CHAPTER 22

Time and Frequency Domain Analyses of Shallow Water Waves on a Slope

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Abstract

Classical time and frequency domain analyses give results with limited physical meaning when applied to shallow water cases. Lower heights and periods than suggested by the actual data are obtained via these analyses. It appears that slope effects make waves less non-linear for steeper than for flatter slopes. One can obtain useful trends by making use of higher-order analyses, utilising higher-order, asymmetrical waves of the covocoidal type as an example.

Introduction

Most coastal engineering applications are done in water depths shallower than 20 m depth, either inside or outside the breaker zone and nearly invariably on a sloping sea bed.

Scientific and technological advances over the past decade have led to a substantially increased understanding of the physical processes taking place in the nearshore environment. This in turn has led to more sophisticated mathematical descriptions of these physical processes, which form the basis of the extensive numerical treatment of most of these processes.

In sediment transport research, rigorous advances have been made towards understanding the relationship between the driving forces and the resulting sediment movement. Nevertheless, it is still necessary to make, sometimes sweeping, simplifications in order to get workable numerical models of, e.g. cross-shore sediment transport. Baillard (1981) and Nairn (1988) have, amongst others, shown that, in order to correctly predict near-bottom sediment flows, it is essential to take cognisance of the higher, uneven moments of the orbital flow due to non-linear surface waves. The question that now arises is to what extent the asymmetry of the surface waves in the nearshore region, which are clearly discernable on wave records and with the naked eye, would influence the cross-shore transport models. Similarly, we believe the time has come to critically analyse the saturation spectrum concept for nearshore situations, again with reference to asymmetrical, non-linear, nearshore surface waves.

With this understanding as a platform, we believe that the substantial body of research on nearshore waves and their related phenomena can serve as a bridge to take currently used predictive techniques for sediment transport and other wave-induced processes, such as wave forces, to new levels of sophistication with associated improved predictive skills. At all times, however, one has to be careful not to become more sophisticated just for the sake of it. The additional benefits have to be quantified.

3. Background

Nelson (1981) conducted a series of controlled laboratory experiments to establish the relationship, if any, between wave properties on a sloping bottom and the bed slope itself. His experiments were run on bed slopes ranging from horizontal to 1 in 10. He came to the following conclusions:

(1) integral wave properties, such as wave length and wave energy, are for all practical intents and purposes *independent* of the bed slope; and

(2) time-dependent wave properties, such as wave shape, orbital velocities and orbital accelerations, are very definitely dependent on bed slope.

Most traditionally used wave theories suffer from one of two problems, namely, it either predicts a linear wave shape or it is higher order, that is, non-linear, but symmetrical around the wave crest. Typically, as the wave non-linearity increases, the numerical complexity also increases. This led Swart (1978) to develop a wave theory with a fairly simple expansion in terms of a higher power of the cosine function, the so-called vocoidal wave theory, which was subsequently shown (Swart and Loubser, 1979) to be as accurate in predicting nearshore wave parameters as Dean's stream function theory (Dean, 1974).

The wave shape in the variable order cosine theory, or vocoidal theory, is defined by:

$$\eta/H = voc(P,X) - \eta_{*t} \qquad (1)$$

and $voc(P,X)$ = the vocoidal function

$$= \{\cos^2(\pi x)\}^P \qquad (2)$$

where η/H = surface elevation, P is the order of the function, with $P \geq 1$, X is the location within the framework from wave crest to wave crest, non-dimensionalised by the wave length, and η_{*t} is the trough depth, relative to the still-water level and non-dimensionalised by the wave height.

On the basis of Nelson's results, referred to above, the thought occurred that it may be possible to, in a similar fashion as was done for vocoidal theory, obtain a function which is on the one hand simple and on the other hand closely resembles the actual behaviour of waves on a sloping sea bed.

As was the case for vocoidal theory, it was again intended to start from first principles, that is, the conformity equation and the equations of motion, with suitable kinetic and dynamic boundary conditions, and to then proceed to derive the wave properties, given the qualitative vocoidal (or in this case covocoidal) wave shape.

After some investigation, the following asymmetrical wave shape was defined (Swart and Crowley, 1989):

$$\eta/H = h_\alpha[voc(P,X)-\eta_{*t}]+0,5r_\alpha[cov(P,X)(1+\cos\pi X)-(1-\eta_{*t}) \quad 1 \quad (3)$$

$$\text{and} \quad cov(P,X) = 1-voc(P,X) \qquad (4)$$

where the first term in Equation (3) gives a symmetrical shape, basically the vocoidal wave shape, and the second term adds the asymmetry around the wave crest. Furthermore, the order P was assumed to be the same as for the similar case on a horizontal sea bottom, and h_α and r_α are functions of P and the bed slope tan α. These functions had to be determined in the derivation. Figure 1 contains spatial and temporal views of the wave shape.

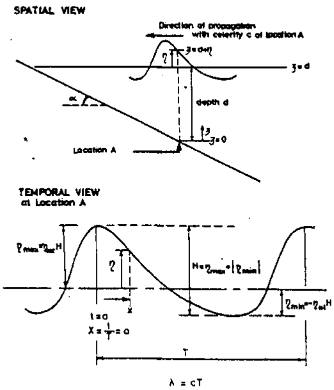

Figure 1. Axis-notation for development of generalised vocoidal theory on a sloping bottom.

A comparison to Nelson's laboratory data showed a very reasonable agreement (Swart and Crowley, 1989) and paved the way for further analyses with the covocoidal theory. It is not intended to submit that the vocoidal and covocoidal theories are the ultimate answers for nearshore, shallow water applications on a horizontal and sloping sea bed respectively, they are definitely not. However, these theories are very useful to investigate trends in data, specifically because of their good adherence to laboratory and field data. Using these theories allows the establishment of numerical techniques which would have been more difficult to construct with more complicated wave theories.

"Uncontrolled" field experiments of surface elevation variations in the nearshore environment in an exposed bay on the southwestern coastline of South Africa, are used in this paper to within the framework of covocoidal theory perform time and frequency domain analyses. These are then used to make some general deductions about the necessity for future research regarding waves on a slope.

4. Experimental set-up

The CSIR has over the past four years repeatedly performed extensive field measurement programmes in the shallow water in a flat bay, ca 20 km long and exposed to the dominant southwesterly swells off the southwestern coast of South Africa.

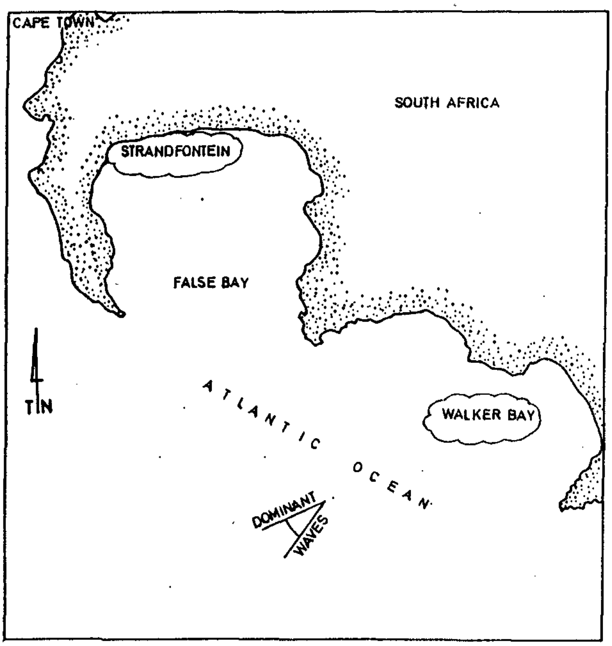

Figure 2. Location of field measurement site.

Four major field exercises have been performed to date, namely in March and September 1986, November 1987 and March 1990.

The September 1986 data had the best initial conditions to allow a comparison of depth effects and slope effects within the same data set. The three measurement sites that were used are characterised in the table below.

Site	Water depth	Slope	Type of instrument used
1	10-11 m	1 in 220	Pressure transducer
2	1- 2 m	Horizontal	Wave staff
3	1- 2 m	1 in 15	Wave staff

Some typical results obtained at each of these three sites are given below to familiarise the reader with the obvious differences between these sites.

Site 1

Figure 3 contains a few typical time records of wave profile and also a typical energy density spectrum. The wave trace shows a high degree of symmetry around the wave crest and also elements of wave grouping. This is supported by the energy density spectrum which contains to clear peaks, the dominant one at a frequency of 0.087 s^{-1} (11.45) and a secondary peak, more than one order of magnitude lower, but clearly discernable, at 0.014 s^{-1} (70s).

Site 2

Figures 4 and 5 show two sets of results which typify the range of variables found at this very nearshore location, where the bottom is for all practical intents and purposes horizontal in the area around it.

The time record of the wave profile in Figure 4 shows mostly a fair symmetry around the wave crest, as well as some clear indications of surf beat. The frequency domain analysis below shows three distinct peaks, now in descending importance with increasing frequency. The peak frequencies are at 0.011 s^{-1}, 0.045 s^{-1} and 0.07 s^{-1}. It is interesting to note that the tail of the frequency spectrum remains practically constant for frequencies greater than 0.08 s^{-1} at a value around 0.04 s^{-1}.

Figure 5, on the other hand, shows some clear signs of asymmetry around the wave crest, and furthermore a somewhat greater decomposition than in Figure 4, although the same three peaks can still be seen in the energy-density spectrum.

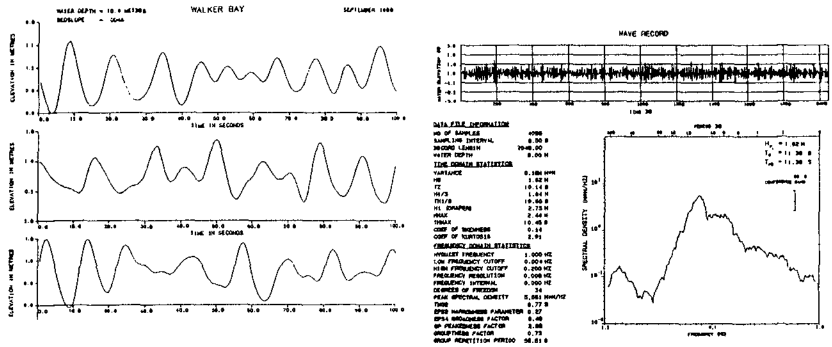

Figure 3. Typical time and frequency domain results for Site 1.

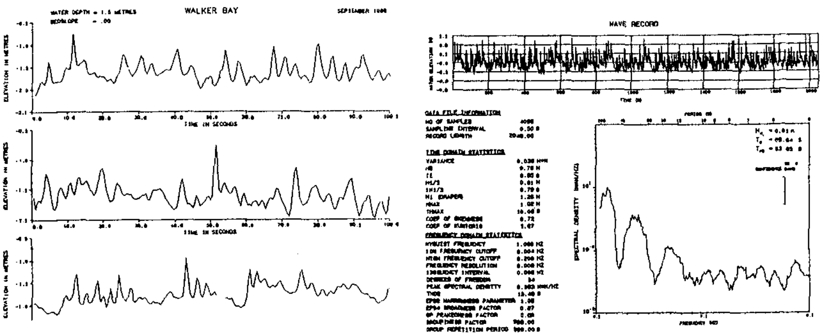

Figure 4. Typical time and frequency domain results for Site 2.

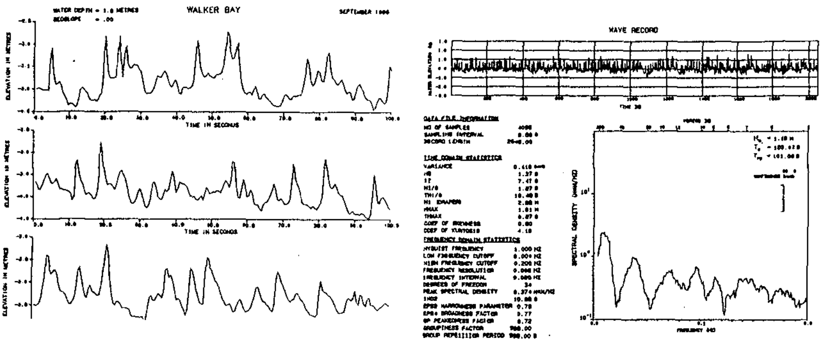

Figure 5. Typical time and frequency domain results for Site 2.

The wave groupiness exhibited in the time domain data is quite substantial.

Most of the data for this site are closer in resemblance to Figure 4 than to Figure 5.

Site 3

The data in Figure 6, for the same general water depth as for Figures 4 and 5, are in quite striking contrast to those for the earlier two figures. Whilst the time domain data also show signs of wave groupiness, the asymmetry of the waves around the wave crest is really very marked. This is general for all data collected at this site.

The energy density spectrum again has three distinct peaks, in order of importance at 0.005 s^{-1}, 0.023 s^{-1} and 0.048 s^{-1}. However, the spectrum now has a clearly saturated tail. Again, this is fairly common for all data collected at this site.

5. Time Domain Analyses

In this section, comparisons will be made of wave height calculated via time domain and frequency domain analysis techniques. The following two time-domain parameters are defined:

H_s; which equals four times the square root of the variance

$H_{1/3}$; which is the average of the highest one-third waves, using the down-crossing method.

These two time-domain estimates of significant wave height are compared with the spectral estimate H_{mo} of the significant wave height. All of these apply to the short wave component of the wave trace, defined by f>0.033.

The results are shown in Figures 7 and 8. The results show that there is close correspondence between $H_{1/3}$, H_s and H_{mo} at Site 1, at 10 m depth. However, at the two shallower sites, both $H_{1/3}$ and H_s are substantially higher than the spectral estimate H_{mo} of the significant height.

The results are summarised in the table below.

Site	Median value of	
	H_{mo}/H_s	$H_{mo}/H_{1/3}$
1	0.99	0.96
2	0.80	0.90
3	0.80	0.65

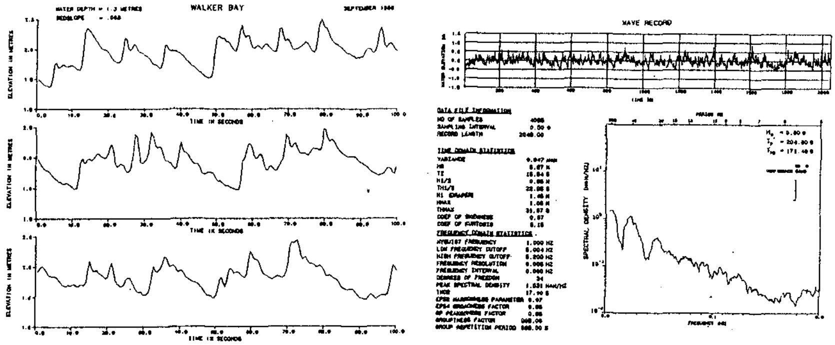

Figure 6. Typical time and frequency domain results for Site 3.

The reason for H_{mo} being less than H_s and $H_{1/3}$ for shallower water depths, is discussed in Section 6 below. It is noteworthy, however, that H_s is generally slightly higher than $H_{1/3}$.

Logically, because $H_{1/3}$ reflects the average of the highest one-third of the waves in the trace, it should be the reference value. Because of the non-linear nature of the waves in shallow water, the H_s value normally should deviate lower than $H_{1/3}$ progressively more as the non-linearity increases. The data seem to indicate the reverse. The reason for this is not clear.

5. Frequency Domain Analyses

Traditionally, spectral analysis of water waves has implied that a linear analysis was performed, whereby sinusoidally-shaped component waves are extracted. Over the years, various variations have been implemented, basically aimed at using bigger computing power to speed up the analysis. Various verification studies have been performed and it is generally accepted that this is a very sound procedure, yielding reliable, robust results, which can be replicated by any other researcher using standard techniques. However, in shallow water, this is not the case. Swart (1982) and Thompson and Vincent (1985), amongst others, have shown that, due to the fact that shallow water waves are non-linear, spectral estimates of wave height and period are too low in this area. This happens because a monochromatic non-linear wave train would, when analysed with a standard spectral technique, yield a spectrum of "apparent" wave heights rather than a single height at the appropriate frequency.

Thornton (1977) showed, by using both the kinematic and dynamic wave breaking criteria, that waves at higher frequencies in the wave energy spectrum are saturated with energy and that energy falls off in the region with a slope of f^{-2}, where f is the wave frequency. Banner and Phillips (1974) showed that, in

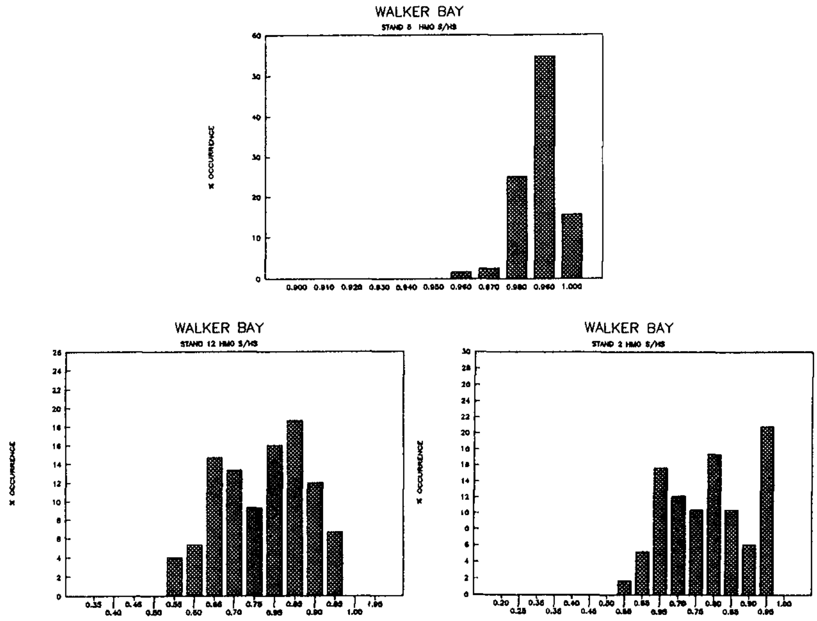

Figure 7. Histograms of H_{mo}/H_s values at Sites 1 to 3.

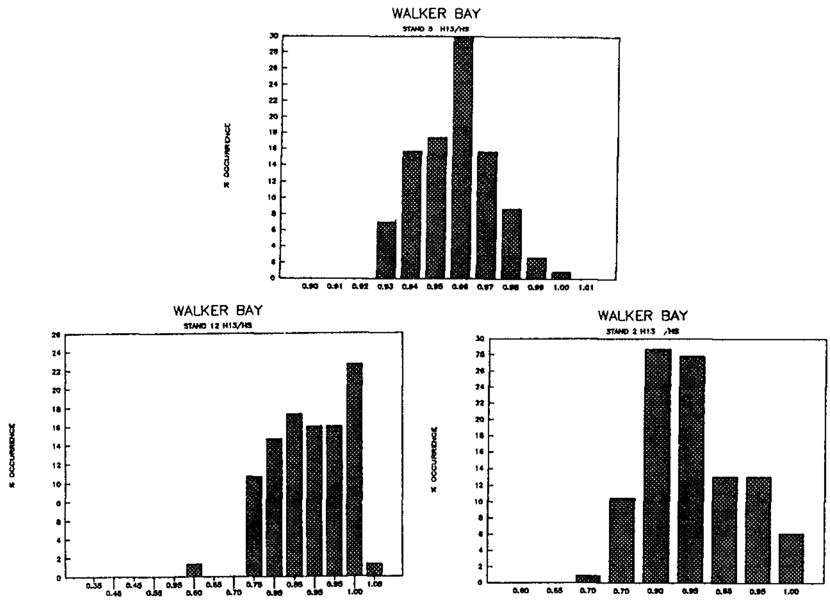

Figure 8. Histograms of $H_{1/3}/HS$ values at Sites 1 to 3.

deepwater, the critical parameter associated with deepwater wave breaking is the wave celerity which leads to a saturation slope in this area of f^{-5}. Thornton (1977) presented data for 20 m and 1.5 m depth, respectively, and showed that the wave spectrum indeed exhibited saturation at f^{-5} and f^{-3}, respectively. However, vertical orbital velocity spectra at both depths had a saturation slope of f^{-3}. Thornton (1976) contains similar spectra for primarily breaking waves, which have a saturation slope of $f^{-7/3}$, which seems to tie in with theoretical work by Phillips (1966) for cases where surface tension is important. On the other hand, Huntley, Guza and Bowen (1977) present wave run up data which has saturation slopes of f^{-3}.

In all this work, the magnitude of the slope is basically explained in terms of the ease with which energy can be transferred from lower to higher frequencies, thus, by a physical constraint. Thornton (1977) remarks that all the spectra he observed for depths of 20 m and shallower contained some degree of saturation. He remarks that "it is not understood whether the apparent saturation region was the residual of an earlier saturated wave condition, or the transfer of energy by non-linear interaction of wave components".

In the present paper, it is shown that another explanation for the saturated tail in the energy spectrum can be sought in the decomposition of non-linear waves into sinusoidal components, thus, due to a numerical imposition, especially at the shallow water depths.

Swart (1982) came up with the concept of extracting, via a Fourier analysis, non-linear wave shapes instead of sinusoidal wave shapes. Using an iteration procedure, Swart extracted appropriately scaled vocoidal waves (Swart, 1977) from shallow water wave trains. Via numerical simulation, he showed the accuracy with which the original input non-linear wave height spectrum is returned via this method, as well as the corresponding drop in wave height due to spectral decomposition of the non-linear wave shapes when a traditional linear method was used. One could employ any wave shape in this method of analysis, provided that the shape can be described numerically in terms of the initial conditions, typically water depth, wave period (frequency) and wave height. It is due to the last parameter that an iteration is required, because the purpose of the analysis is to determine wave height.

In this paper, the non-linear spectral analysis was performed on sample data sets to establish whether the method yields meaningful results and whether this is an avenue of research worth pursuing for random waves on a slope. The analysis was done for both vocoidal and covocoidal waves.

Examples are given here for the two shallow water stands in Figures 9 and 10. It is immediately apparent that the normal Fourier analysis in the top graph of each figure yields a spectrum which is very flat with an appreciable portion of the energy resident at frequencies greater than 0.2. On the other hand, the lower two graphs,

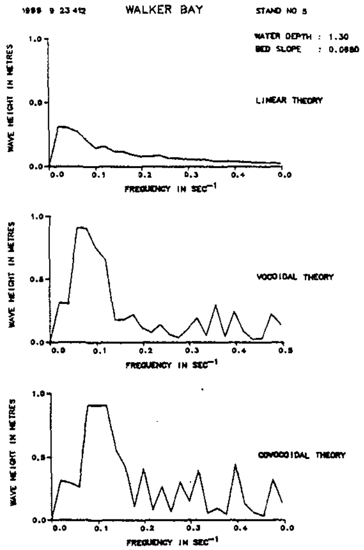

Figure 9. Linear and non-linear Fourier analysis in shallow water for Site 3.

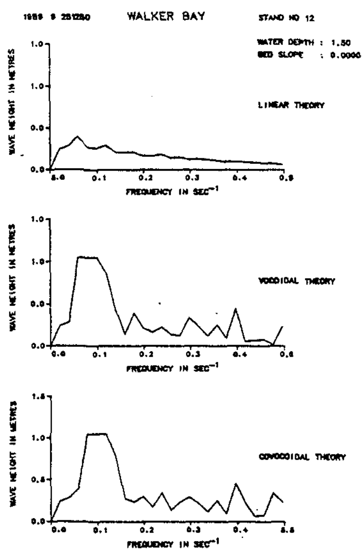

Figure 10. Linear and non-linear Fourier analysis in shallow water for Site 2.

which represent the higher order analysis, show spectra which are much more representative of deepwater spectra. At first sight, it would appear as if the energy associated with the lower two graphs is much higher than that for the upper graph. However, it should be borne in mind (Swart, 1978) that the total energy coefficient for these shallow water waves in a higher order theory is much lower than 0.125.

Furthermore, preliminary calculations show that the tail of a JONSWAP spectrum, of which the wave heights define shallow water wave shapes, falls off at a slope of between $f^{-2.5}$ and $f^{-3.5}$ when the resulting wave trace is subjected to a normal Fourier analysis. This supports the concept of the numerical method leading to the down position of the actual wave.

6. Conclusions

The following is a summary of the main conclusions reached during this study:

- Classical time and frequency domain analyses give results with limited physical meaning when applied to shallow water cases.
- Heights and periods are lower in these analyses than are suggested by actual data.
- Slope effects would appear to make waves less non-linear for the steepest slopes.
- Useful trends can be established by making use of simulations with higher-order, asymmetrical wave shape.
- Higher-order Fourier analysis "pulls" energy from higher frequencies towards lower frequencies, whereby the "saturation" shape disappears and the shallow water spectra then resemble deeper water spectra.
- Heights obtained from higher-order spectral analysis are more in tune with the wave trace than is the case for a linear analysis.

On the basis of these findings, we would suggest a critical review of the physical meaning of shallow water wave spectra and a more indepth analysis of the ramifications of using a non-linear spectral analysis technique.

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