CHAPTER 73

SCALE EFFECTS IN BREAKING WAVES

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Abstract

A breaking wave model, which is partly physical and partly analytical, is proposed. This model is based on observations that up to a certain moment the wave presents a long, smooth, horizontal, cylindrical edge, which then segments due to surface tension effects. A disturbance on a cylindrical surface, withdrawn from the influence of gravity, becomes unstable when its wavelength exceeds the circumference of the cylinder. The rate of growth of the instability, is a function of the radius of the cylinder and the wavelength of the disturbance. Using the theory describing the evolution of the assumed hyperbolic shape of the tip of a breaking wave, the radius of the cylindrical edge is approximated to the radius of curvature of the hyperbola. The model describes the three-dimensional evolution of the curling wave crest. Scale effects are then derived which show good agreement with experimental results.

Introduction

During the overturning process of a breaking wave, the plunging jet is in a state of free fall. Assuming that the wave is two-dimensional, with no transverse inertial forces, and air friction is negligible, surface tension is the predominant force acting on the fluid particles.

Under similar ambient conditions, the behaviour of the overturning face of the breaking wave is the same to that of a liquid cylinder. If a liquid cylinder is constructed and liquid is drawn out gradually, the diameter of the cylinder decreases until the length of the cylinder becomes just about three times as great as its diameter. Soon afterwards instability begins, and the cylinder alters its form; it narrows at the waist, so passing into an unduloid and the deformation progresses quickly until the cylinder breaks in two, and its halves become portions of spheres. This behaviour is due to surface tension effects. Plateau (1863) established experimentally that the distance between the spheres is proportional to the diameter of the cylinder. In the case of a very long cylinder, when many spheres are formed, the parts between consecutive

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dilations or constrictions undergo identically and simultaneously the same modifications as described above (fig. 1).

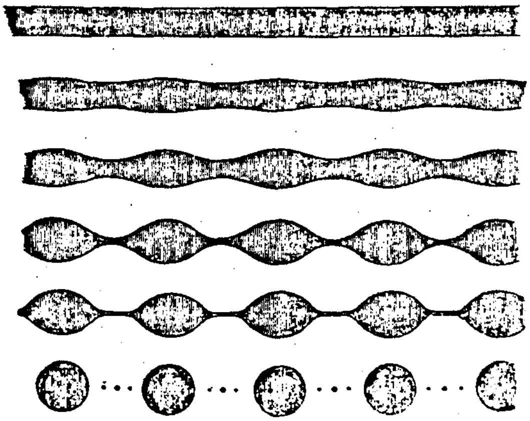

Fig. 1 Stages of the segmentation of a liquid cylinder

Plateau (1863) also noted that the phenomenon of the formation of lines and their resolution into spherules is not confined to the case of the rupture of the equilibrium of liquid cylinders; it is always manifested when one of the liquid masses, whatever may be its figure, is divided into partial masses. The phenomenon is also produced when liquids are submitted to the free action of gravity.

Worthington (1908) discussed the effect of surface tension on breaking waves. He observed that a wave that has just impetus enough to curl over and break, up to a certain moment it presents a long, smooth, horizontal, cylindrical edge, from which, at a given instant, are shot out a large array of little jets which speedily break into foam, and at the same moment the back of the wave, hitherto smooth, is seen to be furrowed and combed (fig. 2). He proposed that these jets are due to the segmentation of the cylindrical rim according to Plateau's law, and the ridges between the furrows mark the lines of easier flow determined by the jets. Further observations of this phenomenon were made by Toumazis (1989) from video and photographic records.

Fig. 2 Diagrams of a breaking wave (Worthington 1908)

Regarding scale effects in breaking waves, Fuhrboter (1986) studied impact loading by breaking waves on slopes and found that scale effects do exist.

Skladnev and Popov (1969) measured impact pressures induced by waves breaking on a slope in a more systematic way. Waves of the same steepness but of wave height ranging between 3 and 120 cm were tested. Prototype conditions were considered the ones with the 120 cm wave height. The results were presented in graphical form (fig. 3). The abscissa is the ratio of the model to the prototype (120cm) wave heights and the ordinate is the ratio of the model to the prototype impact pressure factor K_p, defined as the ratio of the measured pressure to the static pressure ρgh, where h is the wave height. From these results it is evident that, it is not permissible to use Froude criteria to scale up experimental results on breaking waves with model breaker height less than about 0.5m.

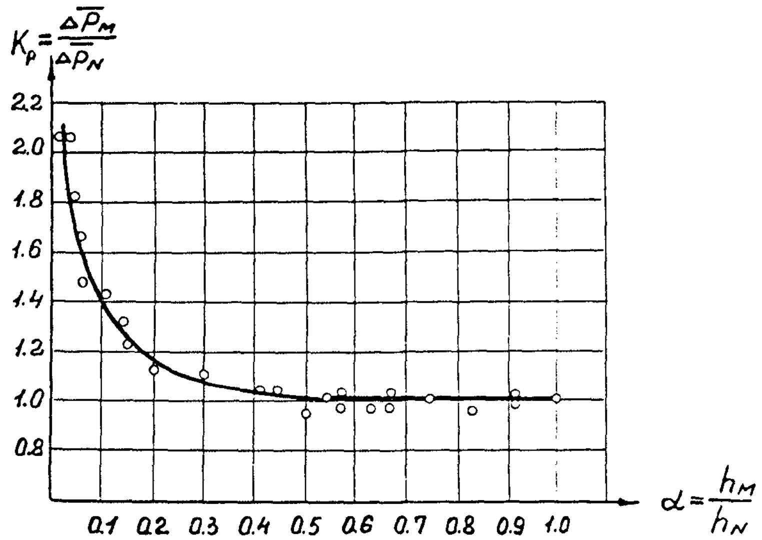

Fig. 3 Effects of wave height on the relative wave pressure value in the wave impact zone. (Skladnev and Popov, 1969)

Stive (1984) reached the same conclusion, stating that in order to avoid disturbing influence of the surface tension on the amount of air absorption in the broken wave, the incident wave height in the model must exceed approximately 0.5m.

The primary objective of this work is to formulate a model for the effects of surface tension on breaking waves and to establish a quantitative threshold beyond which scale effects are negligible. It will be shown that this threshold agrees well with available experimentally derived indicators.

Surface Tension Effects on Free Falling Liquid Cylinders

Any small disturbance on a liquid cylinder may be considered, according to Fourier's theorem, as the summation of a series of sinusoidal wave disturbances. The surface area of the cylinder will change whereas the volume of the liquid remains the same. As surface tension results in contracting the volume to the minimum possible surface area, an increase of the surface area due to the disturbance will be opposed by surface tension, whereas a contraction of the surface will be enhanced. The criterion for stability is therefore the change of surface area for a given volume after the appearance of a small disturbance. In the following derivations the effects of only one sinusoidal disturbance will be considered, bearing in mind that the principle of linear superposition must be applied when a full range of Fourier componets must be taken into account.

If a cylinder with radius r is slightly deformed so that

$$y = r + a \cos \frac{2\pi}{L} x \qquad (1)$$

where x is measured parallel to the axis, y is the distance normal to the axis, a is the amplitude of the deformation, and L is the wavelength of the deformation. Plateau (1863) showed that the cylinder is stable if

$$L < 2 \pi r \qquad (2)$$

The theory is hereby extended to consider a sector of a liquid cylinder with a slight sinusoidal deformation

$$y = r + a \cos(kx) \quad , \text{where } k=2\pi/L \qquad (3)$$

The mean surface area between two consecutive crests, σ, is then

$$\sigma = r \theta + \frac{1}{2} r \theta a^2 k^2 \qquad (4)$$

where θ is the angle between the radii of the sector. Assuming the volume of water in the cylindrical sector remains the same, the mean volume S is

$$S = \frac{1}{2} \theta r^2 + \frac{1}{4} \theta a^2 \qquad (5)$$

Combining the two expressions for σ and S,

$$\sigma = \sqrt{2 \theta S} + \frac{\theta}{4} \frac{a^2}{r} (k^2 r^2 - 1) \qquad (6)$$

The surface area of the undisturbed liquid (a=0) is,

$$\sigma_0 = \sqrt{2 \theta S} \qquad (7)$$

Therefore, the change in the surface area is,

$$\sigma - \sigma_0 = \frac{\theta a^2}{4 r} (k^2 r^2 - 1) \qquad (8)$$

If k.r > 1 the surface area increases and hence the liquid is stable, while if k.r < 1 the surface area decreases and instability results.

The obtained result is independent of the angle θ. The cylinder is a special case of the considered case, with $\theta=2\pi$. Plateau (1864) and Rayleigh (1878) reached the same result by considering the cylindrical shape as the initial condition.

The mode of falling away from unstable equilibrium necessarily depends on the small deformations to which the system is subjected. In an ideal situation of no deformations, even if $L>2\pi r$ the cylinder would still be in equilibrium. In practice however, some kinds of disturbances are always present. Although all disturbances corresponding to $L>2\pi r$ are unstable, the rate of change of the shape due to each wavelength, L is different.

Assuming that the amplitude of the disturbance, a, once unstable, grows exponentially with time, that is

$$a = a_0 \, e^{qt} \tag{9}$$

where a_0 is the initial amplitude, Lord Rayleigh (1878) showed that for a cylindrical surface,

$$q^2 \approx \frac{T}{\rho r^3}\left[x^2 - \frac{9}{8} x^4 + \frac{7}{2^4 \cdot 3} x^6 - \frac{25}{2^{10}} x^8 + \frac{91}{2^{11} \cdot 3 \cdot 5} x^{10} \right] \tag{10}$$

where $x=kr=2\pi r/L$. For given r, all disturbances with wavelength, L, less than $2\pi r$ $(2\pi r/L>1)$ are stable. For L greater than $2\pi r$ the disturbances grow with time, the fastest growing one having $L\approx 9r$.

Gravity and Inertial Effects

The mathematical description of the free surface area of a two-dimensional overturning wave, as governed by inertial and gravitational forces, was presented by Longuet-Higgins (1980). A frame of reference was used in which the x-axis is longitudinal horizontal, in the same direction as the propagation of the wave, the y-axis is transverse horizontal and the z-axis is vertical. The saddle-point, the point where the pressure gradient vanishes, follows a free fall trajectory. Its motion therefore satisfies

$$x = x_1 + U\,(t-t_1) \tag{11}$$

and

$$z = z_1 - 1/2\, g\,(t-t_1)^2 \tag{12}$$

where x_1, z_1, t_1 and U define the initial coordinates, time and horizontal velocity respectively. The tip of the wave is approximated by the hyperbola

$$\frac{x^2}{\alpha^2} - \frac{z^2}{\beta^2} = 1 \tag{13}$$

Taking γ to represent the angle between the asymptotes, δ the angle of rotation of the principal axes, and r the radius of curvature at the tip of the breaker (fig. 4),

$$\tan\frac{\gamma}{2} = \frac{\beta}{\alpha} \qquad (14)$$

$$r = \left|-\frac{\beta^2}{\alpha}\right| \qquad (15)$$

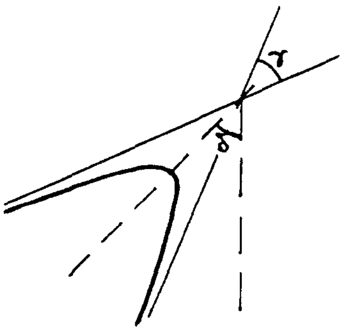

Fig. 4 Hyperbola approximating the tip of the wave.

Given the initial conditions, the two-dimensional evolution of the crest of the wave as depicted in Fig. 5 may be completely described, as shown by Longuet-Higgins (1980). It should be noted that in this reference there is a misprint in equation 5.8; the term f^2 should be included in the numerator of the right hand side.

The Proposed Model for the Breaker

Worthinghton's proposition (1908) is adopted as the fundamental assumption on which the presented model is based. That is, the crest of the overturning wave presents a long, smooth, horizontal, cylindrical edge.

The horizontal cylindrical edge, in the state of free fall, is modified in a manner governed by surface tension principles. The radius of the cylindrical edge is assumed to be independent of surface tension effects and only governed by inertial and gravitational forces. The formulations derived by Longuet-Higgins (1980) are therefore applicable to this problem. The radius of the cylindrical edge (Worthinghton, 1908) is equated to the radius of curvature of the hyperbolic edge of the wave (Longuet-Higgins, 1980). The proposed model is therefore a combination of two distinct analytical descriptions. The problem is studied by considering the evolution of monochromatic sinusoidal disturbances on the tip of an overturning wave.

Consider a small sinusoidal disturbance of amplitude a_0, and wavelength L, on a liquid cylindrical edge. Surface tension considerations suggest that the amplitude of the disturbance will start growing when the wavelength, L, exceeds $2\pi r$, with an exponential rate of growth, $a_t=a_0 \exp(qt)$ (equation 9). The value of q is obtained from equation 10. For a cylindrical edge with a continuously varying radius, r, it is more appropriate to rearrange equation 9 so that,

$$a_{t+\delta t} = a_t \exp(q\ \delta t) \qquad (16)$$

Given the initial conditions of the overturning wave, that is the conditions at breaking point, the evolution of the breaker shape is computed. For any time t, the values of the variables δ and γ as defined by Longuet-Higgins (1980) are thus obtained. These values together with the coordinates of the point of intersection of the axes of the hyperbola, which are determined from equations 11 and 12 describe completely the evolution of the tip of the breaker in the two-dimensional plane of the wave. The radius of curvature of the tip of the breaker is equal to

$$r = \left| \frac{\beta^2}{\alpha} \right| = r_0 \tan^{3/2}(\gamma/2) \qquad (17)$$

Assuming that the rate of increase of the amplitude of a disturbance on the tip of the breaker is the same as that for a cylinder, as derived by Rayleigh (1878), the amplitude of a disturbance of wavelength L is then obtained using equations 16 and 10.

Figure 5 shows the time dependent characteristics of six distinct sinusoids with the same initial amplitude. It is clear that, as the radius of curvature of the curling wave crest decreases with time, the long disturbances start growing first. The rate of growth of the amplitude decreases with increasing wavelength of the disturbance (eqn. 10). A long wavelength starts growing early and slowly, whereas a short wavelength starts growing late and rapidly. There is consequently a particular wavelength which attains the maximum amplitude at the instant of hitting the water surface. This predominant wavelength is therefore a function of the type and size of the breaker (initial radius of curvature, breaker height and horizontal velocity) and the form of the initial disturbance.

Short wavelengths are in general associated with small breaker heights and small radii of curvature at the tip of the breaker. The duration of overturning is short. Sinusoidal disturbances of long wavelengths although unstable, grow very slowly and the short duration of overturning does not allow them to become dominant. The smaller the breaker, the shorter the predominant wavelength.

On the other hand, big breakers have large radii of curvature even at plunging point. Disturbances of short wavelengths do not become unstable. As a result, the bigger the breaker, the longer the predominant disturbance.

Derivation of Scale Effects from the Proposed Model

As it has already been noted, in breaking waves apart from the gravitational and inertial effects, surface tension also plays a significant role. In scaling surface tension effects the ratio of the inertial to the surface tension forces must be the same. The Weber number

$$W_e = u \sqrt{(l\rho/T)} \qquad (18)$$

must be the same in both prototype and model conditions, where ρ and T are the density and surface tension of the fluid, u and l are velocity and length scales respectively.

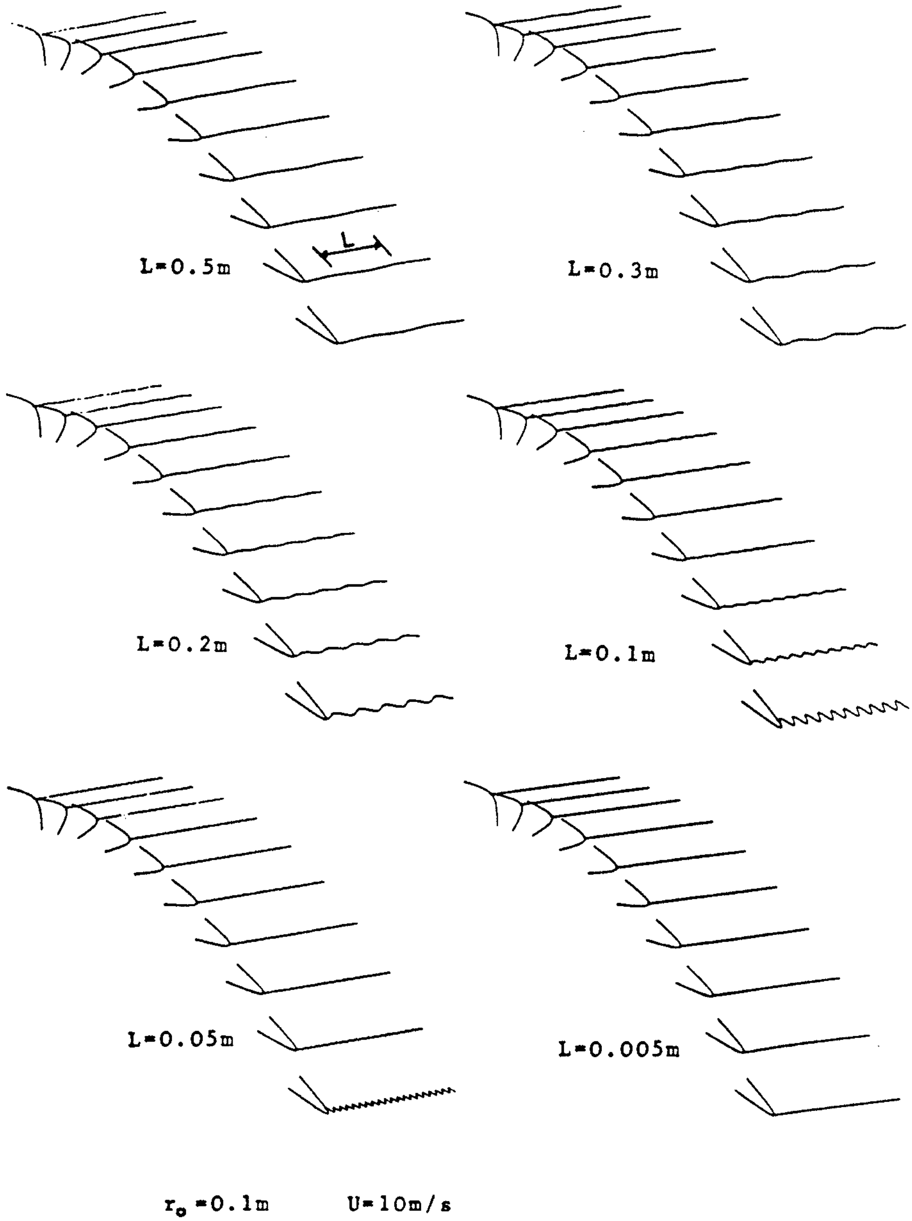

Fig. 5 Evolution of six sinusoids, of different wavelength, on the tip of an overturning wave.

It is evident that the Froude and Weber criteria cannot be satisfied simultaneously. In model tests involving breaking waves, Froude scaling is usually the adopted criterion, as it models satisfactorily the overall conditions, although the details of the breaking process are not fully modelled. It is of vital importance, in the interpretation of the model test results, to know how the non-scaling of surface tension effects affects the prototype conditions.

The developed physical/computational model, which takes into consideration both gravitational and surface tension effects, is now used to quantify scale effects in breaking waves. Consider a wave breaking in shallow water (fig. 6), which has the following characteristics

$$L_b \sim T\sqrt{gh} \tag{19}$$

and

$$H_b \sim h \tag{20}$$

where $\sim$ means order of, L_b is the wavelength of the breaking wave, T is the wave period, h is the water depth at breaking point, and H_b is the breaker height.

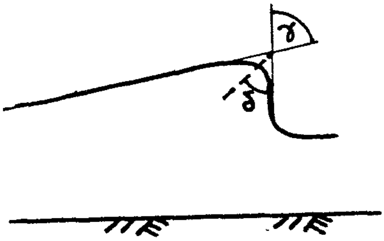

Fig. 6 Breaking wave in shallow water depth.

In order to determine the angle γ when the forward face of the wave becomes vertical, the assumption is made that the backward face of the wave is straight such that,

$$\tan\gamma \sim \frac{L_b}{H_b} = \frac{T\sqrt{gH_b}}{H_b} = T\sqrt{\frac{g}{H_b}} \tag{21}$$

The initial conditions are therefore

$$\gamma_0 \sim \tan^{-1}\left(T\sqrt{g/H_b}\right) \tag{22}$$

$$\delta_0 \sim 1/2\ \gamma_0 \tag{23}$$

$$r_0 \sim 1.4\ H_b \tag{24}$$

The duration of overturning is equal to the time required for a free fall from height H_b

$$t_b \sim \sqrt{2H_b/g} \tag{25}$$

For a given breaking wave height, H_b, and wave period, T, the evolution of a transverse sinusoidal disturbance on the wave crest, with wavelength L, and amplitude a_0, is traced up to plunging point. The ratio of the amplitude of the disturbance at plunging point, a_p, to the initial amplitude, a_0, i.e. a_p/a_0, is a measure of the surface tension effect on the wave breaking process for the assumed wavelength of initial disturbance.

When the above procedure is followed for a range of wavelengths of disturbance, the maximum value of a_p/a_0, corresponding to the predominant wavelength of disturbance, is deduced. Any given breaker is therefore associated with a predominant wavelength of disturbance, whose amplitude at plunging point is amplified by the factor a_p/a_0.

Figure 7 presents the computed values of the ratio a_p/a_0 for a wide range of wave conditions. The variation of this ratio reflects the effects of scaling. A constant value for a_p/a_0 implies that the shape of the breaker does not change between large and small waves.

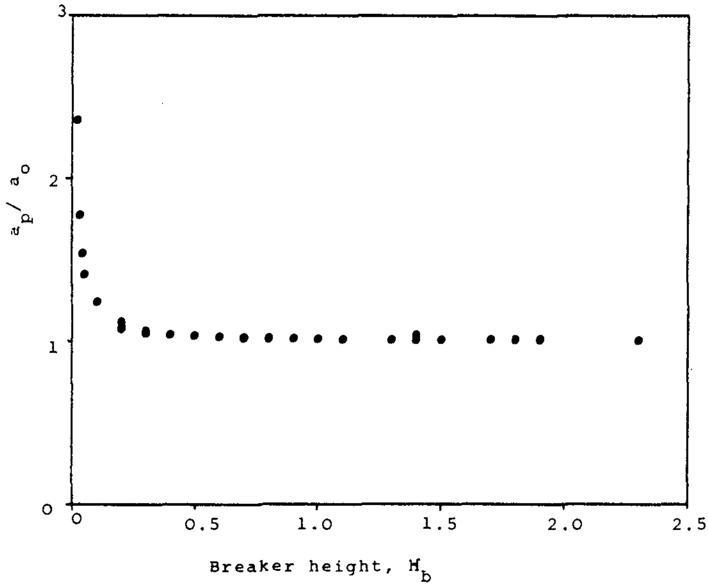

Fig. 7 Model results showing the relative growth of predominant disturbance as a function of breaker height.

It is evident from this plot that, surface tension effects are the same for breaking waves bigger than about 0.5m in height, whilst they become increasingly important for smaller waves. This observation suggests that, assuming the curling face of the wave retains a hyperbolic shape, big waves, which have small curvatures, do not feel the surface tension forces. This is in agreement with the broad characteristics of surface tension; surface tension is a molecular effect which becomes important for high curvatures.

The presented model is in agreement with the experimental results of Skladnev and Popov (1969) and Stive (1984), who both found that scale effects do exist for breaking waves smaller than 0.5m high. The results from the present model and the available experimental observations, however, appear to disagree with regard to the following point. The present model predicts no segmentation of the tip of the wave for big waves, whilst the experimental results suggest that big waves are highly aerated and spayed, whilst in small waves aeration is negligible or does not occur at all (Skladnev and Popov, 1969). This apparent contradiction may be justified by considering the 0.5m cut-off point as the point at which the physical process of wave overturning changes. For breaker heights up to 0.5m, the global surface tension effects are more significant than the microscopic ones. For bigger waves, as the results from the proposed model suggest, the global surface tension effects are negligible.

The radius of curvature of an assumed irregularity on the tip of the wave is very small compared to that of the tip of the breaker. When unstable, the short wavelength disturbances grow much faster than the longer ones. These properties imply that, the short sinusoidal transverse disturbances, which are stable from the macroscopic point of view, are now unstable and fast growing. They thus prevail on the tip of the wave. The described process is illustrated in figure 8. The evolution of the same monochromatic disturbance is presented, as predicted from the numerical model, for four different radii of curvature. The fast growth of the disturbance associated with the small radii of curvature is evident. Irregularities on the tip of the breaker, therefore tend to produce closely spaced jets. These jets are easily observed on the curling crest of two-dimensional overturning waves, breaking in both deep and shallow water conditions.

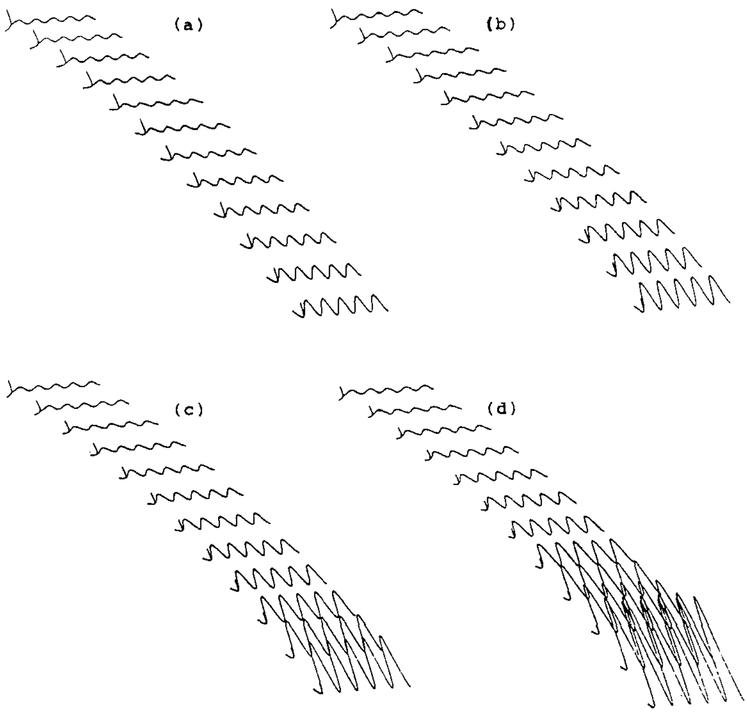

Fig. 8 Evolution of a transverse disturbance (L=40mm), on 4 waves of different curvatures: r_0= a) 14mm, b) 12mm, c) 10mm, d) 8mm. Time interval = 0.02 seconds.

Critical Assessment of the Proposed Model

The accuracy of any model depends on the validity of the assumptions inherent in the modelling. In the proposed model two distinct theories are used, the first one dealing with the segmentation due to surface tension effects, and the second with the stretching of the tip of the plunging jet, due to gravity and inertia effects.

The formulations on surface tension used in the proposed model are based on linear theory. Non-linear solutions have been developed since the original formulations by Plateau and Rayleigh. Bogy (1979) presented a literature review on the problem of the segmentation of liquid jets. Recent theories take into account finite amplitude disturbances, non-linear interactions, viscosity and air friction. The approximations inherent in the proposed model are such that the use of linear theory in describing surface tension effects is quite reasonable.

Regarding the formulations derived by Longuet-Higgins (1980) on the change of form of the tip of the curling wave, no experimental work has been done to test the theory. The reason for the absence of such a work becomes apparent when one looks at photographs showing the overturning process. The curling face loses its continuity, as it spreads into jets and the free surface boundary is no longer two-dimensional, making the description of the cross sectional profile a most difficult task. The theory and assumptions however, on which the derivations are based are reasonable and, overall, the results are broadly in agreement with observations.

The assumptions inherent in the proposed model introduce their own limitations. Firstly, it is assumed that the tip of the overturning wave behaves like a horizontal liquid cylinder. The surface tension formulations for liquid cylinders are based on the assumption that there is no flow across the surface of the cylinder, a condition which is not necessarily valid in a breaking wave. The effect of this assumption is that the model is strictly applicable to the very tip of the wave crest.

The ridges formed at the back of the wave are not treated by the proposed model. Their formation may be explained by the physics on which the model is built. The back of the wave has its own radius of curvature and any transverse disturbance on its surface will grow or diminish according to surface tension principles. The segmentation of the tip of the wave spreads in the vicinity of the crest. The long wavelengths of the enhanced transverse disturbances are further amplified, as the radius of curvature at the back of the wave is longer than at the tip. These disturbances appear as ridges, or channels of easier flow according to Worthington's (1908) terminology, spaced at wider intervals as their distance from the tip of the breaker becomes greater.

A further assumption built in the model is that the two theories used, namely surface tension and gravity, do not affect each other. That is, the radius of the cylinder is not modified by the growth of the disturbance. This assumption affects mostly the short disturbances which rapidly form thin jets, in which case the horizontal cylinder loses its physical meaning.

Conclusions

Surface tension has been shown to play an important role in the overturning stage of a breaking wave. During this stage, any unstable disturbances on the tip of the breaker will grow as the curling face follows a free fall trajectory. The effects of the two main forces, namely gravity and surface tension have been combined together to produce an analytical/ numerical model that describes the evolution of the overturning face of a breaking wave.

The initial conditions of the overturning face were related to the breaker height and period, after making certain approximations. The most important result derived from the model is that breaking characteristics are independent of surface tension effects, for breakers larger than 0.5m high, a result which is in agreement with earlier experimental results.

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