CHAPTER 43

TURBULENCE SCALES IN THE SURF AND SWASH

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Abstract

Ocean surface gravity waves breaking on gently sloping beaches generate substantial turbulent velocity fluctuations, both from overturning at the surface bore and from shear stresses at the bottom. We have used measurements made with laboratory-style hotfilm anemometers in the surf and swash on a *natural beach* to determine the relevant length and velocity scales. Battjes (1975) has pointed out the importance of determining the turbulence scales in the surf zone. Modelers, such as Svendsen and Madsen (1984), for example, rely on length and velocity scale estimates to parameterize and solve the complicated equations that govern surf zone flows. We find that turbulence length scales depend essentially on the bore height, and therefore on the local depth, but may decrease sharply under the bore. We also determine that at least the horizontal velocities approach isotropy at frequencies of 2 to 3 Hz, which turn out to also correspond to length scales on the order of the local depth.

Introduction

The determination of "scales" plays an important role in turbulence research, since turbulent flows must be described by their characteristic times, lengths, velocities, kinetic energies, Reynolds stresses, eddy viscosities and dissipation rates. Much of the theory of turbulence is concerned with establishing connections and relationships between these parameters and much experimental effort has gone into guiding these concerns. The basic reason why this approach is necessary, is that turbulent flows typically contain velocity fluctuations at a broad range of length scales, particularly small ones, so that direct analytical or numerical solution of the governing equations is unmanageable. The parameterizations that arise more or less naturally from the flow conditions then become the description of that flow. The two key scales are turbulence length and velocity, and we concentrate on characterizing these from measurements.

In a shear flow, any sharp vertical gradients in horizontal velocity, $U(z)$, makes the flow unstable and causes turbulent velocities (u,v,w) to grow in all three directions. The turbulence serves to smooth the mean velocity profile by exchanging momenta, and $\overline{uv}$, $\overline{uw}$ and $\overline{vw}$, which are respectively proportional to the Reynolds stresses, τ_{xy}, τ_{xz} and τ_{yz} determine how efficient this is. The Reynolds stresses cannot be calculated in closed form using the original equations of motion (this is the closure problem of turbulence) and they are difficult to measure, even in laboratory flows.

Some of the earliest efforts to understand turbulence were aimed at parameterizing the Reynolds stresses in terms of mean flow quantities (Boussinesq, 1877; Prandtl, 1925; von Karman, 1930), so that:

$$\frac{\tau_{xz}}{\rho} = -\overline{uw} = \nu_t \frac{dU}{dz}, \tag{1}$$

where ν_t represents the turbulent eddy-viscosity, ρ is the fluid density, and dU/dz is the mean shear. This formulation introduced the parameter ν_t which is analogous to the kinematic viscosity and has the dimensions L^2/T, or λq, where λ and $q = \sqrt{k}$ represent respectively turbulence length and velocity scales, and k is the turbulent kinetic energy scale. The turbulent kinetic energy dissipation rate turns out to be $\varepsilon = q^3/\lambda$ (Tennekes and Lumley, 1972). Svendsen and Madsen (1984) use (1) and the form $\nu_t = k^2/\varepsilon$, as the basis for a model of flow in a surf zone bore.

In considering the role of horizontal turbulent eddies in smoothing the profile of longshore currents in the surf zone, Battjes (1975) arrived at a relationship very similar to (1). He argued that the mean shear (cross-shore gradient in longshore current, in this case) must give rise to anisotropy that is measured by $-\overline{uv}/q^2$ and decays at a rate λ/q, balanced only by the mean shear rate itself, or

$$-\overline{uv} \approx q\lambda \frac{\partial V}{\partial x}. \tag{2}$$

Many different formulations of ν_t are possible, involving different combinations of λ, q, k and ε, and each implies a different solution procedure which determines exactly how the closure is made. A spectral method (due to Kolmogorov) can also be used to make estimates of ε based on assumptions of local isotropy in an equilibrium range of length scales (see, for example Grant, et al, 1961) but we do not pursue this in the present paper. In principle, such determinations could serve as consistency checks on estimates made in other ways.

We describe our instrumentation and calibration and deployment procedures in the next section. This is followed by a discussion of several experiments we have conducted in the surf zone over the last few years, and the results that are relevant to the determination of turbulence scales.

Instrumentation and Calibration

Experiments have been performed over a number of years at the moderate-energy, gently sloping beach adjacent to Scripps Institution of Oceanography at La Jolla, California. Wave conditions on this beach are highly variable, but we are confined to carrying out measurements when breaking wave heights are under about 2 m. Instruments are generally mounted on pipe supports cantilevered from other pipes which are jetted into the sand and braced to avoid vibration, as illustrated in Figure 1. We deploy instrumentation in the surf zone during times of low tide and then take data when the tide has brought the waves to the sensors. Tide changes of 1 to 2 m are common during spring tide times of the month, particularly in winter and summer, when local tide ranges are highest. This approach also serves to allow measurements at different mean water depths, or relative position in the surf zone, without the need to move the instruments.

Figure 1. Instrument mounting frame hardware showing typical set-up for measurements in the inner surf zone.

We have deployed current meters, pressure sensors, wave staffs, thermistors and two styles of hotfilm sensors. The current meters were standard, 2-axis, spherical, electromagnetic instruments 4 cm in diameter manufactured by Marsh-McBirney, Inc. The calibration of these meters is linear and relatively stable, the only disadvantages being that the frequency response is limited to about 3 Hz, and there is a severe response distortion when the probes are within a few diameters of the free surface (Guza, et al, 1988).

We have made bottom pressure measurements using standard, strain-gauge type sensors with a resolution of several parts per thousand in equivalent water elevation. The pressure signal is a relatively smooth version of the actual sea surface profile, since the high frequency portions of the velocity potential decay with depth. Pressure signals have been mainly used to have a continuous depth record and a qualitative indication of the wave elevation and phase available for comparison with the higher frequency, turbulence signals. Surface piercing, resistance wire wave staffs have been used extensively in the laboratory, and when carefully prepared and cleaned, can achieve resolutions of better than 1 part per thousand (Flick, et al, 1981). We have found resistance wire wavestaffs to be generally unsatisfactory for routine, unattended data acquisition in the surf zone since debris floating on and in the water, particularly kelp, often stretches or even breaks the wires.

We have used thermistor probes to continuously record water temperature for use in calibrating the hotfilm anemometers. Our probes were "Fastip" model FP-07 microbead thermistors, manufactured by Thermometrics, Inc. The calibrations were done in a water bath and are linear and reproducible to about ± 0.1° C, absolute. Relative temperature changes are accurate to about ± 0.02° C, with a half-power frequency response cutoff of about 22 Hz.

Hotfilm Probes

We have successfully adapted laboratory-style hotfilm probes for use in the natural surf zone. We have used Thermo Systems, Inc. (TSI) model 1230-NaCl, quartz-coated platinum, cone shaped sensors, as well as the less rugged model 1210-NaCl cylinder-shaped probes. Despite many difficulties, particularly with breakage and calibration drift reminiscent of the critical and pessimistic review of Frey and McNally (1973), we believe that hotfilm technology is the only one currently capable of providing high-frequency, small scale measurements of velocity fluctuations in the surf. The presence of bubbles and debris in the surf zone greatly inhibits the use of either laser-doppler or acoustic measurement systems, assuming these could be made small enough to avoid serious flow interference, as with fiber optic techniques.

One of the serious drawbacks of single hotfilm sensors is that they do not discriminate direction. In fact, their response is closer to omnidirectional than it is to a much more convenient cosine of the angle of attack. The rectification of the velocity signal is an advantage, however, in one sense, since it selects out the longitudinal component of the turbulence fluctuations u, in the presence of a strong, although oscillating "mean" flow, U. We have had some success in de-rectifying calibrated hotfilm time series in selected instances, by either using current meter records or high shutter-speed video camera images to identify the time of flow reversal. The cone probes were generally deployed pointing offshore, into the oncoming wave flow. The cylinder probes were oriented with their axes aligned in the longshore direction, thus minimizing interference from longshore directed currents.

Hotfilm anemometry rests on the principal that the heat conducted away from the probe by the moving fluid is proportional to some function of the flow speed, S past the probe and the difference in temperature (overheat) of the sensor, T_p and the water, T_w (King, 1914). The sensor is kept at a constant temperature (in our case) by a bridge feedback circuit (TSI model 1755) that provides an overall frequency response of several hundred Hz. The output (bridge voltage) E is related to the flow speed by King's law

$$E^2 = (A + B\, S^n)\,(T_p - T_a), \tag{3}$$

where A, B and n are calibration constants.

Probe calibrations do not stay stable sufficiently long to simply calibrate the sensors in the laboratory and then shift them to the surf. We have consequently constructed a scheme where we calibrate the hotfilm sensors continuously by comparison with a co-located current meter. Results have been very satisfactory for deployment off the end of Scripps Pier, in 5 m water depth well outside the surf zone. With minor adjustments to this method, we have achieved calibration accuracies of about ± 5 cm/sec in peak flows of up to 200 cm/sec, even in the surf zone.

Operating temperatures of the hotfilm probes were generally set to about 25° C above ambient water temperature, which at La Jolla varies from about 14 to 21° C. This resulted in a relatively low overheat ratio of between 5 and 8% to prevent the formation of bubbles on the probes. The equivalent velocity sensitivity, ΔS, of the probes to fluctuations in ambient water temperature can be determined from relation (3) to be

$$\frac{\Delta S}{S} = -\frac{\delta}{\Delta T}\,\frac{1}{n}\,(1 + \frac{A}{B\, S^n}),$$

where $\delta/\Delta T$ is the fractional change in temperature relative to the overheat, and is assumed small. Assuming E is measured in volts, S in cm/sec and the temperatures are in ° C, typical values for the calibration coefficients are $A = 0.2$, $B = 0.1$ and $n = 0.5$. For an overheat of $\Delta T = 30°$ C and water temperature fluctuations of $\delta = 0.1°$ C, the fractional speed error, $\Delta S/S$ is less than about 2%, so long as flow speeds are larger than about 10 cm/sec.

Surf Zone Turbulence Scales

Figure 2 shows time series data from two hotfilm cone probes and a (rare) wavestaff signal measured in the surf zone at a depth of about 35 cm, with a bore about 25 cm passing the sensors. These data were taken on 21 April 1988, at a sampling rate of 2560 Hz. No current meter data are available, so we were forced to rely on laboratory calibrations of the hotfilm probes, which we believe to be accurate to ± 25%. The hotfilm data are labelled "upper" and "lower" and represent, respectively, the probe at elevation 38 cm off the bottom and the probe at 20 cm off the bottom. The lower probe was essentially submerged over the entire data run, while the upper probe was out of the water until arrival of the bore at time 12:58:19.

Four seconds of data from this run have been analyzed and Figure 3 shows the autocorrelation functions for the upper and lower probe signals. Note that these functions do not, to first order, depend on the probe calibrations. The temporal lag correlations, $\theta(\tau)$ have been adjusted to equivalent spacial lag correlations, $\theta(r)$, by

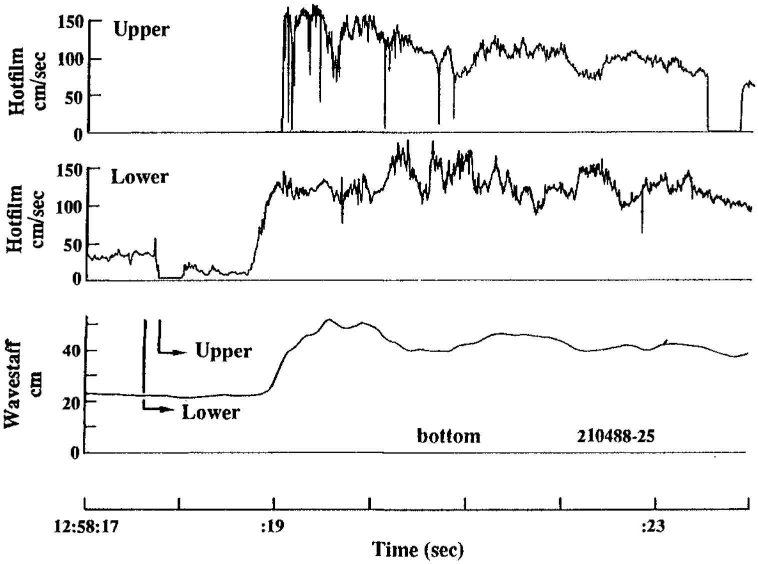


Figure 2. Time series from upper (38 cm) and lower (20 cm) hotfilm probes (upper two traces) and wavestaff signal (lowest trace).

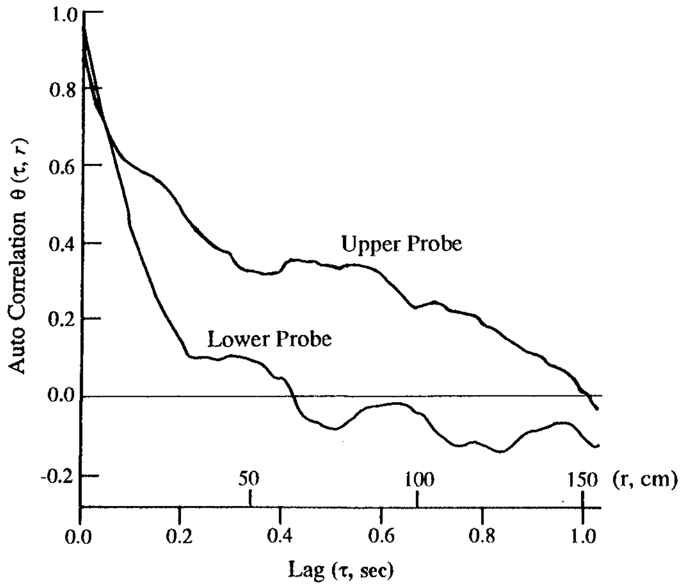


Figure 3. Autocorrelation function from upper and lower hotfilm data (Figure 2) Length scale derived from time scale with U=150 cm/sec.

choosing a mean advective speed, $U = 150$ cm/sec and invoking Taylor's hypotheses, $r = U\tau$. The upper and lower rms turbulence levels are about 30 and 20 cm/sec, respectively, so that $q_{rms} \ll U$ is satisfied. We can use the correlation function $\theta(r)$ to estimate the integral scale of the turbulence, defined as

$$\lambda_i = \int_0^\infty \theta(r)\, dr.$$

Results show a decrease in both decorrelation scale and integral scale with depth in the water column. Decorrelation length at the upper probe is on the order of 150 cm, while the integral scale is about 50 cm. The respective lengths at the lower probe turn out to be about 60 and 25 cm, about half the upper values. This supports the intuitive idea that the turbulent eddy length scales are on the order of the bore height in the bore, and that they are therefore of the same order of magnitude as the local depth. It also suggests that length scales and turbulence levels are both smaller below trough level than in the actively overturning bore. With respect to the turbulence intensity, this conclusion is not new or surprising, but there may be some new information in the suggestion that turbulence length scales decrease from the surface downward.

Note that the actual values of these estimates *do* depend on the hotfilm calibrations (through U in Taylor's hypotheses). However, the *relative difference* in length scales would be exaggerated for any plausible calibration corrections, since the mean advective velocity at the upper probe could not be smaller than that at the lower probe. This would *increase* the length scale estimate at the upper level, relative to the lower one. The same argument holds for the turbulent velocity scale.

Corresponding estimates of dissipation rates, $\varepsilon = q^3/\lambda$ range from about 100 to 300 cm^2/sec^3 at the lower sensor, and from about 200 to 400 cm^2/sec^3 at the upper sensor, depending on the choice of length scale. It is noteworthy that any of these values far exceed corresponding estimates of dissipation rates in tidal channels, which range from 10^{-3} to 1 cm^2/sec^3 (Grant, et al, 1961). By comparison, it has been estimated that the dissipation rate of oceanic tide motions is on the order of 10^{-5} cm^2/sec^3 (Grant, et al, 1961).

Figure 4 shows the first 32 sec of a 256 sec time series of hotfilm data (dark line) and the cross-shore component of a current meter signal (dashed line). These data were taken on 14 March 1990, at a sampling rate of 4096 Hz. The current meter was mounted 64 cm above the bottom, and total water depth during the data run varied between 75 and 100 cm, with breaking wave heights of about 1.2 m. The current meter record has been offset downward by 20 cm/sec for clarity. The hotfilm measurement has been calibrated and de-rectified using the current meter data as a standard. The agreement is quite good.

Figure 5 shows the ratio of longshore to cross-shore velocity measured by the current meter as a function of frequency. The velocity ratios were calculated from spectral levels of each component. It is clear that V/U increases from values around 0.2 at frequencies corresponding to the incoming wave period of about 10 sec, to

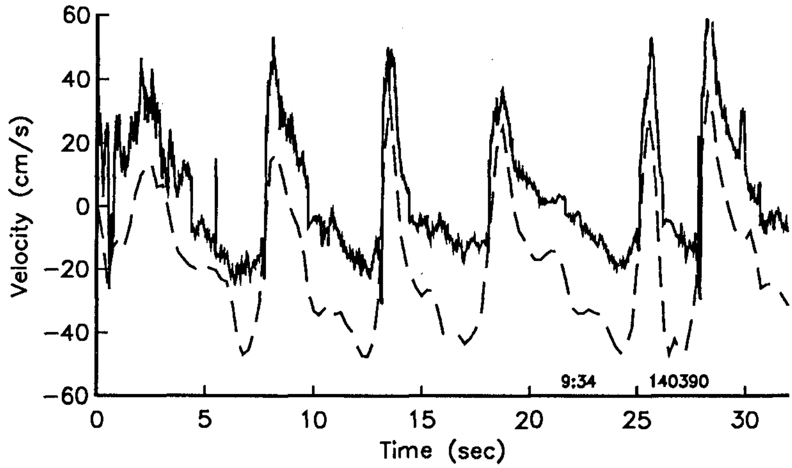
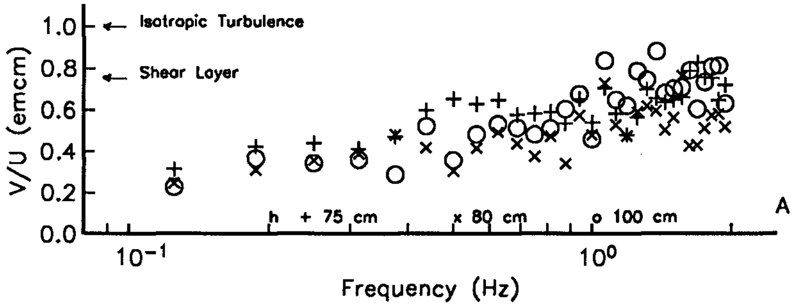


Figure 4. Time series from hotfilm probe (solid trace) and current meter (dashed) in mid surf zone, showing good de-rectification and calibration agreement. Current meter trace is offset -20 cm/sec for clarity.



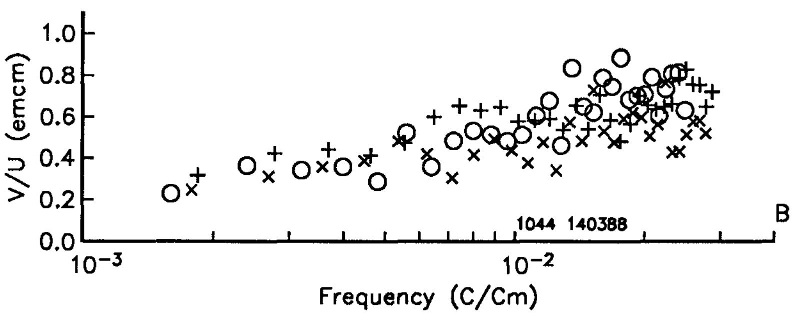


Figure 5. Ratio V/U from current meter data in mid surf zone as a function of frequency and depth, showing approach to isotropy at about 2 or 3 Hz.

values approaching 1 at the current meter cut-off frequency around 2 or 3 Hz. The indicated levels, 0.76, labelled "Shear Layer," and 1.0, labelled "Isotropic Turbulence" correspond to the expected value of V/U for these flows.

Even though fluctuations were only measured in two directions, these data strongly suggest that passing bores generate local turbulent velocities in all three directions. This result should not be surprising, and supports the idea that turbulence under surf zone bores may resemble shear layer turbulence in the transition regime between frequencies associated with wave orbital motions and about 2 or 3 Hz. We would expect the turbulent fluctuations to become increasingly isotropic at higher frequencies, but cannot confirm this with the present measurements. The scatter in the ratio estimates is too large to distinguish a dependence on total depth, which only changes about 20% in the experiment.

Rigorous estimates of the length scales associated with these time scale estimates cannot be made by a straightforward application of Taylor's hypothesis, since the mean flow is unsteady (Figure 4). However, a rough estimate is possible by noting that the peak advective velocities are on the order of $0.25c$, where $c = \sqrt{gh}$ is the shallow water phase velocity, and $g = 980$ cm/sec^2, is the acceleration of gravity. Invoking Taylor's hypotheses with $h = 80$ cm, results in an upper bound estimate of the length associated with the maximum frequency (2 or 3 Hz) shown in Figure 5, which turns out to be about 35 cm. This reveals that the length scales of the eddies at the lowest frequencies that are "fully" turbulent, here assumed to be those approaching isotropy levels consistent with values expected in a shear layer, are on the order of the waveheight. The wave (or bore) height, H in the surf zone is approximately proportional to the local depth, $H = \gamma h$, where $\gamma \approx 0.7$ is an empirical constant. Therefore, we expect that the turbulent length scale is likewise proportional to the local depth.

Considering all the assumptions, not to mention the limited data, this is remarkably consistent with the above derived result from the auto correlation analysis of the "lower" hotfilm probe. There is again nothing particularly surprising about this, indeed, it would be surprising if it were otherwise. The results are noteworthy, however, since these seem to be the first published field measurements that tend to confirm our intuitive expectations. Furthermore, these suggestions also support the idea stated explicitly by Battjes (1975), that turbulent eddies in the surf zone scale according to the depth, or vertical dimension, and *not* a horizontal dimension, such as distance to the shore, or length between bore crests. The horizontal scales are larger than the depth by a factor on the order of β^{-1}, where β is the beach slope. On a gently sloping shore such as La Jolla, this factor typically ranges from 50 to 100.

It is interesting to consider the expected ratio of longshore to cross-shore orbital velocity in a plane wave that has been subject to refraction into water depths equivalent to those in our experiments. Assuming that Snell's law is valid for waves with period 10 sec, in a water depth of only 1 m, it can be shown that a deep water approach angle of 10° is reduced by linear refraction to a shallow water angle of only 2°. This results in $V/U \approx 0.03$, a ratio much smaller than observed.

Swash Turbulence Scales

Measurements were made in the swash region using a cylinder type hotfilm probe on 8 March 1990. The swash region is defined as that part of the beach which is alternately dry and then covered by water as a result of the wave runup. Due to the prevalence of significant low frequency surf beat, and a mild beach slope of about 1:75, the swash region can be tens of meters wide and each bore may or may not reach the sensor location (Guza and Thornton, 1982; 1985). Equivalently, each individual backwash event may or may not drain the beach at the sensor location. The probe was mounted 1 cm off the bed. Figures 6a and 6b show time series from the hotfilm that have been de-rectified using video tape images. We used the 1/4000 sec shutter speed available on our video camera to ensure clear images at the (fixed) 30 frame/sec sampling rate. We were thus able to resolve the flow reversals to approximately 1/30 sec.

Figure 6a shows 64 sec of data which include 4 prominant up-rush and back-wash cycles, but the probe does not get dry until the very last cycle, shortly after t = 64 sec. The maximum water depth, h_0, during each bore was about 30 cm. Unfortunately, we had no direct measure of the depth except what could be estimated from the (oblique) video images. Each swash cycle is characterized by a sharp reversal from offshore (negative) to onshore (positive) flow, followed by a gradual, nearly linear slowing and finally a turning to offshore flow as the wedge of water drains off the beach face. This sequence forms the characteristic sawtooth shape of the velocity time series.

There is turbulent motion with amplitude scales on the order of 5 cm/sec apparent in each of the deceleration phases, but the most prominent occurs starting at about t = 50 sec (Figure 6a). Apparently due to the build-up of water on the beach face over a number of preceeding uprushes, enough offshore momentum was built up to change the character of this particular backwash. It lasted longer than was typical, and as shown in Figure 6a, reached a larger offshore velocity than normal. Viewing of several hours of video tape shows that this type of event occurs at surf beat intervals, which means in perhaps 5 to 10% of the bores.

This portion of record is plotted in Figure 6b on an expanded scale. Inspection of the video tape corresponding to this time reveals that the strong turbulence was accompanied by a thick cloud of suspended sediment in the layer of water which was approximately 15 cm deep at t = 50 sec. The sediment essentially erupted from the bottom at about this time, and remained in suspension at the sensor location until it ran dry. The water surface also noticebly changed character at about $t \approx 52$ sec, from a smooth layer to an extremely rough one. This transition is readily apparent on the video images since reflections of the straight mounting pipes, which are very clear at one instant, suddenly become jagged and then disappear. We interpret this transition in roughness as the surface manifestation of the bottom boundary layer turbulence reaching the water surface.

A low-pass trend shown as a dashed line in Figure 6b was subtracted from the signal to leave an estimate of the turbulence residual. This residual was examined using a scheme that measured the time interval between zero-crossings and averaged

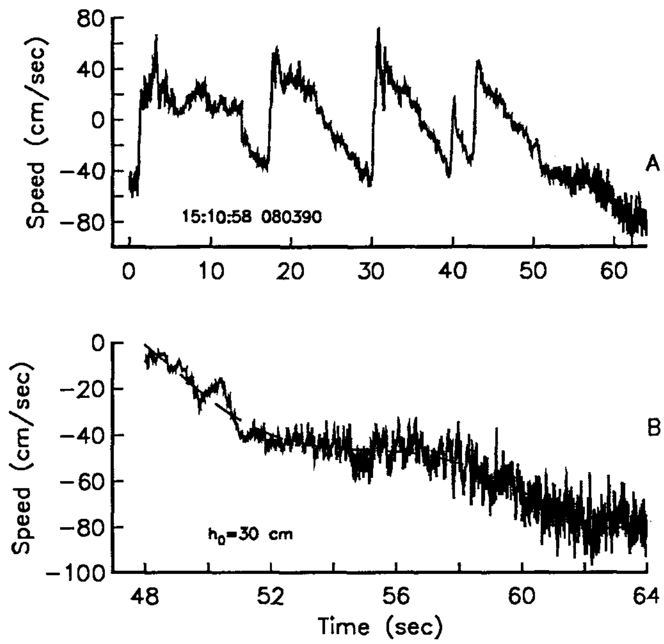


Figure 6. Time series of swash cross-shore velocity measured with hotfilm anemometer. Upper panel (A) shows entire 64 sec record, lower panel (B) shows time 48 to 64 sec. Dashed line is low-passed, mean velocity.

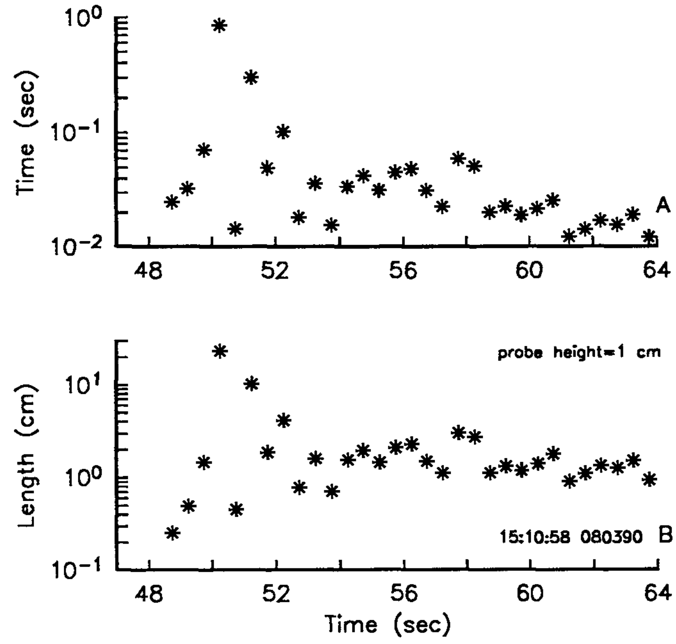


Figure 7. Turbulent time scale (A, upper panel) and length scale (B, lower panel) calculated from zero down-crossings of time series shown in Figure 6b, after removal of low-pass mean. Mean velocity also used to adjust time scale to length scale using Taylor's hypotheses.

the results over 0.5 sec periods. The method was insensitive to the precise shape chosen for the trend filter. The results are shown in Figure 7a, and suggest that the average zero- crossing time decreases as the flow continues it's offshore acceleration.

Application of Taylor's hypothesis to this data, using the low-pass velocity signal mentioned above, yields the results shown in Figure 7b. Here it is revealed that the mean eddy scale is just slightly over 1 cm after $t \approx 52$ sec and remains constant until the end of the record. This result is consistent with the expectation that the eddy scales of flow near a boundary must be on the order of the distance from the boundary. Turbulent eddies cannot transfer momentum in the direction perpendicular to a wall at scales larger than the distance from the wall (Tennekes and Lumley, 1972). This is one characteristic, actually an assumption, that leads to the so-called law-of-the-wall.

Conclusions

Our primary conclusion is that the length scale of turbulence in the surf zone is on the order of the local wave (or bore) height, or equivalently, on the order of the local depth. We also find that turbulent velocity scales are indeed a modest fraction of the local shallow water wave phase speed, $c = \sqrt{gh}$, consistent with laboratory results, but we do not have measurements over a sufficient range of depth to establish the functional dependence, or to quantify the proportionality.

Turbulence length and velocity scales seem to decrease with distance from the surface under a bore. This suggests that the turbulent eddy viscosity, $\nu_t = \lambda q$, and the dissipation rate $\varepsilon = q^3/\lambda$, may be strong functions of vertical position in the water column, as well as functions of depth. These findings may have some bearing on detailed models of both longshore currents and undertow (shore normal currents), since the turbulence quantities enter into the current structure through relations like (1) and (2).

At least the horizontal components of velocity in the surf zone approach isotropy at frequencies on the order of 2 or 3 Hz. This is found to be a gross violation of Snell's law, which predicts that ratios of V/U should be only a few percent. This is a clear manifestation of the generation of turbulence by wave breaking in the surf zone.

Limited data in the swash suggest that the length scales in the boundary layer flow near the bottom during backwash events are proportional to the distance from the bottom. Video tape images of swash flow suggest that fully developed backwash events, where the turbulent bottom boundary layer thickness reaches the full flow depth, occur at beat periods and *not* at individual wave periods.

Acknowledgements

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