

CHAPTER 86

BOTTOM PRESSURES DUE TO LONG WAVES: LABORATORY AND FIELD MEASUREMENTS

Fredric Raichlen¹, Jerald D. Ramsden², and James R. Walker³

1. INTRODUCTION

A convenient and attractive way of determining the nearshore wave environment is through the use of bottom (or near bottom) pressure measurements. However, it has been realized for some time that care must be taken in interpreting such pressure measurements and relating them to the local waves amplitudes in the region where the waves become highly nonlinear and shoal near the shore, e.g., see Grace (1978), Guza and Thornton (1980), Bishop and Donelan (1987), Lee and Wang (1984), Bodge and Dean (1984), and Nielsen (1989) among others. This study is directed to questions raised in connection with the interpretation of the water surface time-history from bottom pressure measurements for highly nonlinear long waves. In particular the waves which were primarily of interest were large solitary and cnoidal waves, i.e., non-periodic and periodic long waves.

The motivation for this work was the requirement to document the water surface-time histories of near breaking waves, nearly solitary in shape, with heights up to 2 m in a large fresh water wave basin. Wave staffs, either capacitance or resistance types, were not readily available for these measurements. In place of this technique, video recordings were made of the water surface at the location of surveyor's staff gages, but data reduction of these video recordings, except for a few cases, was estimated to be excessively time consuming. Hence, pressure transducers mounted near the bottom were installed at the wave staff locations to obtain simultaneous measurements which could be related to the variation of the water surface elevation with time.

To assist in defining the corrections to be used to convert the pressure measurements obtained in the large wave basin to wave heights, an experimental study in the laboratory and an analytical study were undertaken using solitary and cnoidal waves. For solitary waves pressure correction factors based on linear wave theory were not adequate to determine the wave amplitudes from the pressure measurements. Therefore, the correction factors were obtained first from theories

¹ Prof. of Civil Engr., W.M.Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology, Pasadena, CA 91125, USA

² Graduate Student, W.M.Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology, Pasadena, CA 91125, USA

³ Vice President, Moffatt & Nichol, Engineers, P.O.Box 7707, Long Beach, CA 90807, USA

accurate to the second and third approximations as presented by Grimshaw (1971) and given in slightly different form by Fenton (1972). The pressure correction factors for nonlinear periodic long waves were obtained using stream function theory as presented by Dean (1965,1974), and higher order cnoidal wave theories presented by Isobe, Nishimura, and Horikawa (1978) and summarized by Horikawa (1988). The approach of Dean(1974) also was extended for use with solitary waves. This paper will compare the nonlinear theories mentioned for solitary and cnoidal waves to the results of laboratory experiments in addition to the limited field experiments conducted in a large fresh-water wave facility. Some discussion will be given herein regarding the use of pressure transducers for dynamic pressure measurements; this will be presented in Section 3.

As a convenience to the reader, expressions for the solitary wave from Fenton (1972) which are accurate through the third order are presented in this section; these will be compared with experimental results. The wave train is stationary in the x,y plane with the origin of the coordinates at a point on the bottom; x is measured in the direction of wave propagation. The amplitude of the solitary wave can be expressed as:

$$\eta = 1 + \epsilon s^2 - \frac{3}{4} \epsilon^2 s^2 t^2 + \epsilon^3 \left(\frac{5}{8} s^2 t^2 - \frac{101}{80} s^4 t^2 \right) + O(\epsilon^4) \quad (1)$$

where η is the wave amplitude normalized with the depth, h , ϵ is the ratio of wave height to depth, H/h , $s = \text{sech } \alpha x$, $t = \tanh \alpha x$, and:

$$\alpha = \left(\frac{3}{4} \epsilon \right)^{1/2} \left(1 - \frac{5}{8} \epsilon + \frac{71}{128} \epsilon^2 \right) + O(\epsilon^{7/2}) \quad (2)$$

The pressure measured on the bottom can be written as:

$$\frac{p}{\gamma h} = 1 + \epsilon s^2 + \epsilon^2 \left(\frac{3}{4} s^2 - \frac{3}{2} s^4 \right) + \epsilon^3 \left(-\frac{1}{2} s^2 - \frac{19}{20} s^4 + \frac{11}{5} s^6 \right) + O(\epsilon^4) \quad (3)$$

where p is the total bottom pressure and γ is the specific weight of the fluid. The ratio of the wave height to the dynamic pressure head under the wave, accurate to second order, can be obtained from Eq 3 as:

$$\frac{H}{p_d / \gamma} = \frac{1}{1 - \frac{3}{4} \epsilon} \quad (4)$$

and to third order also from Eq 3 as:

$$\frac{H}{p_d / \gamma} = \frac{1}{1 - \frac{3}{4} \epsilon + \frac{3}{4} \epsilon^2} \quad (5)$$

For convenience, in the following, the subscript used in the dynamic pressure, p_d , will be dropped and the dynamic pressure will be denoted simply as: p . Experimental data obtained in a wave tank will be compared to Eqs. 4 and 5.

For the long periodic waves, cnoidal wave theory and stream function theory were used to describe the water surface-time history and the variation with time of the dynamic pressure. Due to the complexity of the resulting expressions, they will not be presented here. In lieu of that, the interested reader is referred to Horikawa (1988) and Dean (1974) for a presentation of the cnoidal wave theory and the stream function theory, respectively.

2. EXPERIMENTAL EQUIPMENT AND PROCEDURES

The laboratory experiments were conducted in a wave tank which is 15.24 m long, 0.38 m wide, and 0.61 m deep, with glass walls throughout and a programmable bulkhead wave generator located at one end. The trajectory of the wave generator was determined from the method described by Goring and Raichlen (1980). In this approach, used for long waves, the position of the vertical bulkhead with time is based on the condition that the velocity of the plate must be equal to the depthwise average of the water particle velocity under the wave at that instant. Thus, the plate trajectory includes nonlinear aspects of wave generation. Voltage-time histories were constructed with this approach and used in the electro-hydraulic servo system of the wave generator for both solitary and cnoidal waves. Resistance wave gages were used to determine the water surface-time history of the wave.

Pressures were measured in the laboratory using a variable reluctance pressure transducer (Pace Model P7D) with a pressure sensing diaphragm which was 0.0267 cm thick and 2.54 cm in diameter. The transducer pressure port was connected directly to the bottom of the wave tank by a 5.28 cm long and 0.46 cm inside diameter brass tube. In unsteady pressure measurements care must be taken to obtain as high a natural frequency of the instrument as is possible relative to the frequency content of the pressure signal while still maintaining adequate sensitivity to insure accuracy. Since air entrainment in a water filled pressure transducer can significantly lower its natural frequency, the pressure gage was filled carefully with water by flushing it for a substantial period of time. An analysis presented by Ippen and Raichlen (1957) showed that the internal dimensions of the tube which connected the transducer to the pressure source could have an important effect on the natural frequency of the gage. By careful attention to this feature of the gage-wave tank connection and to air entrainment, a measured natural frequency of the transducer of about 70 hz was attained.

As implied above, it is not the natural frequency of the gage alone which is important, but the magnitude of the natural frequency of the gage **relative** to the maximum frequency of the important portion of the pressure signal, (or the incident wave). It can be shown that the amplitude spectrum of a solitary wave when analyzed as a linear signal is:

$$\frac{S(\omega)}{S(0)} = \frac{\xi\omega}{\sinh \xi\omega} \quad (6)$$

where:

$$\xi = \frac{\pi}{\sqrt{3}} \sqrt{\frac{h}{g}} \frac{1}{\sqrt{\frac{H}{h} \left(1 + \frac{H}{h}\right)}}$$

and $S(\omega)$ is the amplitude of the frequency component ω , H is the wave height, and h is the water depth, see, e.g., Synolakis (1986). For the wave heights and water depths used in the laboratory experiments, the frequency at which $S(\omega)$ has decreased to about 1% of its value at zero frequency, i.e., $S(\omega)/S(0) = 0.01$, is about 4 hz. Thus, the natural frequency of the gage was significantly greater than the important frequencies in the wave. This resulted in relatively undistorted measurements.

Measurements of water waves were to be made in the field in a large fresh

water wave basin which consists of a wave channel 130 feet wide and 180 feet long with depths varying from 8 feet to approximately 5 feet, opening into a larger basin where the depth decreased uniformly to zero. The latter has a width of about 500 feet and a length in the direction of wave propagation of about 250 feet. The depth variations in the wave channel were as follows: a constant 8 ft depth extending 15 ft from the wave generator, followed by a 40 ft section with a 4.3% slope where the depth decreases from 8 ft to 6.28 ft, followed by 72 ft of constant depth, with the remaining 53 ft of the wave channel having a bottom slope of 2.2% reaching a final depth of 5.11 ft. Waves are generated at one end of the 130 foot wide channel by releasing a fixed volume of water into the channel during a short time interval.

Wave measurements were conducted at three locations in the wave channel; the location which is of interest in this study is in the constant depth region, 6.28 ft deep, about 90 ft from the wave generator. Thus, the position for which the wave measurements will be discussed here was about 35 ft, i.e., nearly six depths, from the beginning of the constant depth section. A pressure transducer was attached to the tripod at a distance from the bottom equal to 0.317 times the depth ($y/h = 0.317$). The pressure gage (Senso-Metric semiconductor pressure transducer Model SP91) had a natural frequency in air of approximately 30 KHz. For a near-breaking solitary wave in this region of the wave channel, 99% of the energy would be at frequencies less than approximately 2 Hz. Although the natural frequency of the transducer in water would be less than in air, little dynamic distortion would be expected using this gage. Simultaneous with the pressure measurements, video recordings were made of the wave as it passed the wave staff. In addition, visual observations were taken of the height of the first major wave as it propagated past the wave staff. In Section 3 theoretical results will be compared to the pressure measurements using the visual observations of the height of the first wave as it propagated past the pressure transducer.

3. RESULTS AND DISCUSSION OF RESULTS

Several theories for long waves will be compared to experimental results obtained in the laboratory and in the field. The variation of both the pressures and the amplitude-time histories of these waves will be presented.

It is useful, first, to look in a general way at the influence of non-linearities and vertical accelerations, i.e., frequency dispersion, on the pressures under solitary waves. (It should be remembered that if linear wave theory were applied to long waves such as solitary waves, the pressures under the waves would be distributed hydrostatically.) Evidence of these effects is shown in a qualitative fashion in Fig. 1. This three-dimensional plot was obtained from Eq. 3 and shows the variation of the wave amplitude normalized by the dynamic pressure head on the bottom, as a function of relative wave height and of relative distance measured from the crest for a solitary wave. The abscissal range of relative wave height is: $0 < H/h < 0.65$. In the ordinate, for convenience, only the value: $\eta/(p/\gamma) = 1$ is indicated. This corresponds to hydrostatic pressure conditions and it is seen to apply only for small relative wave heights independent of location, x/h . However, as the relative wave height increases, the effects of nonlinearities and vertical accelerations become apparent, and linear theory, i.e., a hydrostatic pressure distribution under the wave, is not applicable. For example, at $H/h = 0.65$, under the crest, the dynamic pressure head is less than the wave amplitude. Conversely, at $x/h \approx 2$, the dynamic pressure head is greater than the amplitude. If care were not taken in interpreting the pressure measurements properly for such large waves, the wave height would be significantly under-

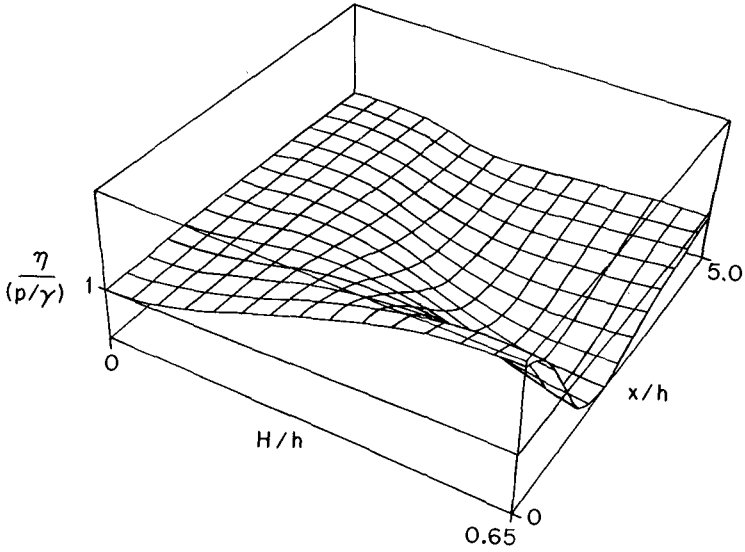


Fig. 1 The Variation of Wave Amplitude Normalized by Dynamic Bottom Pressure with Relative Wave Height and Distance for a Solitary Wave

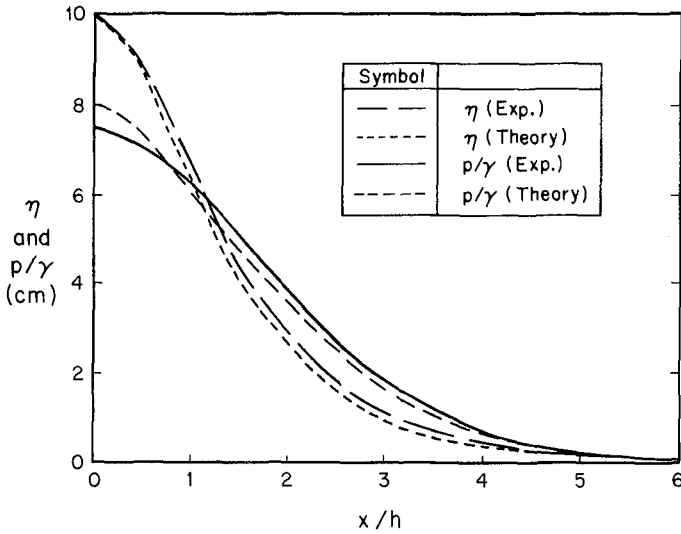


Fig. 2 Variation of Amplitude and Dynamic Bottom Pressure With Relative Distance for a Solitary Wave: $H/h = 0.54$

estimated and the wave shape distorted.

This variation is more clearly shown in Fig. 2 where the results of experiments conducted in the laboratory are presented. The ordinate corresponds to both the wave amplitude and the dynamic pressure head, each expressed in centimeters, and the abscissa is the distance from the crest normalized with respect to the depth. The depth for these experiments was 18.6 cm and the relative wave height was: $H/h = 0.538$. Theoretical pressures and amplitudes as obtained from Fenton (1972), Eqs. 1 and 3, are shown along with the experiments. It is seen that the dynamic pressure under the crest is approximately 75% of the amplitude. Therefore, if simple linear theory were used the wave height obtained from pressure measurements would be underestimated by approximately 25%. As distance from the crest increases, the correction factor becomes equal to unity at approximately $x/h = 1.5$, where the dynamic pressure is equal to the amplitude. For $x/h > 1.5$, because of the vertical water particle accelerations, the dynamic pressure becomes larger than hydrostatic.

The variation of the ratio of the wave height to the dynamic pressure head on the bottom with relative wave height for a solitary wave is shown in Fig. 3. Data are presented from two separate experiments. In Experiment A the pressure transducer used was as obtained from the manufacturer, and in Experiment C the internal pressure ports in the gage were enlarged as much as possible, limited only by the gage dimensions. It appears that the internal gage modifications did not affect the pressure measurements; thus, any change of the frequency response of the gage accomplished by the internal modifications were not important for these waves. No doubt one important reason for this is that for these solitary waves, as mentioned earlier, 99% of the energy was at frequencies less than approximately 4 hz whereas the natural frequency of the gage was about 70 hz.

Three analytical curves are shown in Fig. 3, corresponding to second and third order solitary wave theories and to the stream function theory. The analytical curves obtained from Eqs. 4 and 5 are shown as dotted and dashed, respectively. The solid curve which is labeled stream function theory was obtained from a numerical algorithm based on the development by Dean (1965) and from computations at higher orders than the results presented by Dean (1974). To use this method for solitary waves, first the variation of the ratio of wave height to dynamic pressure head with depth-to-wave length, h/L , was defined for long periodic waves. The curves corresponding to constant H/h so obtained were extrapolated to $h/L = 0$ to define a stream function result which might be applicable to solitary waves. (This will be discussed later in detail.) It is seen in Fig. 3 that the stream function theory agrees well with the experimental results for the full range of relative wave heights which were investigated.

These data have been replotted in Fig. 4 as the ratio of the dynamic pressure head on the bottom to the depth as a function of relative wave height. Experimental data shown in Fig. 4 compare well with the stream function theory and poorly with the linear theory except for $H/h < 0.1$. In essence Fig. 4 emphasizes problems which can occur when interpreting pressure gage measurements under long waves. For example, consider the case of a relatively large wave where the pressure has been measured and the wave height is to be determined. Fig. 4 shows that for solitary waves with $H/h > 0.7$ the dynamic pressure is essentially constant. This means that although the wave height is increasing, the vertical accelerations also change in such a way that the pressure remains relatively unchanged. Thus, for this range of relative wave heights it is difficult, if not impossible, to actually define the solitary wave

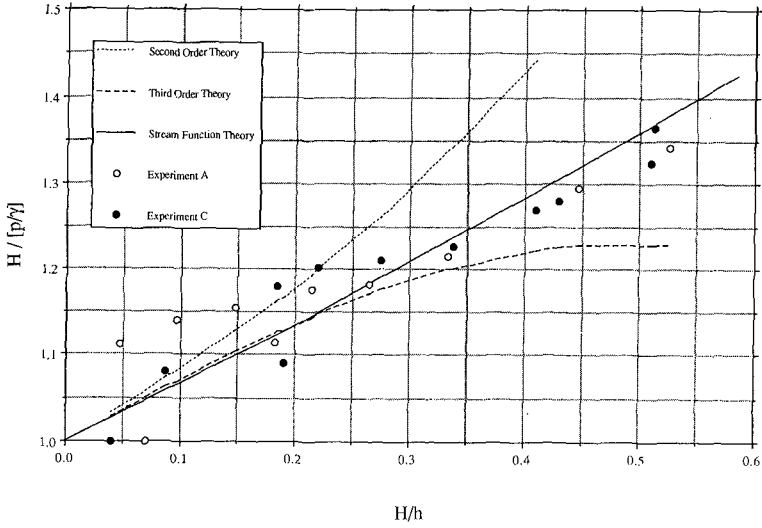


Fig. 3 Comparison of Theories to Experiments for Dynamic Bottom Pressures Under the Crest of a Solitary Wave: $H/(p/\gamma)$ vs. H/h

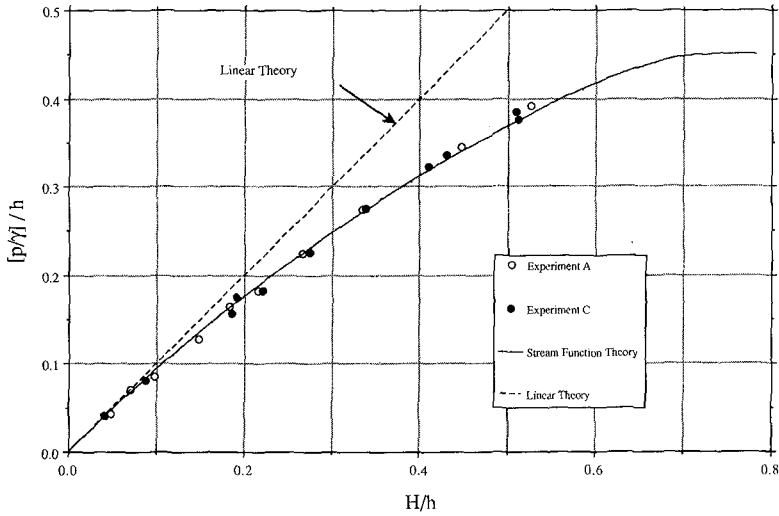


Fig. 4 Comparison of Theories to Experiments for Dynamic Bottom Pressures Under the Crest of a Solitary Wave: $(p/\gamma)/h$ vs. H/h

height from bottom pressure measurements. Unfortunately, at times, it is for just these large waves that one would like to obtain a good estimate of wave height. Thus, for such large long waves, bottom or near bottom pressure measurements may be inappropriate for accurate water surface time histories.

The pressures under large periodic long waves were also investigated in the laboratory and certain of these results are presented in Figs. 5a and 5b. The ordinate in Fig. 5a is the ratio of the wave amplitude to the depth. In Fig. 5b the ordinate is the dynamic pressure measured on the bottom normalized with respect to depth. For both Figs. 5a and 5b the abscissa is the relative distance in the direction of wave propagation. The depth for these experiments was 20.01 cm, the wave period was 1.98 seconds, $H/h = 0.565$, and the non-dimensional period parameter was: $gHT^2/h^2 = 108.6$. The cnoidal wave theory presented in Figs. 5a and 5b is a third-order long wave solution from Horikawa (1988). Included also is the result from the stream function theory carried out to the 21st order. It is seen that the two theories for the wave shape agree reasonably well with the experiments with some divergence in the trough region. The pressures measured are generally less than those predicted by the two theories although the stream function theory agrees better with the experiments near the crest than does the higher order cnoidal wave theory. This result is similar to that observed in the solitary wave phase of the study.

The theoretical and experimental variation of the wave height divided by the total dynamic pressure for periodic waves is shown in Fig. 6 as a function of depth to wave length for constant values of wave height-to-depth. The total pressure is defined in the ordinate as the difference between the pressure measured under the crest and that measured under the trough. The theoretical curves are determined from stream function theory computed to a high enough order so there is little variation in the ordinate value when the order of the computations is increased. For these computations the maximum order was 51 for the longer waves. (It is these curves, for given relative wave heights, which were extrapolated to $h/L = 0$ to define the stream function relationship for solitary waves presented in Fig. 3; the extrapolated curves are shown dashed.) The results from the linear theory, also presented in Fig. 6, appear to converge with the stream function theory for smaller wave height-to-depth ratios only as the depth-to-wave length becomes large. Included in Fig. 6 are the results of four experiments conducted with long cnoidal waves with the relative wave height for each experiment presented in brackets next to the data point. It is seen that the theory agrees reasonably with the experimental data.

The amplitude at the crest and the amplitude at the trough divided by the corresponding pressure head are plotted in Figs. 7 and 8, respectively, as a function of depth-to-wave length and relative wave height. The agreement of the theory with the experimental data appear to be reasonable with respect to the crest amplitude shown in Fig. 7, but appears to be less so for the trough measurements in Fig. 8. The primary reason for the latter is probably experimental error due to the relatively small pressures in the trough region. In both Figs. 7 and 8, the linear theory is shown for comparison, and it is seen, as expected, that for long waves the divergence from the linear theory increases significantly as the relative wave height increases. For example, for $h/L = 0.05$, the so-called limit of shallow water waves, the amplitude at the crest relative to the dynamic pressure head on the bottom for a wave with a relative wave height of: $H/h = 0.6$ predicted by the stream function theory is approximately 1.6 compared to a value of about unity for linear theory. This means that the pressure head under the crest is only about 60% of the amplitude. In Fig. 8 similar data are shown for the trough region where it is seen that, as the relative wave height increases for a given depth to wave length, the wave becomes

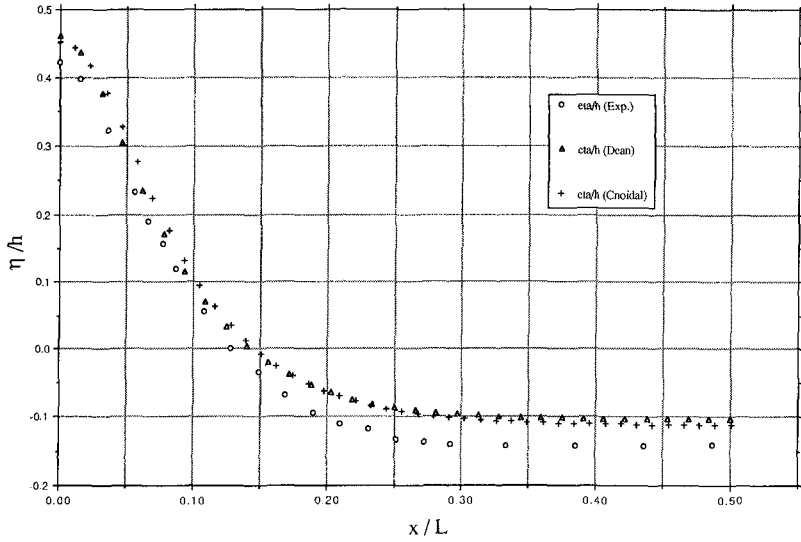


Fig. 5a Comparison of Theories to Experiments for Relative Amplitude of a Cnoidal Wave: $H/h = 0.57$, $gHT^2/h^2 = 108.6$

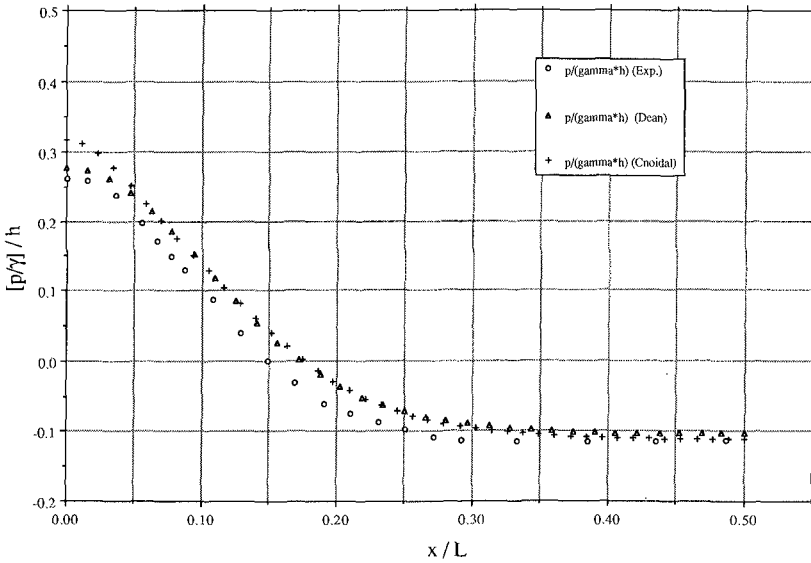


Fig. 5b Comparison of Theories to Experiments for Dynamic Bottom Pressures Under a Cnoidal Wave: $H/h = 0.57$, $gHT^2/h^2 = 108.6$

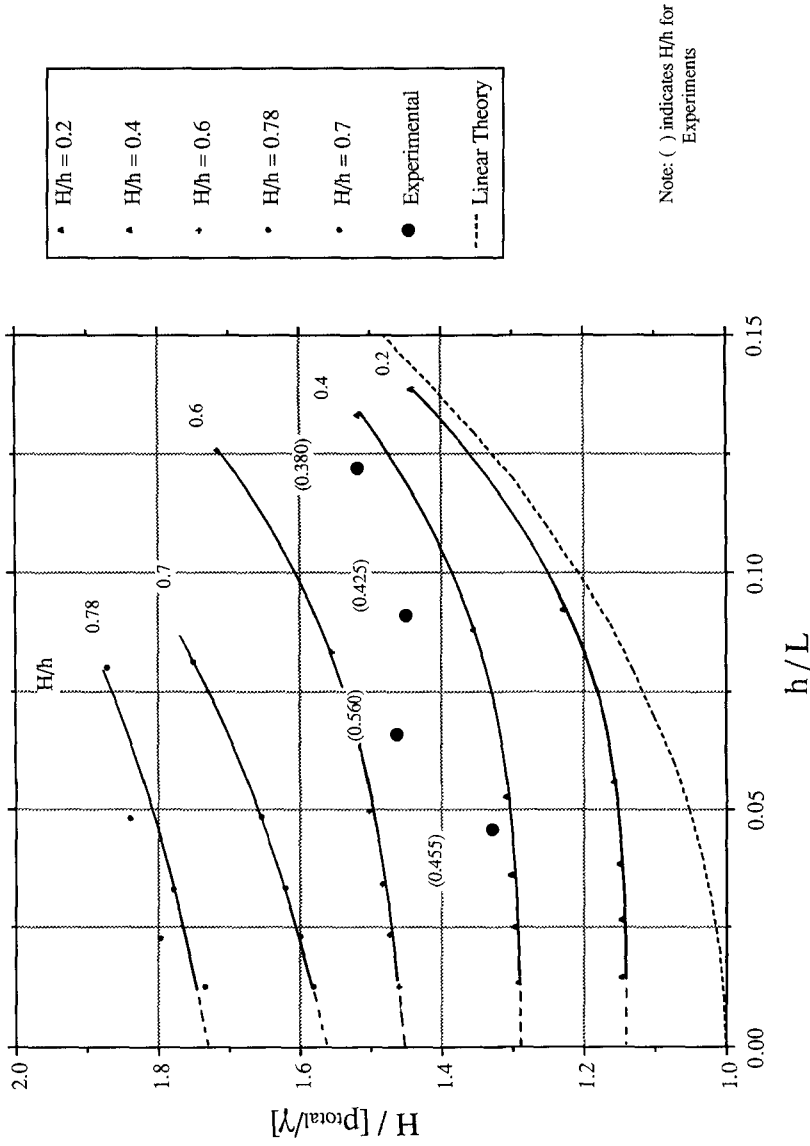


Fig. 6 The Ratio of Wave Height to Dynamic Bottom Pressure as a Function of Relative Depth and Height from Stream Function Theory

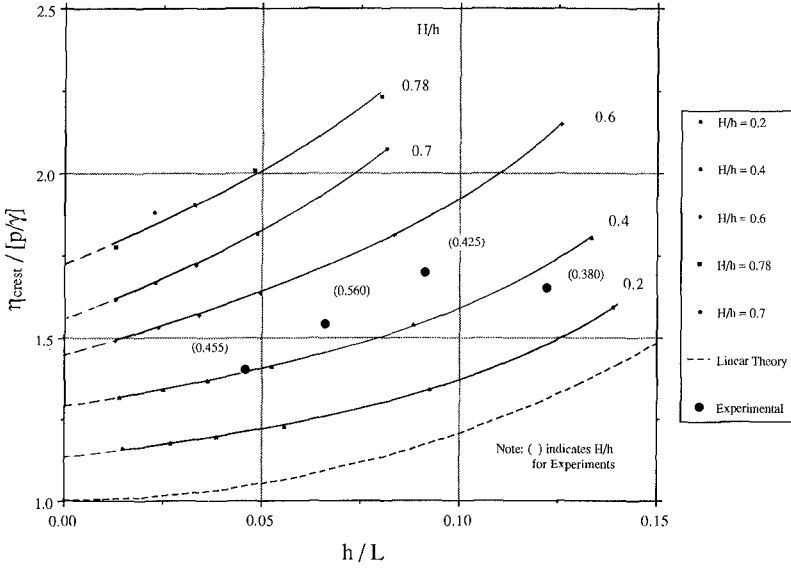


Fig. 7 The Ratio of Crest Amplitude to Dynamic Bottom Pressure as a Function of Relative Depth and Height from Stream Function Theory

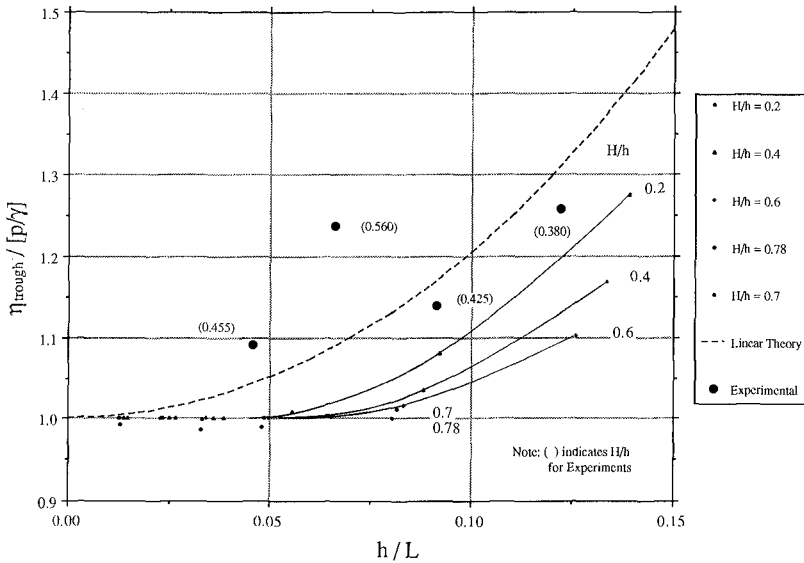


Fig. 8 The Ratio of Trough Amplitude to Dynamic Bottom Pressure as a Function of Relative Depth and Height from Stream Function Theory

more solitary-like so that in the trough region the influence of vertical accelerations becomes less. Thus, contrary to the crest region, the correction factor for a large long wave is nearly unity in the trough region whereas linear theory indicates much larger factors due to the sinusoidal shape of the latter.

To some extent, these data can be summarized as shown in Fig. 9 where the ordinate is the dynamic pressure on the bottom normalized by depth and the abscissa is the wave height-to-depth. Theoretical results are shown for the solitary wave and for two long periodic waves with a relative depth to wave length of: $h/L = 0.05$ and 0.1 . Included also are curves labeled: "Linear (Solitary Wave)" and "Linear ($h/L = 0.1$)". These curves correspond to linear wave theory applied to a solitary wave and to a periodic wave ($h/L = 0.1$). The divergence of the nonlinear theories and the linear theories begins at about $H/h \approx 0.1$ for both the solitary and the periodic waves and increases thereafter. This indicates that care must be taken in applying linear theory to determine the wave height from pressure measurements for long periodic and non-periodic waves. As before, for large waves, the pressure becomes relatively insensitive to increasing wave heights. This again suggests the problem discussed earlier with regard to interpreting wave heights from pressure measurements for H/h greater than 0.6 to 0.7 . It is interesting to note the similarity between the results for periodic waves with $h/L < 0.1$ and solitary waves.

Pressure measurements were made in the field in the large fresh water wave facility described in Section 2 at a distance from the bottom of $y/h = 0.317$. An example of pressure measurements for six different tests with an observed relative wave height of: $H/h = 0.79$ are shown in Fig. 10. The ordinate is the dynamic pressure in feet and the abscissa is the number of data points obtained when sampling at a rate of 16 samples per second. The reasonable agreement among the six runs demonstrates good wave generation reproducibility considering the scale of the waves being generated and the type of wave generator used. It should be recalled from Section 2 that these waves have propagated up a slope of about 4.3% before reaching the horizontal section where the pressure measurements were made. A distinctive solitary wave-like shape of the leading part of this wave is apparent as well as the shelf-like trailing part of the first wave. The latter has been found by others for solitary waves propagating up a slope in the laboratory, e.g., Skjelbreia (1987).

In Fig. 11 data are presented from field experiments conducted in the large wave channel on two different days. The ordinate is the wave height normalized by the dynamic pressure head and the abscissa is the relative wave height. (As mentioned earlier, the wave height which is used corresponds to the height which was observed on the wave staff as the wave propagated past the location of the pressure gage.) Included in this figure is a theoretical curve obtained from stream function theory for $y/h = 0.317$ and $h/L = 0$. Considering the shape of the waves as described by the pressure-time histories presented in Fig. 10, the agreement of the theory and the data are surprisingly good. Relative wave heights corresponding to $H/h > 0.8$ are realized due to the 4.3% slope which precedes the horizontal section where the measurements are made and the short distance from the beginning of that section to the location of the pressure transducer. Thus, to some extent, it appears that the leading portion of the wave can be described by a solitary wave reasonably well.

When the dynamic pressure head normalized by the depth is plotted as a function of relative wave height, as shown in Fig. 12, the variation is similar to that seen in Fig. 4. The field data for large relative wave heights appear to reach an asymptotic value of normalized pressure between 0.45 and 0.5 . These data show, as

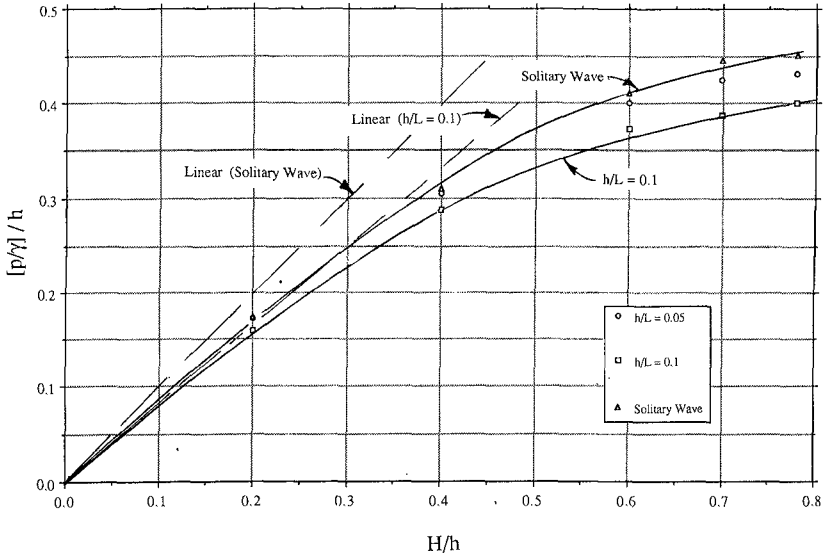


Fig. 9 Variation of Normalized Dynamic Bottom Pressure Under the Crest with Relative Wave Height: Stream Function and Linear Theories

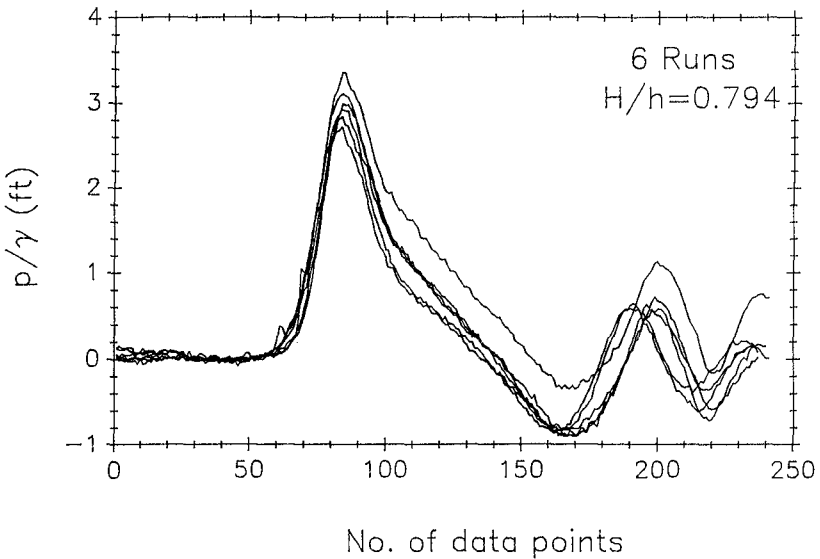


Fig. 10 Field Measurements of Dynamic Bottom Pressure with Time (Data Rate = 16 Samples per sec)

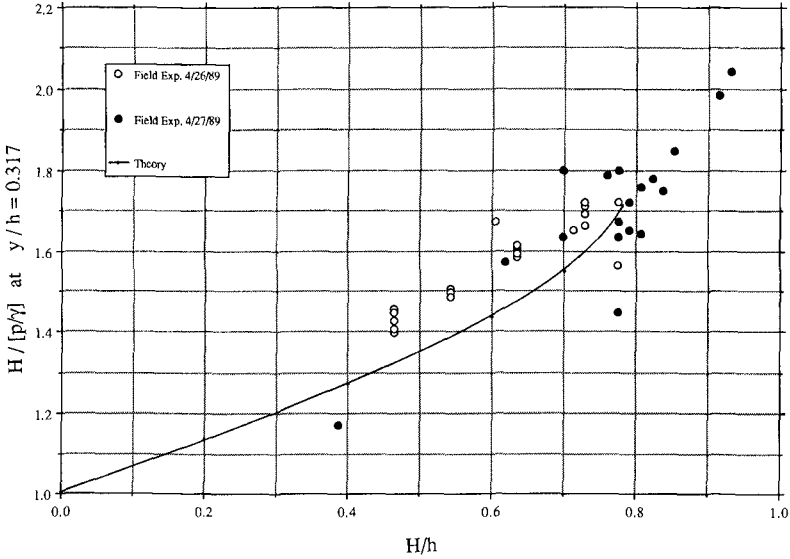


Fig. 11 Comparison of Stream Function Theory as Extrapolated for Solitary Waves to Field Experiments: $H/(p/\gamma)$ vs H/h at $y/h = 0.317$

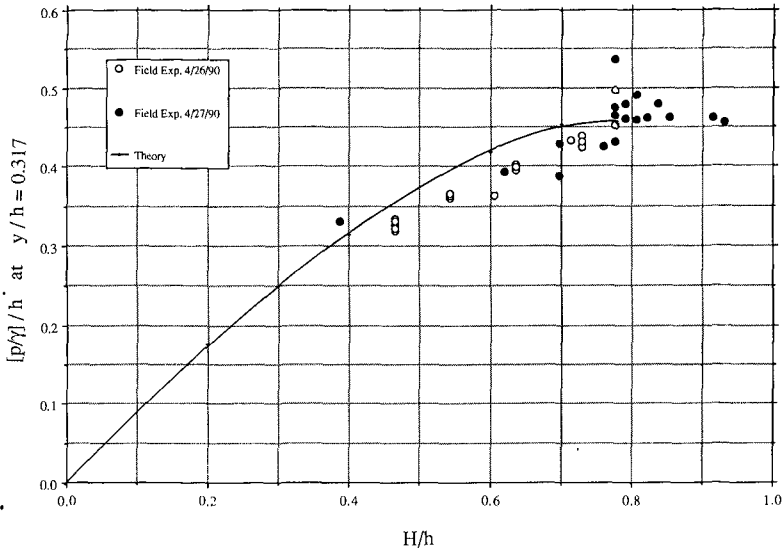


Fig. 12 Comparison of Stream Function Theory as Extrapolated for Solitary Waves to Field Experiments: $(p/\gamma)/h$ vs H/h at $y/h = 0.317$

before for the laboratory results, that for large, long, nonperiodic waves it is inaccurate to define the wave height from the pressure measurements alone.

4. CONCLUSIONS

The following major conclusions can be drawn from this study:

1. For solitary and cnoidal waves, nonlinear effects and vertical accelerations (frequency dispersion) are important in defining the pressures under the waves.
2. For $H/h > 0.1$ linear theory should not be used to evaluate the wave height from pressure measurements for long waves.
3. The extrapolated stream function theory appears to agree best with the experimental data for solitary waves. Solitary wave theories corresponding to the second and third orders in the parameter H/h agree reasonably well with the data only for small relative wave heights. For such non-periodic waves it is inaccurate to define the wave height from pressure measurements alone.
4. For relatively large solitary waves and cnoidal waves the ratio of the dynamic pressure under the wave crest, on or near the bottom, to the depth appears to approach a constant value of 0.45, or somewhat less.

5. REFERENCES

- Bishop, C.T. and Donelan, M.A., "Measuring Waves with Pressure Transducers", Coastal Engr., Vol 11, 1987.
- Bodge, K.R. and Dean, R.G., "Wave Measurement with Differential Pressure gages", Proc. 19th Coastal Engr. Conf., 1984, pp 755-769
- Dean, R.G., "Stream Function Representation of Nonlinear Ocean Waves", Journ. Geoph. Res., vol 70, No 18, pp 4561-4572, Sept. 1965
- Dean, R.G., "Evaluation and Development of Water Wave Theories for Engineering Application", Spec. Report No. 1, USACE, CERC, Nov. 1974
- Fenton, J., "A Ninth Order Solution for the Solitary Wave", Jour. Fluid Mech., Vol. 53, 1972
- Goring, D.G. and Raichlen, F., "The Generation of Long Waves in the Laboratory", Proc. 17th Coastal Engr. Conf., 1980
- Grace, R.A., "Surface Wave Heights from Pressure Records", Coastal Engr., Vol. 2, 1978, pp 55-67
- Grimshaw, R., "The Solitary Wave in Water of Variable Depth", Journ. Fluid Mech., Vol. 46, Part 2, 1971
- Guza, R.T. and Thornton, E.B., "Local and Shoaled Comparisons of Sea Surface Elevations, Pressures, and Velocities", Journ. Geoph. Res., vol 85, C3, March 20, 1980, pp 1524-1530
- Horikawa, K., "Nearshore Dynamics and Coastal Processes", Univ. of Tokyo Press, 1988
- Ippen, A.T. and Raichlen, F., "Turbulence in Civil Engineering: Measurements in Free Surface Streams", Jour. of Hyd. Div., ASCE, Vol. 83, No. HY 5, Oct. 1957

- Isobe, M., Nishimura, H., and Horikawa, K., "Expressions of Perturbation Solutions for Conservative Waves by Using Wave Height", Proc. 33rd Annl. Conf. of JSCE, II, pp 39-43 (in Japanese), 1978
- Lee, D.-H. and Wang, H., "Measurement of Surface Waves from Subsurface gage", Proc. 19th Coastal Engr. Conf., 1984
- Nielsen, P., "Analysis of Natural Waves by Local Approximations", Journ. of Waterways, Port, Coastal, and Ocean Engineering, ASCE, Vol. 115, No. 3 May 1989
- Skjelbreia, J.E., "Observations of Breaking Waves on Sloping Bottoms by Use of Laser Doppler Velocimetry", W.M.Keck Lab. Report KH-R-48, Calif. Inst. of Tech., Pasadena, CA, 1987
- Synolakis, C., "The Run-up of Long Waves", PhD Thesis, Calif. Instit. of Tech., Pasadena, CA, USA, 1986