CHAPTER 19

CONDITIONAL SIMULATIONS IN LABORATORY FLUMES

Marcos H. Giménez[1]
Leon E. Borgman[2]
Robert T. Hudspeth[3]
Josep R. Medina[4]
Carlos R. Sánchez-Carratalá[5]
Hang Tuah[6]

ABSTRACT

A robust algorithm is presented which is capable of embedding a deterministic sequence of waves into a randomly generated wave train without changing the stochastic properties of the random waves.

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[1]Profesor Asociado, Dept. of Applied Physics, Univ. Politécnica de Valencia, Valencia 46071, SPAIN.

[2]Professor, Dept. of Geology, Univ. of Wyoming, Laramie, WY 82070, U.S.A.

[3]Professor, Ocean Engineering Program, Dept. of Civil Engineering, Oregon State University, Corvallis, OR 97331, U.S.A.

[4]Profesor Titular, Dept. of Transportation, Univ. Politécnica de Valencia, Valencia 46071, SPAIN.

[5]Research Asst., Dept. of Transportation, Univ. Politécnica de Valencia, Valencia 46071, SPAIN.

[6]Professor, Bandung Institute of Technology, Bandung, INDONESIA.

INTRODUCTION

A frequent request by sponsors of research projects in maritime laboratories is to analyze the behavior of a structure for a given design sea state. During the years, the testing of physical models in wave flumes has proven to be both cost-effective and reliable. Physical modelling of random waves requires generating random wave trains with a specified variance spectrum.

Funke and Mansard (1987) pointed out that present technology now makes it possible to treat the wave generator and the digital simulation technique as two essentially separate problems in wave generation. During the last decade, digital to analog numerical simulations have become extremely efficient. Several methods are available for generating these numerical simulations. Borgman (1969) identified two fundamental methods: 1) the superposition of sinusoidal waves; and 2) the filtering of white noise. Hudspeth and Borgman (1979) demonstrated the advantages of using FFT algorithms for numerical simulations. Tuah and Hudspeth (1982) introduced deterministic (DSA) and non-deterministic (NSA) spectral amplitude models for random wave simulations based on FFT algorithms. Medina and Sánchez-Carratalá (1988) compared the different methods available for the numerical simulation of random waves.

At the present, the physical simulation of random waves having a specified variance spectrum in a wave flume may be accomplished by using a variety of techniques. However, sometimes research sponsors want to include a deterministic sequence of waves in the random wave simulation used to test structures. For example, sponsors may want to know how a particular wave group that occurs in a random wave simulation that is defined by a specified variance spectrum affects the performance of a structure. This request usually requires a laborious wave by wave analysis of many random wave simulations searching for a sequence of waves that almost resembles the requested one. An alternative method to this searching technique is to use "conditional simulation". This alternative method generates a numerical random wave simulation that has a specified variance spectrum and also includes the deterministic sequence of waves requested by the sponsor. A robust algorithm is derived for this "conditional simulation".

CONDITIONAL SIMULATION

Conditional simulation is a technique for generating a numerical realization that includes a given deterministic sequence of waves and that has a specified variance spectrum. This technique simulates realizations for a stochastic process that is defined by a specified variance spectrum and that contain a deterministic sequence of waves which occur at a prescribed time in the realizations. The method presented here may be used either in the time domain or in the frequency domain. In either case, the method is a two-step procedure.

The first step requires that a random time sequence be simulated that has a specified variance spectrum, $S_u(f)$. This simulation is called an "unconditional time sequence" and is denoted by $\eta_u(t)$. Figure 1-a shows an example of an unconditional time sequence obtained using a DSA algorithm (cf., Tuah and Hudspeth, 1982) and a specified Goda-JONSWAP variance spectrum (cf., Goda, 1985) using: $\gamma=1$, $m_0=1$ m^2 and $f_p=0.27$ Hz. The total number of points in the time sequence is $N=2048$ and the time interval $\Delta t=0.1$ seconds.

The second step is to embed the deterministic sequence of waves into the unconditional time sequence at a prescribed point in time. These two steps may be done either in the time domain or in the frequency domain. The deterministic sequence of waves is called the "embedded sequence" and is denoted by $\eta_e(t)$. The realization that contains the "embedded sequence" is called the "conditional time sequence" and is denoted by $\eta_c(t)$.

Figure 1-b shows a deterministic wave group or the "embedded sequence" that is to be embedded into the unconditional time sequence shown in Fig. 1-a. The time interval Δt of the "embedded sequence" must be the same as the time interval of the unconditional time sequence (e.g., $\Delta t=0.1$ seconds in Fig. 1). In Fig. 1, the first and last values of the embedded sequence $\eta_e(t)$ are to be embedded at the prescribed times $n_1=961$ and $n_2=1088$, respectively, of the unconditional time sequence $\eta_u(t)$. The total difference between the two prescribed time values is $\nu=n_2-n_1=127$. Therefore, the total number of values of the embedded sequence is $\nu+1=128$.

Finally, Figure 1-c shows the conditional time sequence containing the deterministic wave sequence embedded in a random wave simulation having a specified variance spectrum. The total number of points ($N=2048$) and the time interval ($\Delta t=0.1$ seconds) are the same for both $\eta_u(t)$ and $\eta_c(t)$. Near the two ends of the embedded wave group, the conditional simulation algorithm modifies the unconditional time sequence $\eta_u(t)$ in order to maintain the correlation structure in the conditional time sequence $\eta_c(t)$ in accordance with the specified variance spectrum $S_u(f)$.

TIME DOMAIN ALGORITHM

In the time domain, the conditional simulation algorithm inserts the embedded sequence $\eta_e(t)$ into the unconditional time sequence $\eta_u(t)$ between the two points of time n_1 and n_2. The remaining values of the unconditional time sequence are modified in order to maintain the correlation structure that is associated with the specified variance spectrum $S_u(f)$.

The conditional time sequence may be obtained from (cf., Hudspeth et al. 1990):

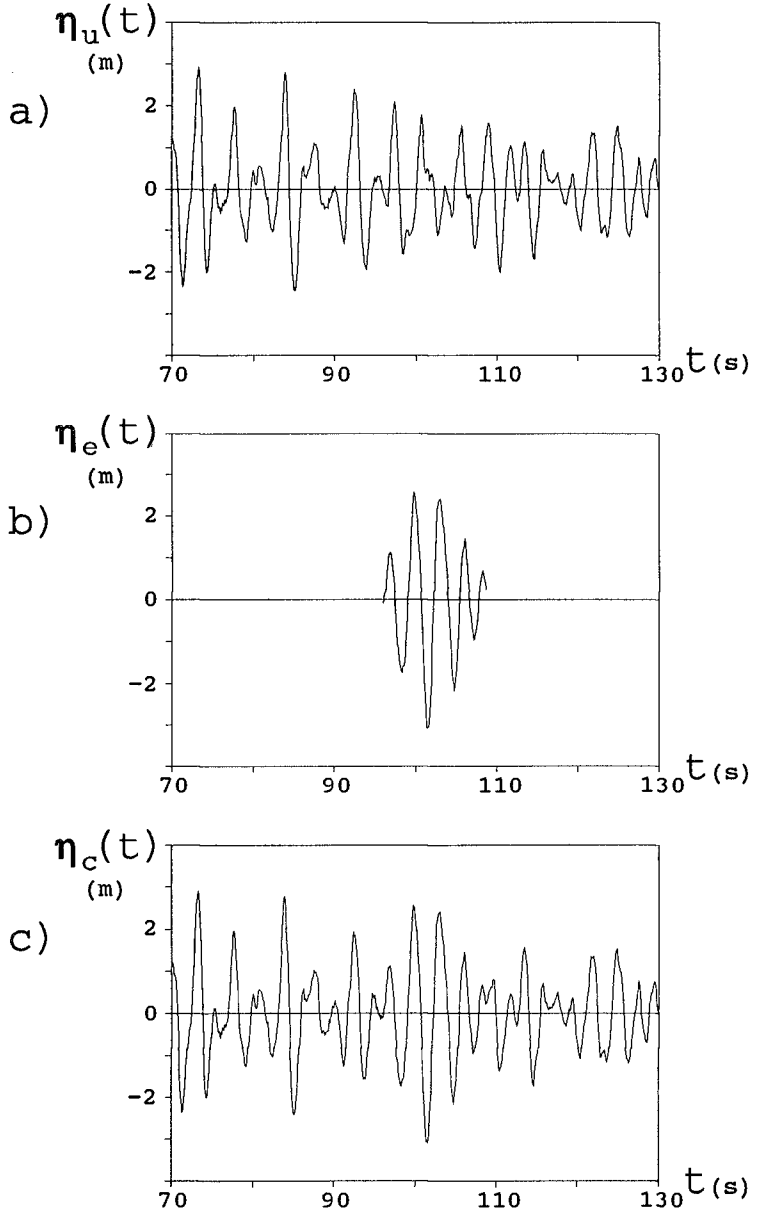

Figure 1. Example of a Conditional Simulation: a) Unconditional Time Sequence; b) Embedded Sequence; c) Conditional Time Sequence.

$$n \in [n_1, n_2] \rightarrow \eta_c(n\Delta t) = \eta_e(n\Delta t) \tag{1}$$

$$n \notin [n_1, n_2] \rightarrow \eta_c(n\Delta t) = \eta_u(n\Delta t) + C_{12}X \tag{2}$$

where:

$$C_{12} = [c_{n-n_1}, c_{n-n_1+1}, \ldots, c_{n-n_2}] \tag{3}$$

where c_k is the covariance at lag $|k|\Delta t$ associated with the specified variance spectrum $S_u(f)$; and X is the solution of the following system of equations:

$$C_{11}X = v_e - v_u \tag{4}$$

where:

$$C_{11} = \begin{bmatrix} c_0 & c_1 & \cdots & c_v \\ c_1 & c_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & c_1 \\ c_v & \cdots & c_1 & c_0 \end{bmatrix} \tag{5}$$

$$v_e = [\eta_e(n_1\Delta t) \ldots \eta_e(n_2\Delta t)]^\top \tag{6}$$

$$v_u = [\eta_u(n_1\Delta t) \ldots \eta_u(n_2\Delta t)]^\top \tag{7}$$

where C_{11} is the covariance matrix; v_e is the vector of values of the embedded sequence; and v_u is the vector of values of the unconditional time sequence at the prescribed time values where the embedded sequence is to be inserted.

The conditional simulation algorithm inserts the deterministic sequence of waves into the unconditional time sequence and modifies only those values of the unconditional time sequence that are near the two ends of the embedded sequence (vide Fig. 1). For the values of the unconditional time sequence that are located far from the interval of the embedded sequence, the values of the time lags $|n-n_1|$, $|n-n_1+1|, \ldots, |n-n_2|$ are large and the values of the covariances in the matrix C_{12} associated with these large time lags are small. Consequently, at these large values of time from the embedded time interval Eq. 2 becomes, approximately:

$$\eta_c(n\Delta t) = \eta_u(n\Delta t) + C_{12}X \sim \eta_u(n\Delta t) + [0 \ldots 0]X = \eta_u(n\Delta t) \tag{8}$$

and the differences are small between the unconditional and the conditional time sequences far from the time interval of the embedded sequence.

FREQUENCY DOMAIN ALGORITHM

In the frequency domain, the conditional simulation algorithm for inserting an embedded sequence $\eta_e(t)$ into an unconditional time sequence $\eta_u(t)$ may be shown to be equivalent to the time domain algorithm (cf., Hudspeth et al. 1990). The advantage the frequency domain algorithm is that FFT algorithms reduce substantially the computer time needed to obtain a long conditional time sequence.

The unconditional and the conditional time sequences may be expressed as a superposition of sinusoidal waves with frequencies which are multiple integers of the discrete frequency interval $\Delta f = 1/(N\Delta t)$ according to

$$\eta_u(n\Delta t) = \sum_{m=0}^{N-1} (a_m - i b_m) \exp(i 2\pi mn/N) \tag{9}$$

$$\eta_c(n\Delta t) = \sum_{m=0}^{N-1} (\alpha_m - i \beta_m) \exp(i 2\pi mn/N) \tag{10}$$

The FFT coefficients of the conditional time sequence $\{\alpha_m$ and $\beta_m\}$ may be obtained from the FFT coefficients of the unconditional time sequence $\{a_m$ and $b_m\}$. These coefficients are related by

$$\begin{bmatrix} \alpha_m \\ \beta_m \end{bmatrix} = \begin{bmatrix} a_m \\ b_m \end{bmatrix} + C'_{12} X \tag{11}$$

where X is the solution of the same system of equations given by Eq.4 that were used for the time domain algorithm; and C'_{12} is given by:

$$C'_{12} = S_u(m\Delta f)\Delta f \begin{bmatrix} \cos(2\pi mn_1/N) & \dots & \cos(2\pi mn_2/N) \\ \sin(2\pi mn_1/N) & \dots & \sin(2\pi mn_2/N) \end{bmatrix} \tag{12}$$

APPLICATIONS

The solution of Eq. 4 requires two constraints: 1) a goodness-of-fit constraint to the specified variance spectrum and 2) an ill-conditioned constraint for matrix C_{11}.

Firstly, the goodness-of-fit to the specified variance spectrum must be constrained by both the length and characteristics of both the embedded deterministic wave sequence and the unconditional time sequence.

Secondly, the covariance matrix may become ill-conditioned for typical ocean spectra because of the relatively low energy content of the specified variance spectrum at both low and high frequencies for relatively long simulations at relatively small time intervals of Δt. This combination of low energy levels and relatively long simulations at relatively small time intervals may produce numerical instabilities when inverting the ill-conditioned covariance matrix C_{11} in Eq.4. A similar numerical instability was described and solved by Medina and Sánchez-Carratalá (1988) using robust AR models for ocean spectra. Therefore, it is reasonable to use their solution in order to obtain a robust method for the conditional simulation of random ocean waves.

The robust method introduces into the specified variance spectrum a very low level of white noise that is acceptable for any practical application. If m_0 is the specified variance, satisfactory results were obtained by introducing a white noise level of $0.0025m_0$ which reduced the original specified variance spectrum, $S_u(f)$, by a factor of 0.9975. The modified specified variance spectrum which is satisfactory for practical engineering purposes is given by $S_u'(f) = 0.9975\ m_0$ + white noise. Finally, the modified specified variance spectrum $S_u'(f)$ is used in Eqs.3,5 and 12 instead of the original specified variance spectrum $S_u(f)$. The relative advantages of the robust method is illustrated below for typical ocean spectra.

In order to illustrate the stability of the robust method, consider removing a short piece of record from an unconditional time sequence and then inserting the same short piece back into the same unconditional time sequence at the same place. In this case, the vectors v_c and v_u in Eq.4 are equivalent and the resulting conditional time sequence η_c is equal to the unconditional time sequence η_u.

If the values of the embedded sequence shown in Fig. 2-a are modified only by a small amount, then the conditional and the unconditional simulations should differ by only the same small amount. However, that is not always the case if the matrix C_{11} becomes ill-conditioned. For example, if the values of the short piece of record shown in Fig. 2-a between 80-85 seconds are truncated to centimeters (a reduction of about 1%) and then embedded back into the unconditional time sequence at the same position in time, the resulting conditional time sequence shown in Fig. 2-b, is unsatisfactory. However, by using the robust method with a level of white noise of only $0.0025m_0$, the resulting conditional time sequence shown in Fig. 2-c differs only slightly from the unconditional time sequence. We note that any wave record from a laboratory simulation or from the ocean may be contaminated by levels of noise equal to or larger than this amount. Therefore, $S_u(f)$ and $S_u'(f)$ are equally acceptable for representing the variance spectrum of ocean waves. Figure 1 illustrates an application of the robust method for embedding a wave group.

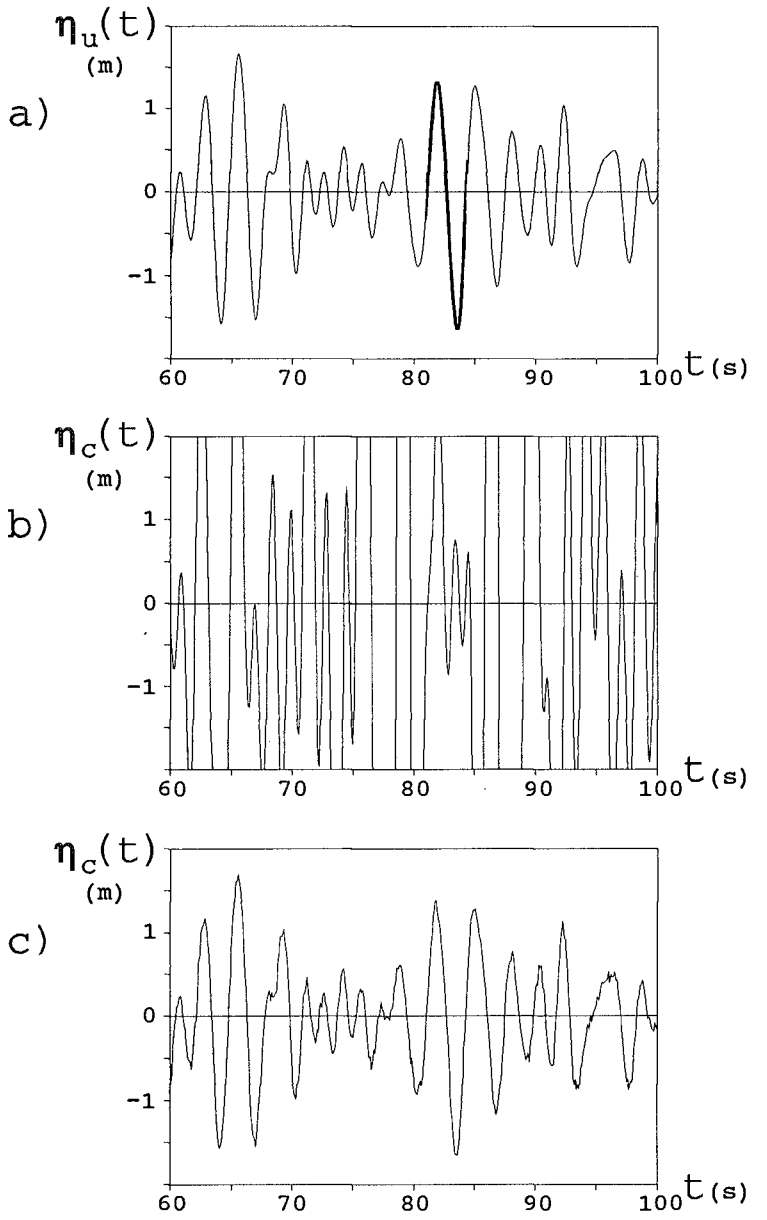

Figure 2. Comparisons of Conditional Simulation Methods: a) Unconditional Time Sequence; b) Example of Unstable Conditional Simulation; c) Example of Stable Robust Simulation.

The goodness-of-fit to the specified variance spectrum is influenced by several variables. First, the lengths of both the embedded sequence and the unconditional simulation must be considered. The wave group ($\nu+1=127$) shown in Fig. 1-b has been embedded into unconditional simulations having total lengths of N = 512, 1024 and 2048. These unconditional simulations are realizations from a specified Goda-JONSWAP variance spectrum having spectral parameters: $\gamma=1$, $m_0=1$ m^2 and $f_p=0.27$ Hz. The variance of the embedded wave group is $2m_0$. Table 1 compares the relative errors of the spectral moments computed from the conditional and from the unconditional time sequences as a function of the total length of the simulation with the length of the embedded sequence held constant ($\nu+1=127$).

TABLE 1. Effect of N on Errors in Spectral Moments Between η_b & η_c

Moments	N=512	N=1024	N=2048
m_0	17.6%	11.6%	4.8%
m_1	16.0%	10.1%	4.3%
m_2	13.9%	8.4%	3.5%

The goodness-of-fit to the specified variance spectrum improves as N increases relative to the length of ν. Figs. 3-a&-b compare the specified variance spectrum for the unconditional simulation with the variance spectrum for the conditional time sequence for N=512 and for N=2048, respectively.

The goodness-of-fit to the specified variance spectrum must also be constrained by the stochastic properties of the embedded sequence with respect to the specified variance spectrum. One of these stochastic properties is the contribution of the embedded sequence to the specified variance spectrum. Table 2 compares the relative errors of the spectral moments computed from the unconditional and from the conditional time sequences for the conditional simulation in Fig. 1 as a function of the contribution of the embedded sequence to the specified variance spectrum for an unconditional time sequence of length N=2048.

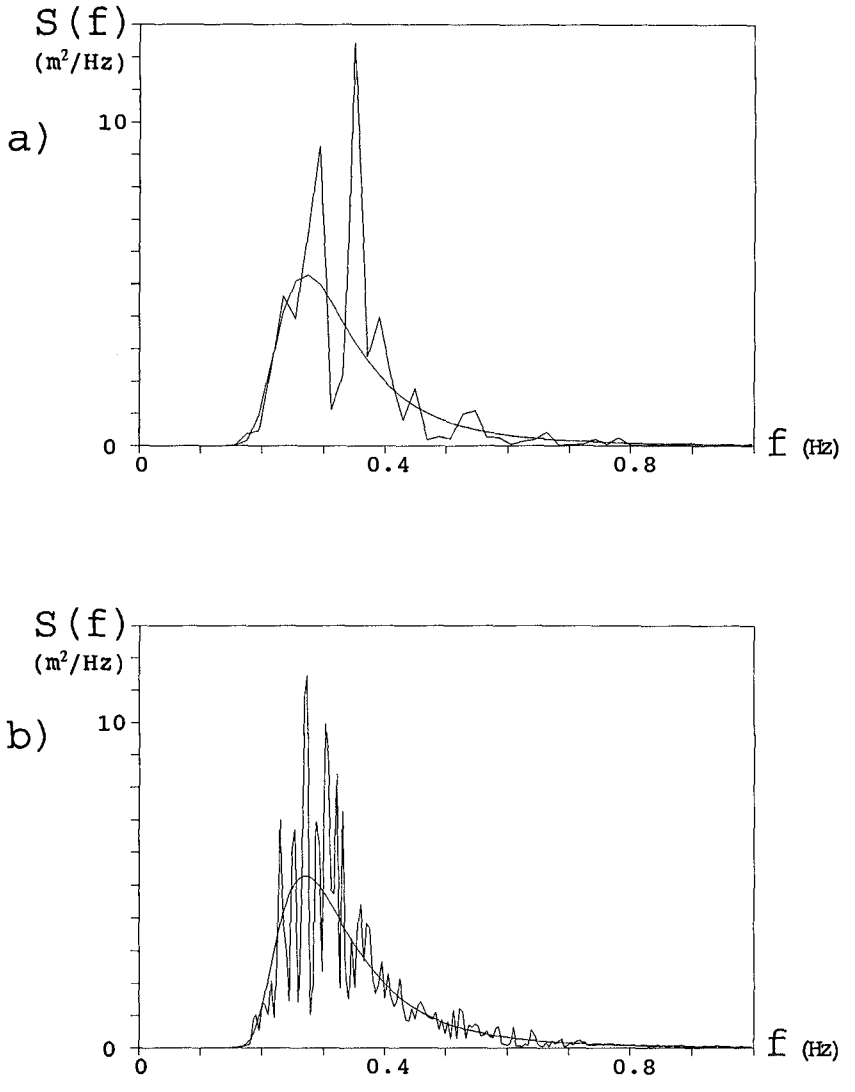

Figure 3. Specified Variance Spectrum and Conditional Time Sequence Spectrum for: a) N=512; b) N=2048.

TABLE 2. Effect of contribution of embedded sequence to specified variance spectrum on errors in spectral moments between η_o & η_c

Moments	$m_0/2$	m_0	$2m_0$	$4m_0$
m_0	-5.6%	-2.1%	4.8%	18.5%
m_1	-5.1%	-1.9%	4.3%	16.8%
m_2	-4.8%	-2.0%	3.5%	14.5%

The goodness-of-fit improves as the contribution of the embedded sequence becomes more consistent with the specified variance spectrum.

SUMMARY AND CONCLUSIONS

A robust method is presented for the conditional simulation of random ocean waves having a specified variance spectrum and containing a deterministic sequence of waves at prescribed point in time. The addition of a low level of white noise (variance $0.0025m_0$) to the specified variance spectrum for the unconditional simulation avoids the instabilities that are due to inverting an ill-conditioned matrix. The effects of the length of the embedded wave sequence $\nu+1$ compared to the total length of the simulation N and of the contribution of the embedded wave sequence to the specified variance spectrum were illustrated for a Goda-JONSWAP spectrum.

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