Directional Random Waves Propagation on Beaches

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Abstract

The paper deals with a method for the simulation of irregular, directional waves over large coastal areas using a linear model based on a energetically equalized discretization of wave spectrum and a parabolic approximation to the Berkhoff equation for wave evolution. A sensitivity analysis based on a comparison with physical model tests gives some ideas about the applicability of the method.

Introduction

Waves on the coast often shows little directional spreading due to the effect of refraction and diffraction. However, there is now sufficient evidence about the critical importance that directionality can have in the transformation of wave characteristics from deep to shallow waters, as shown through numerical experiences by Goda (1985) and Isobe (1987) and in physical model tests by Vincent and Briggs (1989).

Evolution models for spectral parameters including wave refraction and diffraction and assuming no wave generation or dissipation can be made applying the linear superposition of component waves transformed according to Berkhoff's (1974) equation if wave non-linearities can be disregarded for the specific problem to be studied or not important due to particular wave characteristics. However, over the very large areas often appearing in coastal studies, even the application to a single component of the mild-slope equation can be a formidable task. A solution for that can be the use of a parabolic approximation to the Berkhoff equation. In fact, parabolic equation method (PEM) extension for use with irregular directional waves has been done by Isobe (1987) applying the Radder's (1979) wave equation in the ray - front coordinates system. Very recently, Panchang et al. (1990) have also presented irregular directional results for the Vincent and Briggs tests (1989) using Radder's equation over an orthogonal grid, with a good agreement between computed and physical model results.

The present paper deals with the application of higher order, large angle PEM (Booij, 1981), (Kirby, 1986a, 1986b) to directional wave propagation on large coastal areas. At first, a brief review of the methodology employed is given. Some basic

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questions arise about the method for wave spectrum discretization and they are studied suing a comparison with physical model tests by Vincent and Briggs(1989). Then, the method is applied to an idealized coastal area showing some of the implications that directionality can have on coastal environments.

Methodology

Under the assumption of validity for linear superposition as a method for the description of irregular waves, simulation of directional waves can be made trough the application to a number of monochromatic waves representing the whole wave spectrum of a wave equation for the evolution of monochromatic waves over an irregular bed.

Given a wave spectrum $S(f, \theta) = S(f)\ G(f,\theta)$ where $G(f, \theta)$ is a normalized shape function for directional distribution, a representation between given cut-off values, (f_{min}, f_{max}), $(\theta_{min}, \theta_{max})$ in N_f, N_θ components each of which defined by a corresponding frequency, a direction and a wave amplitude $(f_i, \theta_{ij}, a_{ij}, i = 1, N_f, j = 1, N_\theta)$ can be made by a constant step method:

$$\Delta f=\frac{f_{max}-f_{min}}{N_f} \qquad \Delta\theta=\frac{\theta_{max}-\theta_{min}}{N_\theta} \tag{1}$$

$$\begin{aligned} f_i&=f_{min}+(i-0.5)\Delta f \\ \theta_{ij}&=\theta_{min}+(j-0.5)\Delta\theta \\ a_{ij}&=\sqrt{2S(f_i,\theta_{ij})\Delta f\Delta\theta} \end{aligned} \tag{2}$$

The method employed in the present work for an energetically equalized discretization of the spectrum consists of:

$$\begin{aligned} &f_i \textit{ given by: } \int_{f_{min}}^{f_i} S(f)df=\left(\frac{m_0}{N_f}\right)(i-0.5) \\ &\theta_{ij} \textit{ given by: } \int_{\theta_{min}}^{\theta_{ij}} G(f,\theta)d\theta=\left(\frac{1}{N_\theta}\right)(j-0.5) \\ &a_{ij}=\sqrt{\frac{2m_0}{N_f\,N_\theta}} \end{aligned} \tag{3}$$

being m_0 the zero order moment of the frequency spectrum. In the context of wave propagation, a similar approach has been presented by Goda (1985), at least for the frequential part of the spectrum.

Having obtained a representation of the irregular waves as a number of simple monochromatic waves, a PEM is employed for solving the wave potential field over the area of interest. It is possible then to recover some wave characteristics as significant wave height, instantaneous free surface, etc, through linear superposition of results for the components. Given the limitation of PEM that assumes a main wave propagation direction one alternative is to obtain a specific calculation grid over the area modelled as a function of the direction of the incident wave, thus overcoming this limitation. That approach has been taken by Panchang et al.(1990). However, the use of large angle parabolic models such as (Kirby, 1986b), allows for the use of only one calculation grid in some cases even with some quite broad directional spectrum, avoiding the need for an interpolation of results of components in order to recover the irregular wave characteristics of interest.

For his application on wave propagation studies, equalized discretization shows some basic advantages:

1. Grouping of wave components around the spectrum peak, both in the frequency and direction domains (as seen in fig. 1). This is important in connection with PEM, given its intrinsic angular limitations. This makes easier the design and orientation of a calculation grid for a given area and can avoid the need of an arbitrary truncation of the spectrum.

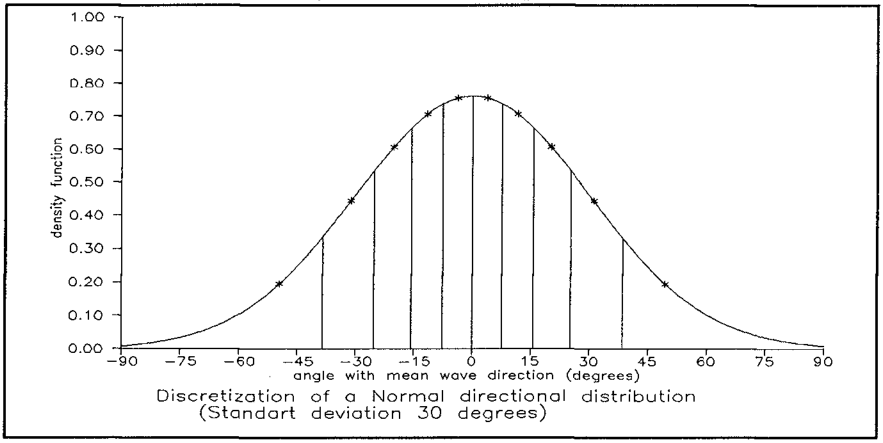

Figure 1.- Equalized discretization for a directional shape.

2. Each one of the equalized components has a same amount of energy and can, in a first approximation, contribute equally to the disturbance in a given point. Then, computational effort is optimized for a given number of wave components.

The main disadvantage is that equalization is more complex and time consuming (when the spectral shape is not directly integrable) than constant step procedures; in the present context, however, equalization is a trivial task in comparison with the effort involved in the propagation of the wave components.

In the present application, solution of equations (3) for the equalized discretization of an arbitrary spectrum shape has been made using standard library routines (Morris, 1987) for evaluation of integrals over finite intervals and equation solving.

From a practical point of view, two main questions should be assessed:

1. Order of magnitude in the number of wave components needed to obtain accurate results, within the intrinsic limitations of the method (linear superposition, approximate PEM).

2.- Optimal distribution of the computational effort between the frequency and directional domain.

That points are going to be examined by comparison between the results applying the proposed methodology and some physical model test.

Comparison with Physical Model Tests

A numerical model built within the methodology exposed has been compared with the results of some tests on irregular directional wave propagation performed by Vincent and Briggs (1989) in a wave basin with a directional irregular wave generator. For the irregular waves target spectrum has been a TMA shape (Bouws et al., 1985) in the frequency domain and a wrapped around the circle normal distribution shape for directions. The bathymetry modelled has been a elliptic shoal similar to the one studied by Berkhoff et al. (1982), but over an horizontal bed instead of a sloping one. Figure 2 shows the layout of the model.

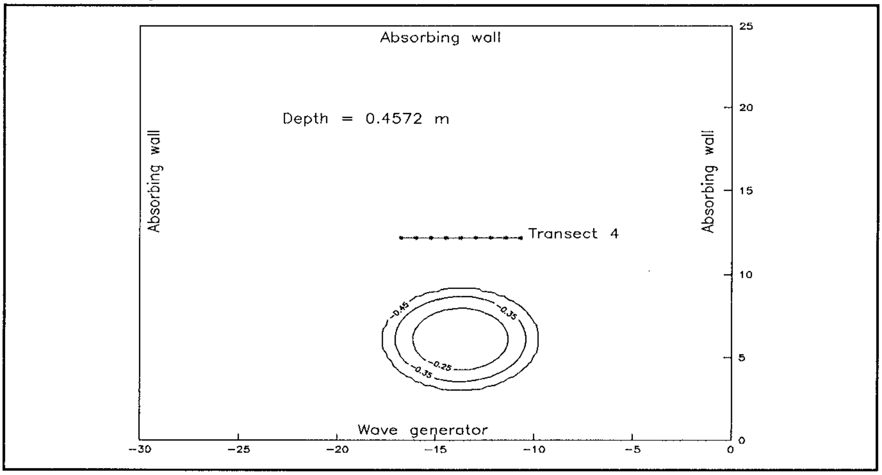

Figure 2.- Layout of the model

Cases selected are shown in table 1, and covers both wide and narrow shapes both in the frequency and direction domains for low steepness waves in intermediate water depth, making at first reasonable the use of linear theory. A monochromatic test and two unidirectional tests have also been computed.

Test Id.	Waves	Peak Factor	Standard Deviation
M2	Regular	-	-
U3	Irregular	2	0
U4	Irregular	20	0
N3	Irregular	2	10
N4	Irregular	20	10
B3	Irregular	2	30
B4	Irregular	20	30

Table 1.- Tests selected for comparison

In the numerical calculation, for the irregular cases the equalized discretization of the spectrum has been made taking only 10 frequency components by 10 directional (when required) components with the same frequency cutoff as in the physical model. Then for the narrow directional spreading tests, initial wave direction over the grid has an obliquity of up to around 16 degrees with the assumed main wave direction, avoiding the need for an arbitrary wrapping of the directional distribution in order to satisfy the PEM assumptions. Otherwise, for tests B3 and B4, maximun initial obliquity is around 49 degrees, and is going to be higher given the expected wave pattern after the shoal, thus some truncation should be applied.

The area modelled has been discretized in a calculation grid of 101 x 121 nodes with node distance of 0.25 ms. Wave propagation for each component over the same grid has been calculated using a Crank - Nicholson finite difference scheme based on higher order, large angle Kirby's minimax parabolic equation (Kirby, 1986b) in a linear version and without friction terms. Initial condition for complex wave amplitude ϕ for each oblique component ij in the first row of the grid are:

$$\phi_{ij}=a_{ij}\ \exp\ (i\ (k_i\ \sin\theta_{ij}\ y\ +\ (S_0)_{ij})) \tag{4}$$

being k_i the wave number corresponding to wave component i,j, and y the coordinate values corresponding to the grid nodes along the first row (x being the main propagation direction). A pseudo random value S_0, equally distributed between 0 and $2\ \pi$ is added to the initial phase function for each component in order to obtain realistic instantaneous free surfaces.

Minimax parameters for the computations have been taken for 70 degrees of aperture, and a dissipative filter (Kirby, 1986a) has been applied to the computed results.

Figures 3 - 4 shows results of significant wave height along measurement section 4, with the reference of physical model results. The continuous line presents numerical results and the broken line physical results from the graphs of Vincent and Briggs (1989).

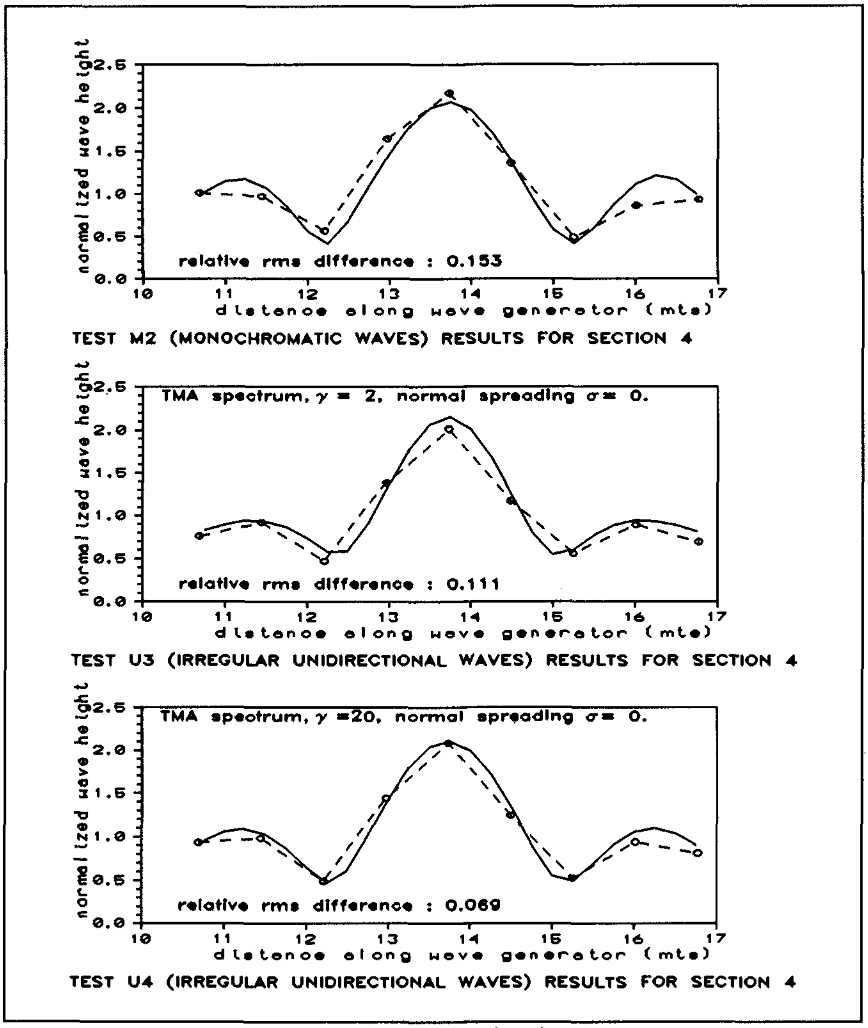

Figure 3.- Results for non directional tests

As signaled by Vincent and Briggs (1989), the non-directional cases show very similar features to the ones in the regular test. However, the qualitative agreement is better in tests U3, U4 than in M2, featuring both numerical and physical results the lateral linear caustics that develops behind the shoal, not present in the physical model results for the monochromatic case.

For the directional tests, the magnitude of the caustic shows an intense decrease, practically disappearing for wide spreading tests. Similarly, numerical and physical results fits well, except in the test B4, with a narrow frequency spectrum and

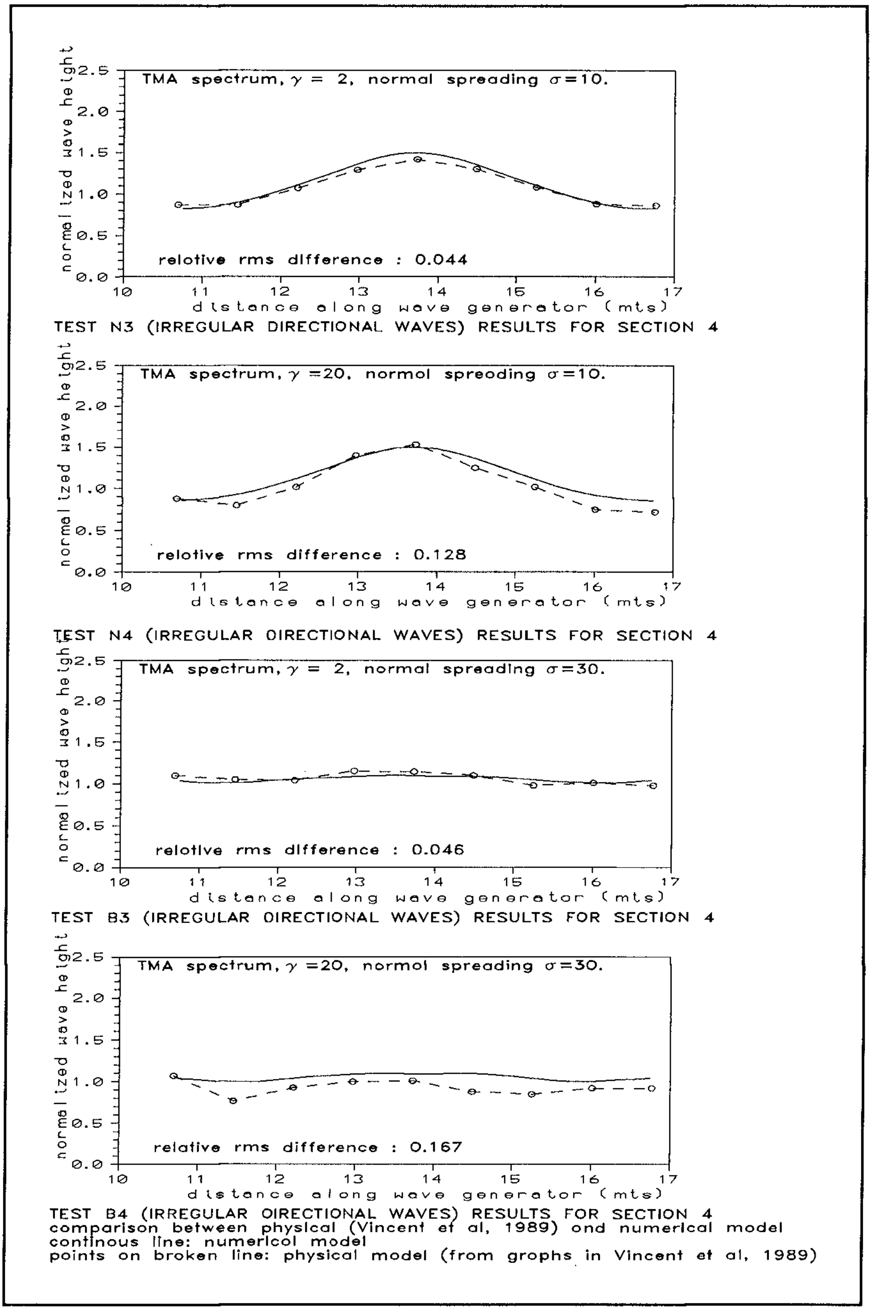

Figure 4.- Test results for directional cases

wide directional spreading, where physical model results shows a systematic trend towards lower values than the numerically computed, perhaps due to some amount of dissipation over the shoal.

A quantitative measure of the quality of results has also been made, using a relative root mean square difference between physical and numerical model for that section:

$$dif_{rms}=\sqrt{\frac{\sum\left(\frac{H_{pm}-H_{nm}}{H_{pm}}\right)^2}{N}} \tag{5}$$

where:
H_{pm} : Wave height in physical model (from graphs in Vincent and Briggs, 1989)
H_{nm} : Wave height in numerical model
N : number of measurement points along section 4

Table 2 summarizes the results.

Test id.	Relative root mean square difference
M2	0.153
U3	0.111
U4	0.069
N3	0.044
N4	0.128
B3	0.046
B4	0.167

Table 2.- Comparison between numerical & physical results

Then it appears that in the cases modelled an irregular linear wave propagation model based on an equalized discretization of the spectrum and the application of a higher order PEM provides a similar quality of results as with PEM applied to monochromatic test.

Sensitivity Analysis

A sensitivity analysis of the results as a function of number of wave components has been made for test case N3, trying to obtain information about which number of wave components is needed for a correct reproduction of irregular propagation in that test and also studying the stability of the numerical model. Results have been evaluated with eq. 5 for an increasing number of discrete components (equal number in frequency and directions) from 4 (2 x 2) to 900 (30 x 30), obtaining the relative difference with physical model tests.

Results are plotted in figure 5 (solid line), together with similar data for results obtained through a constant increment discretization of the spectrum (dashed line).

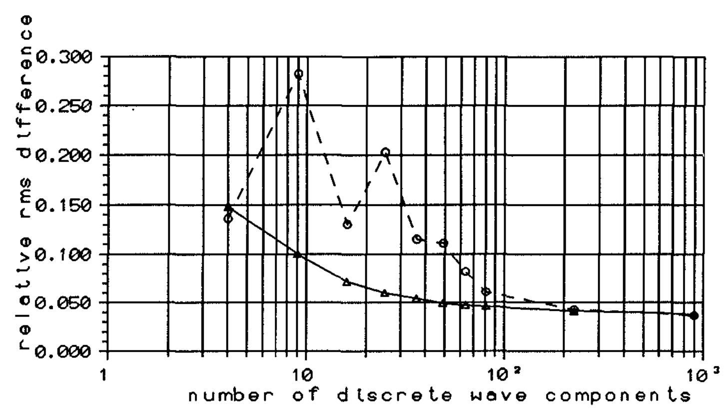

Figure 5.- Sensitivity to number of components (test N3)

As can be seen, results obtained with equalized discrete components converge in a stable way towards the asymptotic solution for the numerical model and a very little improvement is obtained with more than 100 components. Otherwise, results with constant step shows larger errors with a small number of components and an anomalous behavior when the number of intervals in the directional domain is odd, due to the fact that then a component is placed on the spectral peak, with a predominant amount of wave energy which implies results more similar to the monochromatic situation. Of course, with an increasing number of wave components, differences between both approaches vanishes, but it is still noticeable at the number of wave components used in the present work.

Then it can be concluded that equalized discretization gives, from the point of view of wave propagation, a more reliable representation of irregular waves than constant step methods, in particular if the number of wave components used is not high. The need of a low number of components for the obtention of good quality results is a hopefully conclusion, making possible the application of the methodology over large areas with reasonable computing resources, and implies also the possibility of application of more complex models such as the elliptic mild-slope equation using a similar approach.

Another question is the optimum relationship between number of directional and frequency components involved in the computations. That question is to be solved in a case basis, but given the greater importance of directionality over frequency shape at least in cases involving diffraction, it appears likely to use a larger number of directional components. From a computational point of view, it should be noted than the coefficients in the discretized parabolic equation do not have any dependence on the wave direction (except for the independent term that depends also on initial conditions), being a function of wave frequency , wave number and related quantities (phase and group velocity). Then, when a component has been computed for a given frequency, only a minor modification on the independent term of the parabolic equation system is needed in order to setup the equation for a new wave direction with the same frequency. Figure 6 shows results for test B3 computed using double number of directional components (dashed line) that of frequency ones and an equal number (solid line).

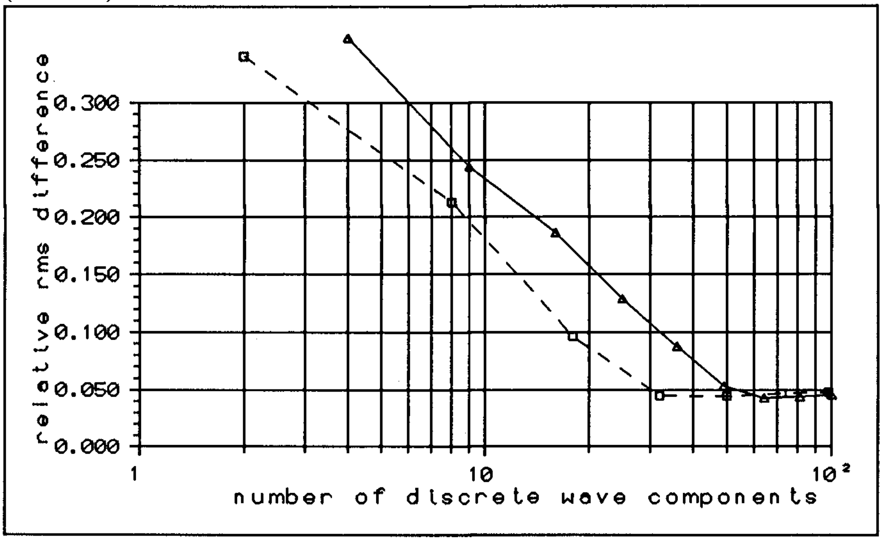

Figure 6.- Comparison of results with $N_\theta = 2\ N_f$ (Test B3)

A similar quality of results is obtained with (N_f = 4, N_θ = 8) and (N_f = 7, N_θ = 7) with the added advantage of the low computational cost of computing an additional directional component for a given frequency as compared with the cost of the setup of the coefficients of the system of equations that arises from PEM, needed if more frequencies are involved.

Application over an idealized bathymetry

An application has been made over an idealized bathymetry given by:

$$\begin{gathered} z(x,y)=-10+0.001*x \\ (0.\leq y\leq 1500.) \\ (0.\leq x\leq 1000.) \end{gathered} \qquad (6)$$

with x, y and z in meters and representing a plane sloping beach. A shore-parallel breakwater is located at a depth of 7. mt and extends from y = 750 to y = 1500 mt.

Three different wave conditions have been tested:

a) Monochromatic waves with period 10 sec. and normal incidence.
b) Swell-like waves, defined by a Jonswap spectrum with γ = 7. and directional spread modelled according to the Mitsuyasu - Goda - Suzuki(Goda, 1985) shape

function with S_{max} = 100.
c) Sea-like waves, with γ = 2. and a broader directional distribution given by S_{max} = 25. at the offshore boundary of the modelled area.

The discretization has been made taking 101 x 151 equally spaced nodes (distance between nodes 10 mt) and the irregular runs have been made with 5 x 15 components. Simulation of wave breaking has been artificially introduced in the model in a way similar to Ebersole (1985), trying to obtain qualitative information about breaking along the beach.

Computed results for significant wave height field are given in figures 7 - 9.

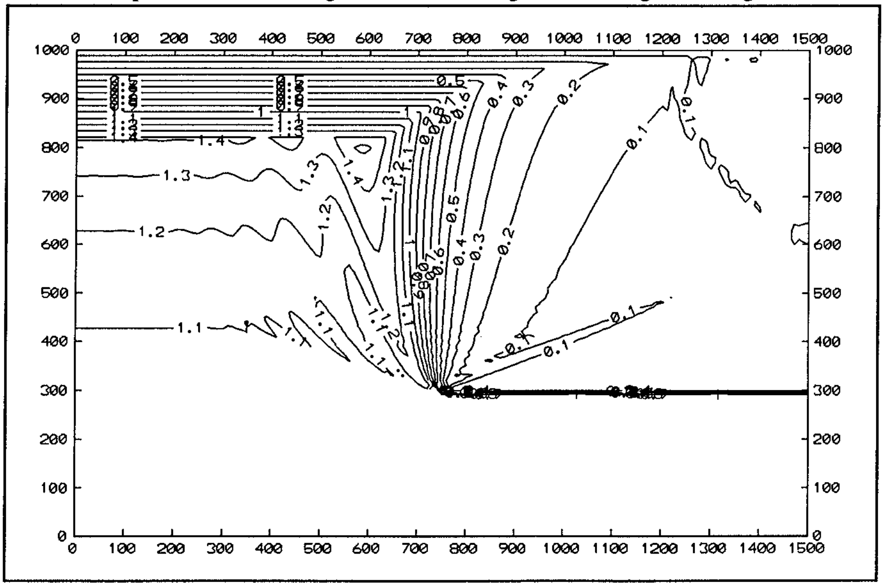

Figure 7.- Wave height field for monochromatic test

As can be seen, there are significative differences between for monochromatic and irregular wave results. At first, wave height field seems to be smoother on the coast in the irregular waves tests (the "wavy" variation in wave height associated to wave diffraction vanishes). It is to be noted that for all the tests, a dissipative interface (Kirby, 1986b) has been applied and is needed due to the numerical noise that arises in higher order parabolic schemes. Also, as signaled by Goda (1985) wave penetration along the line between the toe of the breakwater and the coast is higher for the irregular, broad directional test than for the monochromatic and the swell-like test.

Then, it appears than for narrow-directional distributions, wave irregularity can have a main effect of smoothing some of the special characteristics of monochromatic wave response, having a more fundamental effect for sea wave states.

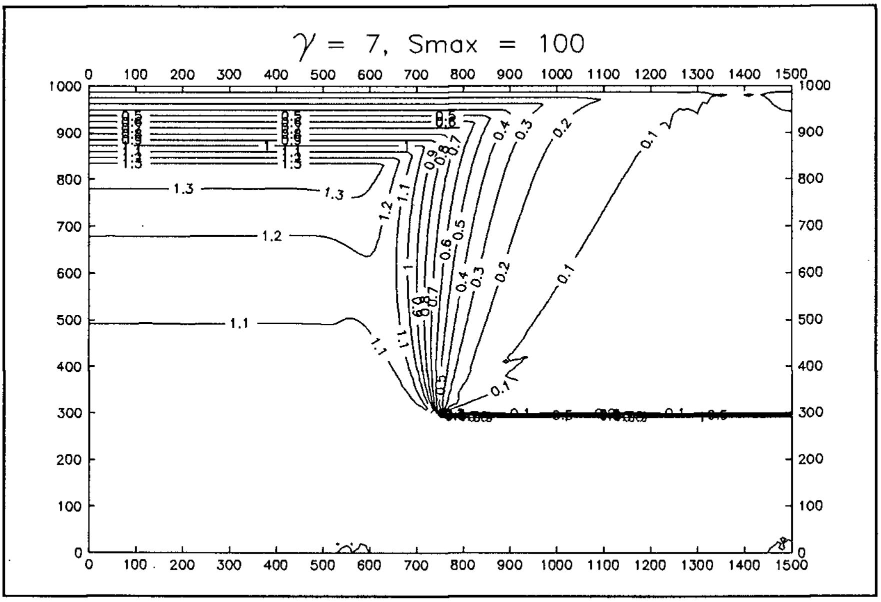

Figure 8.- Wave height field for swell conditions test

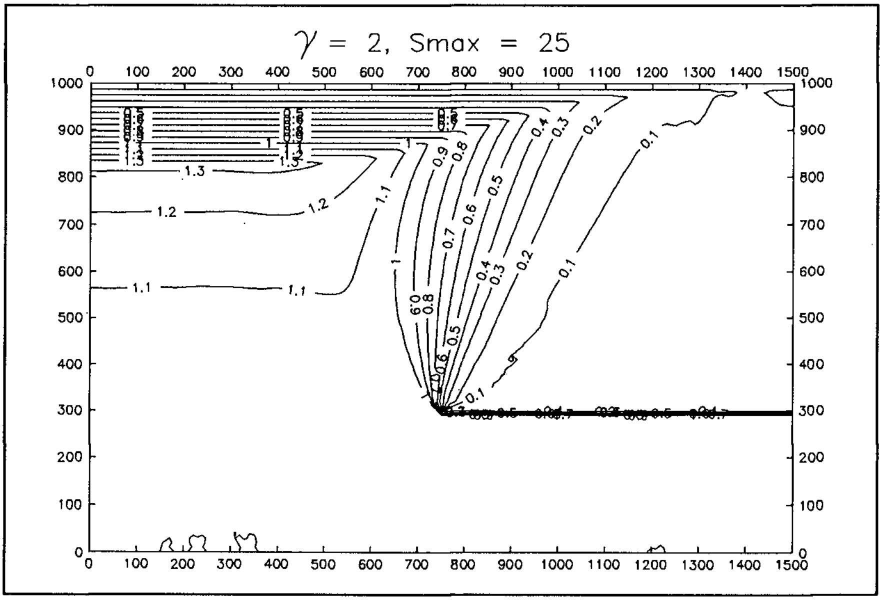

Figure 9.- Wave height field for sea wave conditions

Figure 10 shows breaking wave height along the beach for the three tests, being a main characteristic the more slowly decrease in wave height along the diffraction area for sea waves.

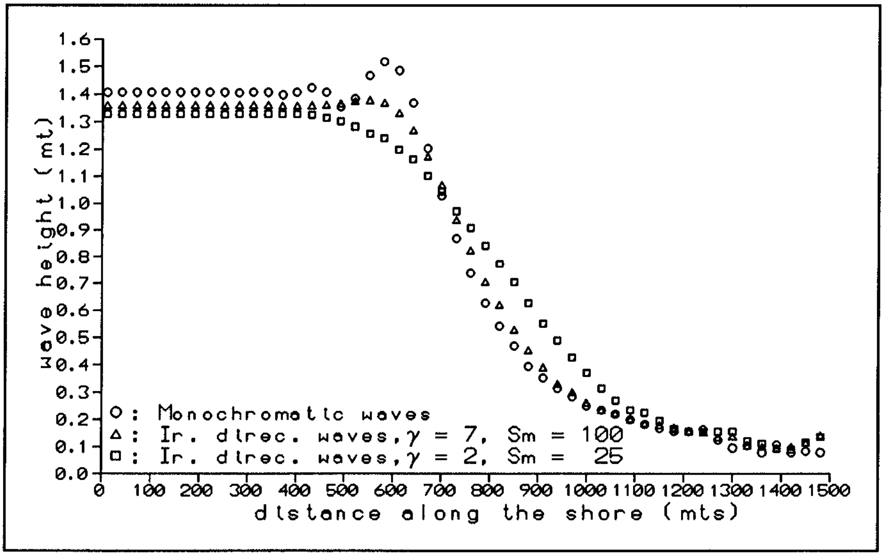

Figure 10.- Breaking wave height along the shore

Conclusions

A methodology for the study of irregular directional waves that can be applied to large areas has been presented, based on linear superposition of energetically equalized components of the target spectrum and the application of higher order parabolic approximations to the Berkhoff equation. Comparison with low steepness physical model tests by Vincent and Briggs (1989) and a sensitivity numerical experiment shows than it is possible to obtain quite accurate results even with a very low number of components, and that it could be more interesting to take a higher number of directional components than frequential ones. That should apply also for models builded from the elliptic mild slope equation.

A more extensive comparison including the high steepness tests of Vincent and Briggs (1989) is to be made for studying the expected degradation of predictive capability of the method for situations in which nonlinearities play an important role. As an alternative, another approach, perhaps based on more general equations for free surface flow simulation should be considered. Unfortunately, however, most of those models cannot be applied over the total range of depths to be considered for wave transformation from deep to shallow waters.

An application for an idealized coastal area shows some interesting features in the wave height field along the shore for the test with sea waves condition, giving results in qualitative agreement with those obtained by Goda (1985) for directional wave diffraction applying semi-analytical models. Aditional work is to be done for incorporating a more reallistic description of wave breaking into the model.

One of the actual weaknesses for application of directional wave propagation to studies on coastal areas is the absence of information about directional wave conditions and propagation characteristics on nature. Then an effort should be made in order to obtain field data and validate numerical models against that information.

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