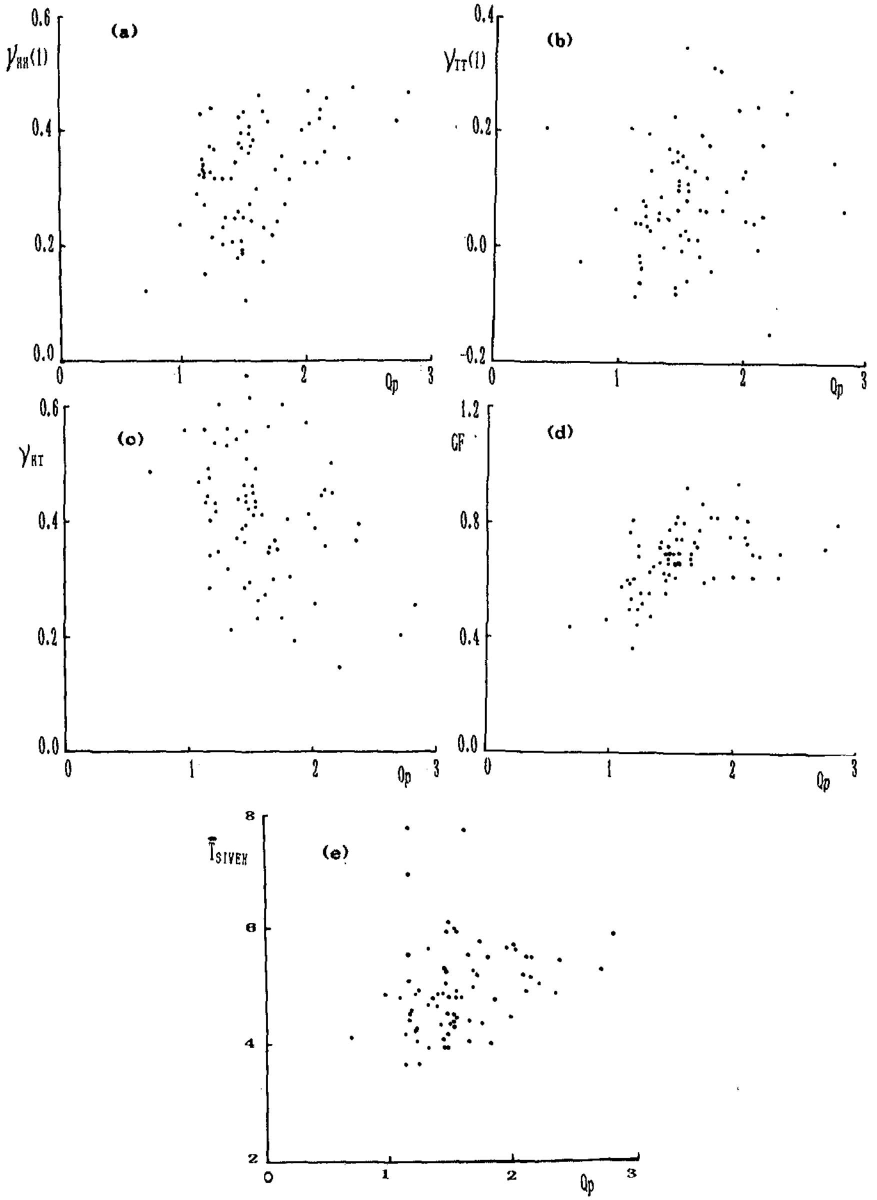

Fig.5 Relationships between wave groupiness factors and Qp

where T_{02} is the mean period calculated using the spectral moments. But the correlation function method is used to estimate the spectrum and the spectrum was smoothed by Hamming window once in this paper. The relationships between Qp, GF and K are given in Fig.6. It is shown that Qp and K can give similar results in our case and both Qp and K can be the spectral width parameters for wave group characteristics.

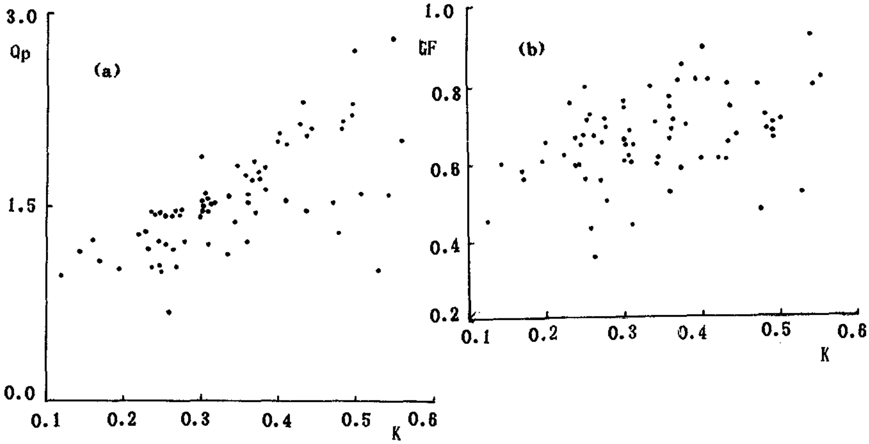

Fig.6 Relationships between Qp, GF and K

4.5 Relationships between wave groupiness factors and GF

The relationships between $\gamma_{HH}(1)$, $\gamma_{TT}(1)$, γ_{HT}, $\overline{T}_{SIVEH}$ and GF are given in Fig.7. All factors are evidently correlated to GF except $\gamma_{TT}(1)$. It means that GF is the main factor for representing the wave groupiness.

5. CONCLUSIONS

(1). At present, there are several methods for studying the wave group phenomenon. The results of analysis from the field data in this paper indicate once again that the wave envelope theory is not available because its results are far less than the field values. The results obtained from the statistical theory of runs are also smaller. Kimura's probability theory of run length based on the correlated wave heights can give the average values of the wave group parameters which agree with the field values quite well. but the distribution of calculated values are more concentrated than that in field.

(2). There are close relationships between the wave groupiness and the spectral peakedness (or width), i.e., the groupiness is strengthened as the spectrum becomes narrower. Most of the factors representing the wave groupiness are correlated to the spectral peakedness Qp to some extent, $\gamma_{HH}(1)$, GF, and $\overline{T}_{SIVEH}$ increase and γ_{HT} decreases with the increase of Qp. K is also a spectral width parameter for wave group characteristics.

(3). Among the factors describing the wave groupiness, the wave groupiness factor GF defined by Eq.(20) is the principal parameter. The other factors are correlated to GF to some extent except $\gamma_{TT}(1)$ which is not available. The

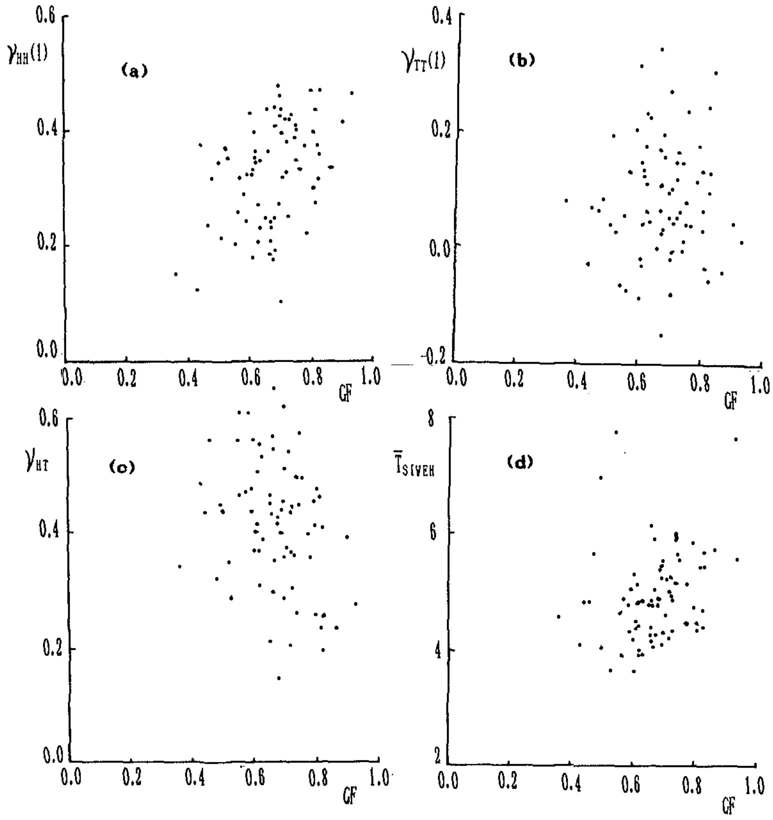

Fig.7 Relationships between wave groupiness factors and GF

mean length of runs (the mean length of wave group) and the mean total length of run (the average number of waves between successive groups) also increases with the increase of GF.

ACKNOWLEDGMENT

The authors wish to express their heartfelt gratitude to the First Design Insitute of Navigation Engineering, Ministry of Communications, for offering gratis the field wave records in Shijiu Port and Mr. Minquan Gu, Chief Engineer of the Institute for his valuable suggestions. Also, the gratitude is owed to prof. Dacuo Zhang and Mr. Weitao Tai , both of Qingdao Oceanographic University and Mr. Muguo Li and Mr. Ningchuan Zhang , both of Dalian University of Technology for their great help during data processing.

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