CHAPTER 40

STOCHASTIC MODELING OF SURFING CLIMATE

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Abstract

A continued increase in the popularity of surfing, coupled with the potential for adverse impacts by coastal construction projects on favored surfbreaks, has recently confronted engineers with the problem of defining and quantifying a 'good' surfbreak. A rudimentary analysis of surfing mechanics demonstrates that conditions can be examined in terms of the joint statistical climate of 1) breaker peel rate, and 2) attainable board speed as characterized by the Irribarren Number. Assuming a planar beach and using linear wave theory, two theoretical models for the joint probability density of peel rate and Irribarren Number can be used to appraise short-term surfing conditions. Results indicate that the short-crested character of real waves plays a primary role in enhancing surfbreak. For practical calculations of long-term surfing climate at specific sites, offshore wave gage data can be transformed across a measured beach profile to predict breaker conditions in the outer surf zone. Application of such a numerical algorithm to the beach at Duck, North Carolina indicates that the waves are suitable for surfing roughly 25% of the time.

Introduction

Often the major impetus for beach nourishment and structural projects is not only erosion control and the protection of upland real estate, but also the preservation of the recreational opportunities afforded by a fronting beach. Surfing is one such activity, and although Morahan

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(1971) and others have assigned substantial monetary value to surfing recreation, the economic and cultural benefits of surfing have usually been overlooked during the development of beach projects. Consequently the physical impact of a particular project on the local surfbreak, and the subsequent impact on surfing recreation, have rarely been considered.

In recent years, surfers have begun to increase public awareness as to the importance of preserving good surfbreaks (see Pratte, 1987), and some engineers and planners have begun to at least take surfing into consideration during project design. This immediately gives rise to the problem of how to define and quantify a 'good' surfbreak and the suitability of a particular beach for surfing. Viable engineering models for beach 'surfability' are required in order to prevent negative impacts on surfing, to guide mitigation if such impacts are unavoidable, and to perhaps even enhance the surfbreak at a project site.

Background

Criterion for a Ridable Wave

According to Walker (1974) and Dally (1989), the qualitative definition of 'surfable' is a wave on which a surfer can maintain a mean speed (termed 'board speed') that is as fast or faster than the rate at which the point of incipient breaking translates along the wave crest (termed 'peel rate'). If the breaking segment of the wave overtakes the surfer, the wave 'closes out' and becomes unsurfable. In the most basic sense, it is the joint statistical climate of attainable board speed and peel rate that determine the surfing climate of a particular beach.

Board Speed

To date there appears to have been no comprehensive studies which specifically address the speeds attainable on surfboards. Using aerial photographs, Walker (1974) was able to infer mean board speed from estimates of the peel rate of waves that were surfed, and found maximum speeds on the order of 12 m/s (27 mph). Although direct measurements of board speed have yet to be obtained, parameters currently found in engineering practice can be utilized to provide insight.

Attainable board speed is a function of the size and shape of the face of the breaker, the weight of the surfer, and board characteristics. For a given board shape and surfer weight, the board speed can be inferred by using the

wave height at incipient breaking (H_b) to quantify the size and the Irribarren Number (I) to represent shape of the wave face. Irribarren Number is defined as

$$I_b = m/[H_b/(gT^2/2\pi)]^{1/2} \quad (1)$$

$$I_o = m/[H_o/(gT^2/2\pi)]^{1/2} \quad (2)$$

in which m is bottom slope, T is wave period, g is gravitational acceleration, and the subscripts "b" and "o" denote conditions at incipient breaking and in deep water, respectively. The Irribarren Number has been shown to characterize breaker type (Galvin, 1968 and Battjes, 1974). Spilling breakers are surfable, but not as desirable as plunging breakers. Collapsing breakers are unsurfable and in fact are quite dangerous to surfers. Table 1 presents surf climate classifications in terms of Irribarren Number and breaker type.

TABLE 1 - Surf Climate Classifications

Irribarren Number	Breaker Type	Surfing Terminology
$I_b < 0.4$ $I_o < 0.5$	Spilling	'mushy'
$0.4 < I_b < 2.0$ $0.5 < I_o < 3.3$	Plunging	'tube' or 'hollow'
$2.0 < I_b$ $3.3 < I_o$	Surging or Collapsing	'cruncher'

Peel Rate

The peel rate, V_{bp}, as defined by Walker (1974) is given by

$$V_{bp} = c_b/\sin(\alpha_b) \quad (3)$$

where α_b is (in planform) the angle between the wave crest and the path scribed by the moving break point. This angle is determined by the gradient in wave height along the wave crest, which has contributions from both oblique incidence and longshore variation in wave height. Referring to Figure 1 for definitions, and assuming the bottom contours are locally straight and parallel, α_b is given by

$$\tan \alpha_b = ds/dn = \tan \Theta_b + ds_*/dn \quad (4)$$

where ds is the incremental path length scribed by the breakpoint projected in the direction of wave propagation. As mentioned, a portion of this distance, ds_*, is associated with the local longshore gradient in wave height, i.e. the short-crestedness of the breakers. This quantity is given by

$$ds_* = (H_b - H_*)/(dH/ds) \quad (5)$$

where H_* is the height of the wave as it passes the bottom contour. If there is no longshore gradient, the break point simply moves along the bottom contour. Assuming that the ratio of wave height to water depth at incipient breaking (K) is spatially uniform, it can be shown that

$$ds_*/dn = \frac{-(dH/dy)}{[(dH/ds)\cos\theta_b] + (K\, m \cos^2\theta_b)} \quad (6)$$

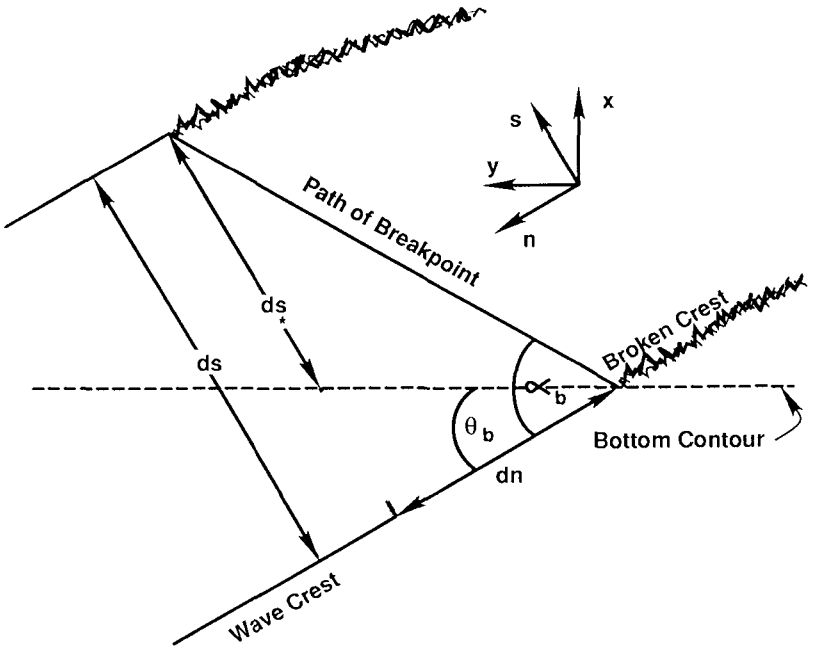

Figure 1 - Schematic diagram of breakpoint mechanics showing movement of breakpoint during an increment of time.

The rate of shoaling, dH/ds, required in Eq.6 can be derived from conservation of energy flux. For a planar beach and adopting shallow water linear wave theory, this is given by

$$dH/ds = (H/4h)\ m \cos \theta_b + (H/2) \tan \theta_b\ (d\theta_b/ds) \quad (7)$$

If it is assumed that $d\theta_b/ds$ is small, at the breakpoint Eq.7 becomes

$$dH/ds = (K/4)\ m \cos \theta_b \quad (8)$$

and Eq.4 becomes

$$\tan \alpha_b = \tan \theta_b + \frac{-(dH/dy)}{(5/4)\ K\ m \cos^2 \theta_b} \quad (9)$$

Based on this background, stochastic models for the joint statistical behavior of board speed (Irribarren Number) and peel rate can be developed to assess the surfing climate at a beach. In order to study the relative importance of wave obliqueness versus short-crestedness in determining peel rate and surfing climate, two stochastic models are derived below. The first one neglects short-crestedness whereas the second neglects obliqueness. As will be seen, they are based on the assumption of stationarity in the incident wave climate, and so may be used to assess surfing climate in only a short-term sense.

Stochastic Model for Short-Term Surfing Climate #1

If it is assumed that the bottom contours are straight and parallel and that there is no longshore variation in wave height, $\alpha_b = \theta_b$ (see Eq.9). From Eq.4, Snell's Law then dictates that V_{bp} is predetermined in deep water, i.e.

$$V_{bp} = c_o/\sin(\theta_o) \quad (10)$$

where $c_o = gT/2\pi$. By utilizing Eqs.2 and 10, surfing conditions can be characterized entirely in terms of deepwater conditions. With a given constant bottom slope, a joint probability distribution function (pdf) of the three random variables H_o, T, and θ_o is required. It is further assumed that wave direction is independent of height and period, i.e.

$$pdf(H_o,T,\theta_o) = pdf(H_o,T) \cdot pdf(\theta_o) \quad (11)$$

From Longuet-Higgins (1983), the pdf(H_o,T) is given by

$$pdf(R_o,\tau) = C_1 (R_o/\tau)^2 \exp\{-R_o^2[1+\epsilon^{-2}(1-\tau^{-1})^2]\} \quad (12a)$$

where

$$R_o = H_o/H_{rmso} \quad (12b)$$

$$\tau = T/\bar{T} \quad (12c)$$

$$C_1 = [4/(\pi\epsilon)^{1/2}][1+(1+\epsilon^2)^{-1/2}]^{-1} \quad (12d)$$

H_{rmso} is the root mean square wave height in deep water, $\bar{T}$ is the mean period, and ϵ is a band width parameter. Adopting a cosine-squared type distribution for pdf(θ_o), i.e.

$$pdf(\theta_o) = (2/\pi) \cos^2(\theta_o - \bar{\theta}_o) \; ; \quad -90^\circ \leq (\theta_o - \bar{\theta}_o) \leq 90^\circ \quad (13)$$

straightforward transformation of random variables leads to

$$pdf(I_o,\tau) = 2 C_1 \frac{m^6 \tau^4}{I_o^7 \bar{S}_o^3} \exp -\{ \frac{m^4 \tau^4}{I_o^4 \bar{S}_o^2} [1+\epsilon^{-2}(1-\tau^{-1})^2]\} \quad (14)$$

$$pdf(\hat{V}_{bp}) = (2/\pi) \frac{\tau}{[1-(\tau\hat{V}_{bp})^2]^{1/2}} \cos^2[\sin^{-1}(\tau\hat{V}_{bp}) - \bar{\theta}_o] \quad (15)$$

where $\hat{V}_{bp}$ is dimensionless peel rate defined by

$$\hat{V}_{bp} = (g\bar{T}/2\pi)/V_{bp} \quad (16)$$

and $\bar{S}_o$ is a deepwater wave steepness defined as

$$\bar{S}_o = H_{rmso}/(g \bar{T}^2/2\pi) \quad (17)$$

The marginal pdf of I_o and $\hat{V}_{bp}$ is found by integrating numerically with respect to wave period.

Sample results are shown in Figures 2a, b, and c. The lines separating surfable from unsurfable conditions are shown only conceptually, with the exception that the upper limit on Irribarren Number is dictated by the exclusion of collapsing breakers. Integrating the volume of probability density contained in the surfable region

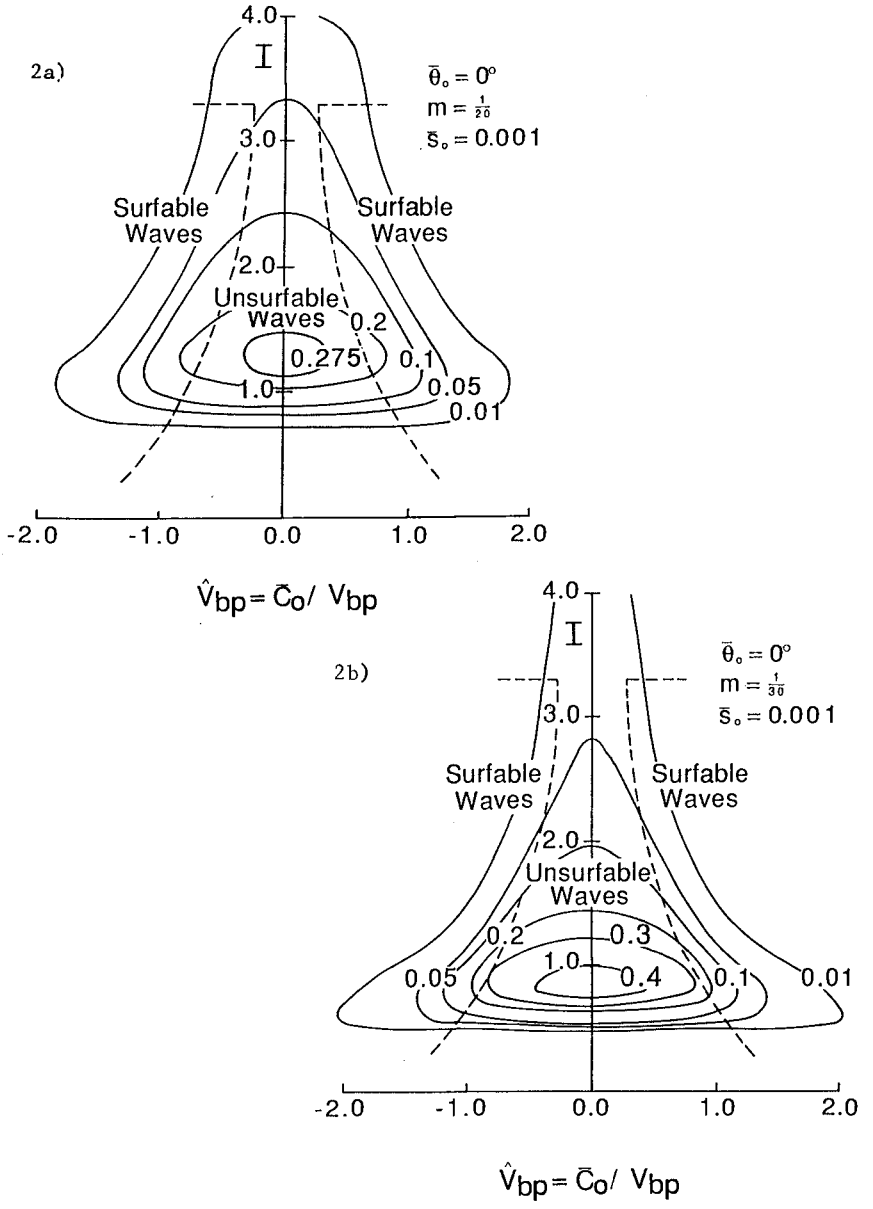

Figure 2 - Sample results of theoretical model for surfing climate given by numerical integration of Eqs.14 and 15. Volume contained in surfable region represents the proportion of incoming waves that are surfable.

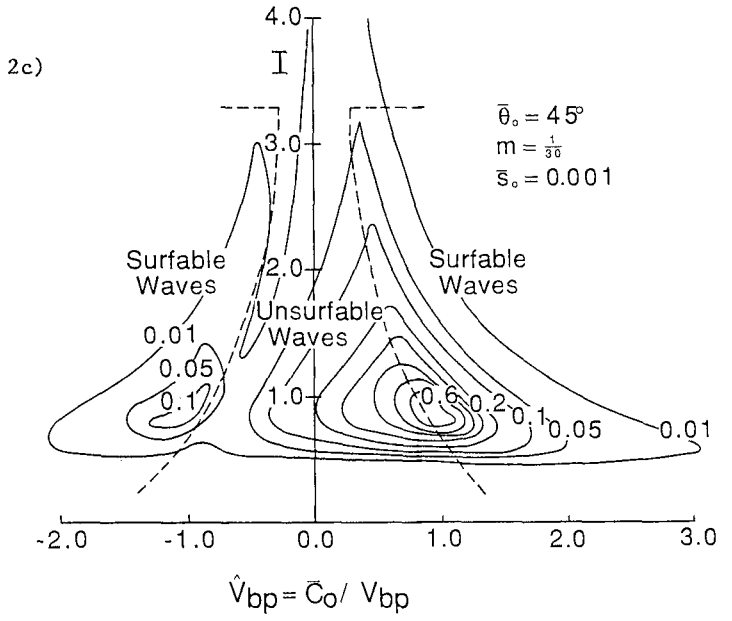

Figure 2 - concluded

yields the proportion of the incoming waves that can be regarded as surfable, and provides a number that can be used to quantify the surfing climate for a particular beach and incident wave conditions.

Figures 2a and b are for a mean angle of incidence of zero, but for different bottom slopes. Note that more probability density is shifted into the unsurfable region if the bottom slope is decreased from 1/20 to 1/30. If the mean angle of incidence is oblique to the shoreline as in Figure 2c, the pdf changes shape significantly and more density falls within the surfable region.

Stochastic Model for Short-Term Surfing Climate #2

In this model it is assumed that all waves break parallel to the bottom contours of a planar beach, and that a single uniform value can be used to represent the longshore gradient in wave height. Starting again from Eq.12, and employing Eq.2 and linear wave theory to shoal waves from deep water to the break point, transformation of random variables leads to

$$pdf(I_o, \hat{C}_b) = C_1 \frac{m^2 \hat{C}_b^4 K^2}{I_o^3 \bar{S}_o^3 \pi^2} \exp -\{ \frac{m^4 \hat{C}_o^4}{I_o^4 \bar{S}_o^2} [1+\epsilon^{-2}(1-\hat{C}_o^{-1})^2] \} \quad (18a)$$

in which

$$\hat{C}_o = \hat{C}_b [(K^2/2\pi^2)(I_o/m)^4]^{1/5} \quad (18b)$$

Using Eqs.3 and 9 yields the relation

$$\hat{C}_b = \hat{V}_{bp}^{-1}[\Omega(1+\Omega^2)^{-1/2}] \quad (19a)$$

in which

$$\Omega = [(-dH/dy)/(1.25\ K\ m)] \quad (19b)$$

and a final transformation leads to

$$pdf(I_o, \hat{V}_{bp}) = C_1 \frac{m^2 K^2}{I_o^3 \bar{S}_o^3 \pi^2} \frac{[\Omega(1+\Omega^2)^{-1/2}]^5}{\hat{V}_{bp}^6} \cdot$$

$$\exp -\{ \frac{m^4 \hat{C}_o^4}{I_o^4 \bar{S}_o^2} [1+\epsilon^{-2}(1-\hat{C}_o^{-1})^2] \} \quad (20)$$

Sample results for a value of dH/dy of -0.005 are displayed in Figure 3. In comparison to Figures 2b and 2c, it is clear that for the same bottom slope and mean deepwater steepness a much greater proportion of waves fall within the bounds of the surfable region. Although the two models are quite simple and heuristic, these results do indicate that the contribution to the peel rate from the short-crestedness of waves is more important than that from wave obliqueness. Future investigations and modeling of surfing climate should be focused accordingly.

Computational Model for Long-Term Surfing Climate

Because the formulations above inherently assume that the incident waves are a stationary random process, they can only be valid on a time scale on the order of several hours to perhaps several days. To quantify the surfing climate of a site on a seasonal or yearly time scale, a long-term model or methodology is required. In addition, surfers usually line up at the outer edge of the surf zone and choose to ride only the higher waves in the incoming groups. Therefore it appears that a reasonable description of the long-term surfing climate would be the joint distribution of board speed and peel rate associated with

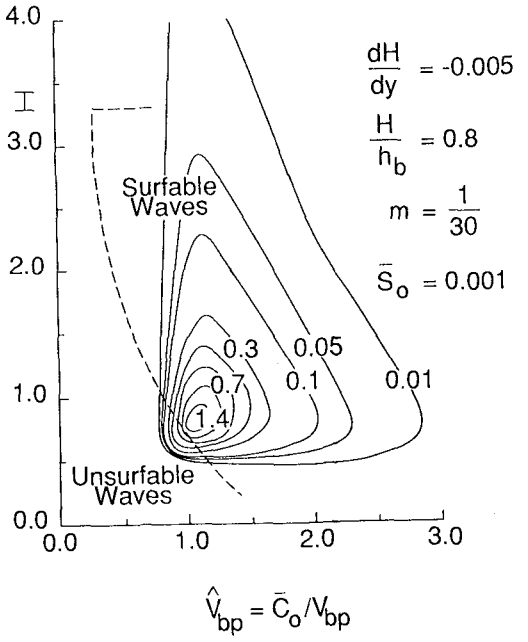

Figure 3 - Sample results of second theoretical model for short-term surfing climate given by Eq.20.

only the larger breaking waves (e.g. $H_{1/10}$), analyzed at intervals of several hours.

Wave height and period provided by long-term wave gage records, hindcast predictions, or even manual observations might be used to calculate Irribarren Numbers. However, wave direction information needed to partially determine peel rate is more difficult to obtain, plus observations of the longshore gradient in breaker height are almost nonexistent. At present, development of usable methodologies to assess long-term surfing climate must therefore be guided by available data.

As an example, detailed, long-term wave information is available in the Monthly Data Summaries published by the Field Research Facility in Duck, North Carolina. The data utilized herein is derived from pressure gage time series collected for 34 min, once every six hours, and is reported in the form of energy based significant wave height, H_{mo}, and associated peak spectral period, T_p. The gage is located at a nominal water depth of 8 m. Concurrent tide measurements are also provided, plus high quality nearshore bathymetry is available from CRAB surveys.

Numerical Algorithm

H_{mo} and T_p can be used as the input required by the joint probability density function of wave height and period derived by Longuet-Higgins (1983) by assuming that $H_{rms}=H_{mo}/\sqrt{2}$ and $\bar{T} = 0.8\ T_p$. With the pdf discretized into a joint histogram, the routine steps landward along a measured beach profile, shoaling each representative wave according to the theory of Shuto (1974):

$$H^2C_g = \text{const. (linear theory)} \quad \text{for} \quad gHT^2/h^2 < 30 \tag{21a}$$

$$Hh^{2/7} = \text{const.} \quad \text{for} \quad 30 < gHT^2/h^2 < 50 \tag{21b}$$

$$Hh^{5/2}[(gHT^2/h^2)^{1/2}-2\sqrt{3}] = \text{const. for } 50 < gHT^2/h^2 \tag{21c}$$

where C_g is the group velocity. Each representative wave shoals until the condition for incipient breaking developed by Weggel (1972) is satisfied:

$$(H_b/h) = b(m) - a(m)\ H_b/gT^2 \tag{22a}$$

where

$$a(m) = 43.8(1.0-e^{-19m}) \tag{22b}$$

$$b(m) = 1.56/(1.0+e^{-19.5m}) \tag{22c}$$

Because the bottom profile at the FRF is irregularly shaped and often contains a bar formation, and estimate of the effective bottom slope is needed in Eq.22. This estimate is determined from the section of profile immediately seaward of the point of interest, by averaging the slope over a distance of one wave length. The routine continues to step across the profile until 10% of the waves have broken. At this point the average incipient breaker height and average of the associated Irribarren Numbers at breaking ($H_{b.1}$ and $I_{b.1}$) are calculated for this 10% of the waves that broke first.

Sample results from this algorithm are presented in Figure 4 for the month of January, 1989, and in Figure 5 for the month of July, 1989. It is interesting to note that the predicted Irribarren Numbers fall predominantly in the range of spilling breakers, and that $H_{b.1}$ and $I_{b.1}$ are inversely correlated - unfortunate from the surfer's point of view.

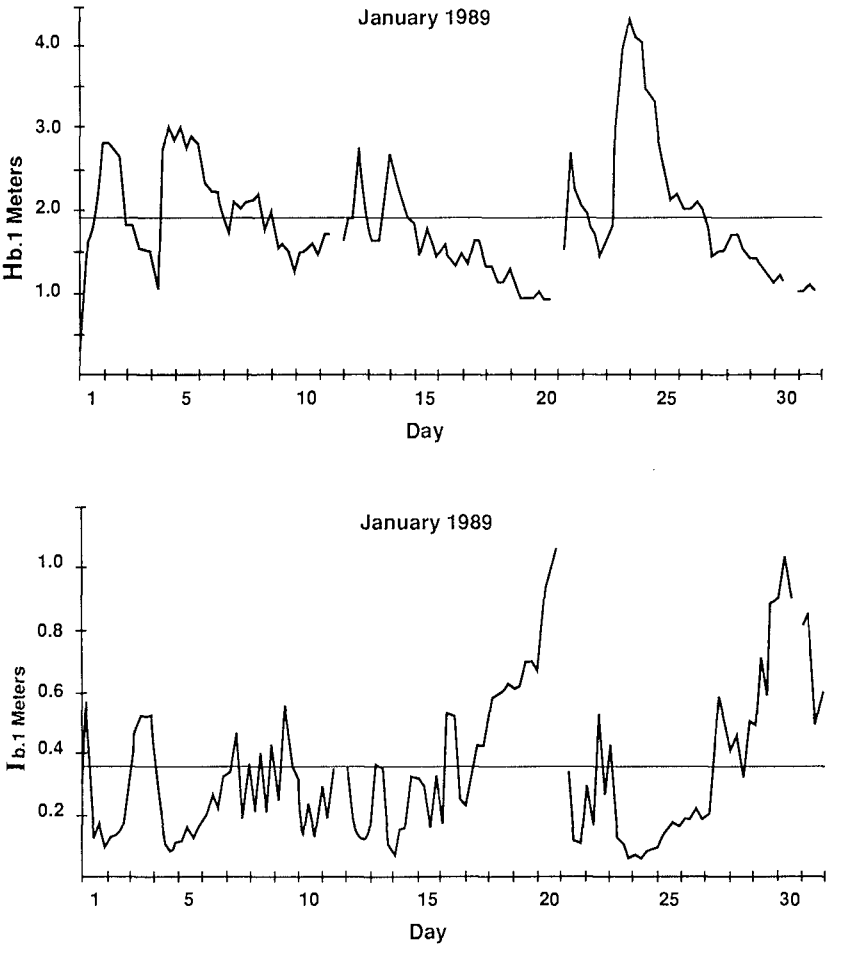

Figure 4 - Results from the numerical algorithm for the long-term surf climate at Duck, North Carolina during the month of January, 1989. The correlation between $H_{b.1}$ and $I_{b.1}$ is -0.71.

If a criterion for surfable conditions is imposed such that $H_{b.1}>1.25$m and $I_{b.1}>0.3$, the waves were surfable roughly 35% of the time in January and 33% of the time in July. Raising the requirement on $H_{b.1}$ to 1.5m reduces January to 20% surfable, and July to 15%.

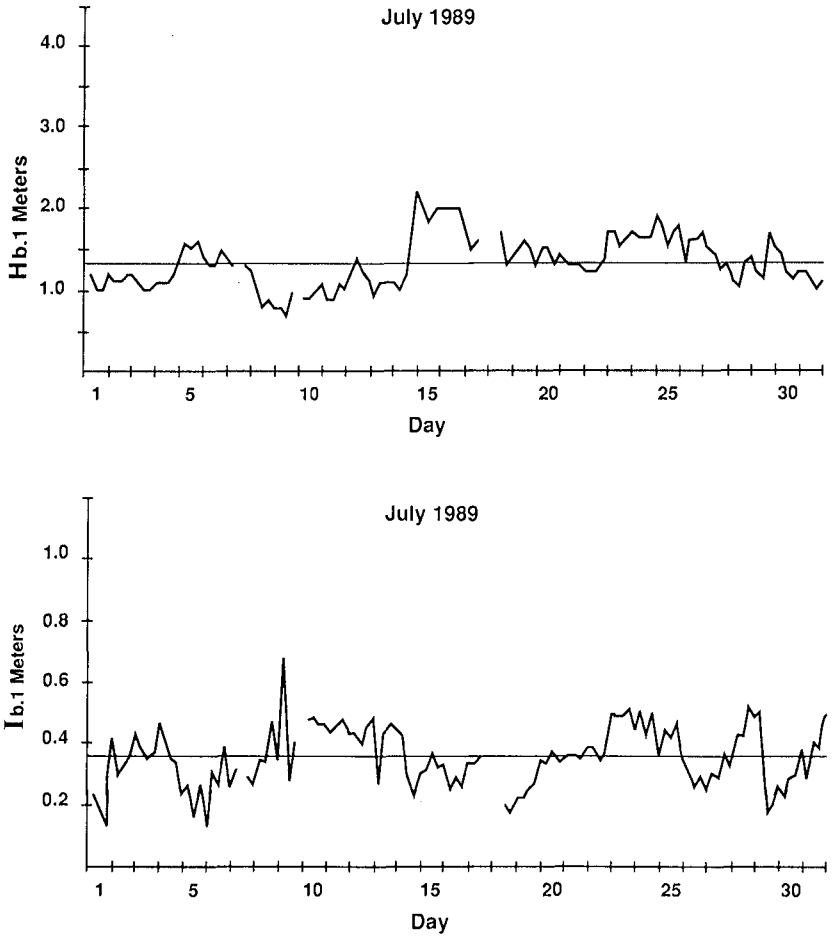

Figure 5 - Results from the numerical algorithm for the long-term surf climate at Duck, North Carolina during the month of July, 1989. The correlation between $H_{b.1}$ and $I_{b.1}$ is -0.23.

Summary and Conclusions

Viable engineering models and methodologies that can be used to evaluate the surfing climate at a beach are needed to preserve the recreational benefits of the coast. A rudimentary analysis of surfing mechanics demonstrates that the basic parameters in need of study are 1) the peel

rate of a breaking wave, and 2) the attainable board speed. Assuming a planar beach and using linear wave theory, two heuristic models have been presented that can be used to appraise short-term surfing conditions, given beach and incident wave characteristics. Results indicate that it is essential to include the short-crested character of real surf in model formulations. Also presented is a numerical algorithm which utilizes arbitrary beach profiles and measured offshore wave data to evaluate surfing conditions on a seasonal or yearly time scale.

Future investigations of surfing should include a better parameterization of attainable board speed in terms of wave and wind characteristics, with verification to direct measurements. Another important aspect to consider is the paddling effort required in order to catch a wave, i.e. the interplay of the mechanics of catching a wave and the transformation of the individual wave as it approaches breaking. This can play a major role in assessing the surfbreak on a particular day.

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