EFFECTS OF WAVE GROUPS ON THE STABILITY OF RUBBLE MOUND BREAKWATERS

by

Josep R. Medina[1], Claudio Fassardi[2], and Robert T. Hudspeth[3]

ABSTRACT

The stability of the armor layer of rubble mound breakwaters has been shown to be highly influenced by some wave grouping characteristics. The measured damage functions are dependent on the Envelope Exceedance Coefficient, α, and also on the Groupiness Factor defined as: $GF=\sigma[H^2(t)]/8m_0$.

INTRODUCTION

Wave groups have been identified as playing an important role in a variety of coastal problems. Medina and Hudspeth (1990) reviewed most of the parameters and methodologies commonly used, as well as the engineering problems associated with wave groupiness. Although the effects of wave groups on the stability of the armor layer of mound breakwaters have been considered by many, a rational method for incorporating wave groups into the design of mound breakwaters has yet to be developed.

Current design methods for armor layers follow the design methodology proposed by the Shore Protection Manual (SPM, 1984) which relates the design of the armor layer to a single representative wave height, H_{10}, which corresponds to a design sea state. The SPM methodology does not incorporate the effects of

---

[1]Prof. Titular, Dep. of Transportation, Univ. Politécnica de Valencia, Camino de Vera s/n, Valencia 46071, SPAIN.
[2]Ass. Res., Dep. of Civil Engineering, Oregon State University, Corvallis, OR. 97331, U.S.A.
[3]Professor, Dep. of Civil Engineering, Oregon State University, Corvallis, OR. 97331, U.S.A.

storm duration, wave period or wave groupiness from the design sea state.

Bruun (1985) reviewed the design of mound breakwaters and emphasized that the stability of these structures is sensitive to wave groups; and, therefore, wave grouping characteristics should be incorporated into their design. On the contrary, Van der Meer (1988) proposed new formulae which considered the influence of duration and wave periods; however, he concluded that wave grouping characteristics and spectral shape have only a minor influence on the stability. Recently, Medina and McDougal (1990) reanalyzed the experimental data from Van der Meer and Pilarczyk (1988); and found that rubble mound breakwaters were more stable against sea states with long wave groups, which appears to be contrary to intuition.

As noted above, there is an apparent contradiction regarding the effects of wave grouping on the stability of rubble mound breakwaters. In order to evaluate the influence of wave grouping on the stability of rubble mound breakwaters and revetments, a series of large scale experiments were conducted at the O.H. Hindsdale Wave Research Facility (OHH-WRF) at Oregon State University. The main goal of these experiments was to resolve the controversy regarding the dependence of damage on the spectral shape and/or the characteristics of wave groups.

ENVELOPE AND WAVE HEIGHT FUNCTIONS

Rye (1982) reviewed the methods for analyzing wave groups and concluded that wave groups measured from field data compared quite well with those obtained from numerical simulations using linear algorithms. The validity of the linear hypothesis was also obtained by Goda (1983) and by Battjes and Vledder (1984) for wave groups in non-shallow waters.

In his classic treatise on random noise, Rice (1954) developed an extensive theory that may also be applied to linear surface gravity waves where the envelope theory appears to be appropriate for analyzing wave groups. Medina and Hudspeth (1987) and Hudspeth and Medina (1988) used the envelope theory of Rice and focused their attention on only some of the more important aspects for analyzing wave groups. The envelope theory, originally applied to 1-D wave analyses, may also be extended rather easily to 2-D and 3-D waves and to wave groups.

Assuming that the sea surface elevation at a point is an ergodic Gaussian stochastic process having a variance spectrum $S_\eta(f)$, a realization may be approximated by

$$\eta(t) = \sum_{m=1}^{M} R_m \cos(2\pi f_m t + \theta_m) \tag{1}$$

where M = the total number of wave components in the realization; R_m, f_m, and

θ_m = the amplitude, the frequency, and a random phase angle, respectively, of the m^{th} wave component. The random phase angle is uniformly distributed in the interval U[0,2π]. The Hilbert transform, $\hat{\eta}(t)$, of $\eta(t)$ is given by

$$\hat{\eta}(t) = \sum_{m=1}^{M} R_m \sin(2\pi f_m t + \theta_m) \tag{2}$$

and the analytical function, AF(t), by

$$AF(t) = \eta(t) + j\hat{\eta}(t) = A(t)\exp(j[\theta(t) + \phi]) \tag{3}$$

where $j = \sqrt{-1}$; A(t) = the envelope function; and $[\theta(t) + \phi]$ = the instantaneous phase angle defined by

$$A(t) = \sqrt{\eta^2(t) + \hat{\eta}^2(t)} \tag{4}$$

$$\theta(t) + \phi = \mathrm{ARCTAN}\left[\frac{\hat{\eta}(t)}{\eta(t)}\right] \tag{5}$$

In the complex plane, Hudspeth and Medina (1988) identified AF(t) as an orbital movement consisting of a vertical displacement of a point floating in the sea surface $\eta(t)$ and a horizontal displacement $\hat{\eta}(t)$. An instantaneous wave height, H(t) = 2A(t), and a local radian frequency, $\Omega(t) = d\theta(t)/dt$, were defined. The statistical properties of these two functions were evaluated and related to the characteristics of wave groups.

The envelope theory may be extended to 2-D experiments in wave flumes using space (x) and time (t) as the independent variables. The water surface elevation in a wave flume may be expressed as

$$\eta(x,t) = \sum_{m=1}^{M} R_m \cos[2\pi(\lambda_m x - f_m t) + \phi_m] \tag{6}$$

where ϕ_m is a random phase uniformly distributed in the interval U[0, 2π]; and λ_m is the inverse of the m^{th} wave length computed from by

$$f_m^2 = \left(\frac{g}{2\pi}\right)\lambda_m \tanh(2\pi\lambda_m h) \tag{7}$$

where h is the water depth, and g is the acceleration due to gravity. The Hilbert transform of $\eta(x,t)$ in the space and time domains may be defined as

$$\hat{\eta}(x,t) = \sum_{m=1}^{M} R_m \sin[2\pi(\lambda_m x - f_m t) + \phi_m] \tag{8}$$

where x and t are considered to be space and time parameters for time and space domain calculations, respectively. The envelope and wave height functions in the wave flume are given by

$$H(x,t) = 2\,A(x,t) = 2\sqrt{\eta^2(x,t)+\hat{\eta}^2(x,t)} \tag{9}$$

The variance spectrum in the space domain, $S_\eta(\lambda)$, may be related to the variance spectrum in the time domain, $S_\eta(f)$, according to the linear dispersion relationship and

$$S_\eta(f)\,df = S_\eta(\lambda)\,d\lambda \tag{10}$$

The properties of waves and envelopes in the time domain may be projected to the space domain to obtain

$$4\,S_A(\lambda) = S_H(\lambda) \approx (8-2\pi)m_0\Gamma_\eta(\lambda) \tag{11a}$$

$$S_{H^2}(\lambda) \approx 64 m_0^2 \Gamma_\eta(\lambda) \tag{11b}$$

$$\Gamma_\eta(\lambda) = \frac{2}{m_0^2}\int_0^\infty S_\eta(x+\lambda)S_\eta(x)dx \tag{12}$$

where $S_{H^2}(\lambda)$ and $S_H(\lambda)$ are the variance spectra of $H^2(x,t)$ and $H(x,t)$ in the space domain (t fixed); and $\Gamma_\eta(\lambda)$ is the corresponding spectral density function (unit variance). The envelope spectral density functions in space and time are not related by the linear dispersion relationship.

The spectral density functions for waves and envelopes in space and time may be used to estimate the mean wave velocity and mean group celerity for irregular waves. The flux of energy in a wave flume is approximately proportional to $C_g H^2(x,t)$, where C_g is the mean group celerity. Therefore, it is reasonable to test if $H^2(t)$ at the toe of a rubble mound breakwater affects stability.

Realizations of different discrete waves with the same flux of energy and local wave height and period characteristics may be expressed by

$$\eta_\psi(x,t) = \sum_{m=1}^{M} R_m \cos\left[2\pi(\lambda_m x - f_m t) + (\phi_m - \psi)\right];\ 0 \le \psi \le 2\pi \tag{13}$$

where ψ is a constant phase shift given to each component. Digital-to-analog simulations make it possible to simulate a desired wave train at the toe of the structure.

DESCRIPTION OF EXPERIMENTS

The effects of wave groupiness on the stability of the armor layer of rubble mound breakwaters do not appear to have been treated in a consistent maner in previous studies. The published results of these experiments do not always give a precise indication of the wave grouping characteristic that significantly affected the stability. Johnson et al.(1978) indicated that wave trains with long wave runs and high GF are more damaging. However, Burchart (1979) suggested that sea states with short wave runs are the most damaging. Finally, Van der Meer(1988) could not identify any significant differences between wave trains with high GF, long wave runs and narrow spectra and wave trains with low GF, short wave runs and broad spectra.

The apparent contradiction from these published experimental results may possibly be resolved by the wave grouping characterization model proposed by Mase and Iwagaki (1986). They considered two independent characteristics: a) a wave groupiness factor and b) a run length. Our experiments exploit the model of Mase and Iwagaki (1986) using linear wave theory. Our final results will demonstrate those wave grouping characteristics that significantly influence the stability of the armor layer of rubble mound breakwaters.

In order to analyze the influence of wave groups on the stability of the armor layer, two different design methodologies were considered as being representative of the state-of-the-art methods for design: 1) the S.P.M. method (1984); and 2) the highly elaborated Van der Meer method (1988). In both of these two methods, the stability of the armor layer is controlled mainly by

$$N_s = \left[\frac{\rho_r H_s^3}{W\left(\frac{\rho_r}{\rho_w} - 1\right)^3}\right]^{\frac{1}{3}} \tag{14}$$

where N_s is a stability number; H_s is the significant wave height; W is the median value of the mass distribution of rocks in the armor; and ρ_r and ρ_w are the mass densities of the rocks and water, respectively.

The following secondary factors also affect the stability (or damage function): slope, thickness of armor layer, permeability of the structure, number of waves in a run, surf similarity parameter or mean period, roughness and placement of the armor rocks, and wave groupiness. In our experiments, most of these secondary factors were held constant in order to compare the stability number and the wave grouping characteristics of wave runs with damage (or erosion of the armor layer). The 3.7 m. wide wave channel was divided into two equal halves in order to test simultaneously the same rubble mound breakwater profile but with two different armor layer rock sizes (viz., W_L=13 kg. and W_S=10 kg.). Consequently, each run provided two different surf similarity parameters in order to evaluate the influence of wave period on stability.

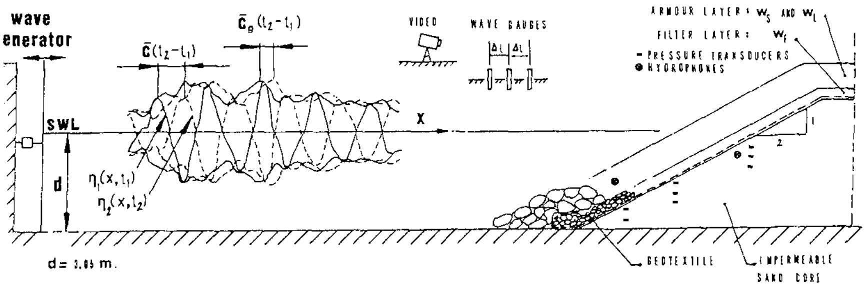

Fig. 1. Schematic of Wave Channel Used for Experiments.

Profiles of the armor layer were recorded after each run in order to measure damage (erosion). A surveying rod with an articulated foot (1 ft. diameter) was used to measure the armor layer profile at every foot. Two hydrophones and nine pressure transducers were placed inside the structure in order to detect rock movement and to measure pressures in the secondary layer.

Two fundamental wave grouping characteristics were considered: 1) the spectral shape related to the mean run length and 2) the energy flux exceedance pattern related to the groupiness factor. The spectral shape and mean run lengths were evaluated using random waves simulated from a Goda-JONSWAP spectra having the same H_s and T_{01} parameters but different peak enhancement factors; viz., $\gamma=1$ and $\gamma=10$. Because breakwaters are subject to damage only from wave heights above a certain threshold level, our experiments were designed to attack the breakwater structure with energy levels above a specified threshold level. Irregular wave trains with envelopes having different energy flux exceedance parameters (related to the groupiness factor) were selected from Monte Carlo realizations of random waves.

Two different realizations of $H^2(x,t)$ were selected for each spectral shape (viz., $\gamma=1$ & $=10$). These two realizations had relatively high and low envelope exceedance coefficients, α, computed from

$$\alpha = \frac{\alpha'}{E[\alpha']} \; ; \; \alpha' = \frac{1}{N}\sum_{n=1}^{N}\left[\frac{H(n\Delta t)-H_{10}}{H_{10}}\right]^2 \delta(n) \tag{15}$$

where $\delta(n)=1$, if $H(n\Delta t)>H_{10}$ and $\delta(n)=0$, if $H(n\Delta t)<H_{10}$; N is the number of data points in the time series; Δt is the discretization time interval; H(t) is the wave height function measured at the toe of the structure; and $H_{10}=1.27\ H_{m0}$. The low and high values for α were selected from one-hundred DSA random wave realizations simulated from two Goda-JONSWAP spectra ($\gamma=1$ & $=10$), with $T_{01}=3.0$ sec., $f_{min}=0.7\ f_p$, and $f_{max}=2.5\ f_p$. Four simulations with the maximum and minimum values of α for each spectral shape were selected for the experiment. Initially, a threshold level of H_{10} was used in Eq. 15; however, changing H_{10} to H_s did not change the statistics of α significantly.

For each of the four envelopes, the constant phase shift used in Eq. 13 was $\psi=0$, $2\pi/3$, $4\pi/3$, and 2π. Because wave trains having $\psi=0$ & $=2\pi$ are identical, the differences in the measured damage for $\psi=2\pi/3$ & for $\psi=4\pi/3$ means that only the wave height and wave period functions are controlling the stability. Therefore, sixteen wave trains (unit variance) of 650 waves were simulated corresponding to: 1) two spectral shapes ($\gamma=1$ & $=10$); 2) two envelope exceedance coefficients (high and low α); and 3) four phase shifts (viz., $\psi=0$, $2\pi/3$, $4\pi/3$, and 2π). The levels of energy in the physical simulations were increased in discrete increments in order to make the stability numbers for the small and large rocks equal in consecutive runs. Seven runs of sixteen wave trains were simulated with significant wave heights determined from

$$H_s(k) = 0.43\left(\frac{W_L}{W_S}\right)^{\frac{k-1}{3}} \; ; k=1,2,...,7 \qquad (16)$$

where $H_s(k)$ is the significant wave height in meters corresponding to the kth run.

Three ultrasonic wave gauges were placed 10 m. from the toe of the structure in order to obtain two records of the incident and reflected wave trains (vide Fig. 1). The wave board did not have direct-digital-control in order to cancel the waves reflected from the structure. Consequently, the measured incident wave trains contained some multi-reflected waves. The method used to separate the incident and reflected wave trains is a modification of the method proposed by Goda and Suzuki (1976).

EXPERIMENTAL OBSERVATIONS

Rock profiles, wave records, video records of run-up, and visual observations of rock movements were recorded during the experiments. The rock profiles were used to calculate the damage defined as the normalized erosion of the armor. The wave records obtained from the three ultrasonic wave gauges located 10 m. in front of the structure were used to measure the waves incident on the structure.

The armor damage was calculated from the eroded area of the mean measured profiles corrected for small errors in measurements and for settlement. Distances along the profiles were normalized by the equivalent cube size (Iribarren) or nominal diameter (Van der Meer), $D_n=(W/\rho_r)^{1/3}$. The Eroded Volume Function (EVF) is defined as the corrected cumulative sum of the differences between the eroded profiles and the original profile. The damage is defined as the maximum value of the EVF.

Two spectral shapes ($\gamma=1$ & $=10$); two envelope exceedance coefficients (high & low α); four phase shifts ($\psi=0$, $2\pi/3$, $4\pi/3$, & 2π); two rock sizes ($W_L=13$ kg. and $W_S=10$ kg.); and seven wave runs of increasing energy (Eq. 16) gave a total number of 512 rock profiles and 224 values of damage. Damage observations from runs $\psi=0$ & $=2\pi$ were used as replicates in order to estimate the statistical variability of the experiments. A comparison between the results obtained from $\psi=2\pi/3$ & $=4\pi/3$ demonstrated that no significant difference was observed by changing the phase shift, ψ. Therefore, the experiments contain four replicates for each of the seven runs with the values of the parameters γ, α, & W held constant.

Measurements of damage at different levels of wave energy were recorded. In order to compare with the definition of damage in the S.P.M. (1984), the breakwater crest elevation was assumed to be equal to the design wave height. The active zone of damage for the experiment ($K_D=4$, cotan $\beta=2$, and $\rho_r/\rho_w=2.74$) was about $31D_n^2$. The data for damage given in the S.P.M. (1984) may then be reasonably approximated by

$$\frac{H_s}{\left[\frac{\rho_r}{\rho_w}-1\right]D_n} = N_s = 1.15\ [\cot\beta]^{\frac{1}{3}}\sqrt{D} \tag{17}$$

where D is the damage or normalized eroded armor; β is the slope; N_s is the stability number; and $D_n=(W/\rho_r)^{1/3}$ is the nominal diameter. We note that Van der Meer and Pilarczyk (1987) have proposed a different equation.

It would be possible to compare the damage function proposed by Van der Meer (1988) using values for P=0.2 (low permeability), $\xi_z=2.2$, and M=1100 waves with the damage function estimated from Eq. 17. However, the Van der Meer formulae (1988) were not used for comparisons because they depend critically on the value of the permeability parameter which is too difficult to estimate for design. Van der Meer and Pilarczyk (1987) suggest typical values for P for typical cross sections. However, the two cross sections used in our experiments have values for $0.1 < P < 0.4$. Consequently, estimates of damage may vary by as much as 50% about the average value of P=0.2. Because the selection of the parameter P is too subjective to use for design, we have not included the Van der Meer formulae in our comparisons.

Both Eq. 17 and the Van der Meer formulae (1988) suggest a fifth power relationship between damage and the significant wave height. Figure 2 compares the following experimental data for the small size rock: a) envelopes E1 ($\gamma=10$, $\alpha=1.8$) and E3 ($\gamma=10$, $\alpha=0.5$); b) envelopes E2 ($\gamma=1$, $\alpha=1.6$) and E4 ($\gamma=1$, $\alpha=0.5$); c) envelopes E1 and E2; and d) envelopes E3 and E4. The damage function estimated by Eq. 17 and 95% confidence intervals for the experimental data are also included in Fig. 2. Comparisons for the large size rock were similar.

Figure 2 demonstrates that damage produced by envelopes E1 and E2 were similar and clearly higher than damages produced by envelopes E3 and E4. This implies that the mean run length and spectral shape are not relevant for estimating armor stability. On the contrary, the envelope exceedance pattern significantly affects the stability. In addition, the surf similarity parameter significantly affected the stability because all of the damage functions for small size rocks gave systematically higher values than those for large size rocks (with a ξ_z 4.5% lower).

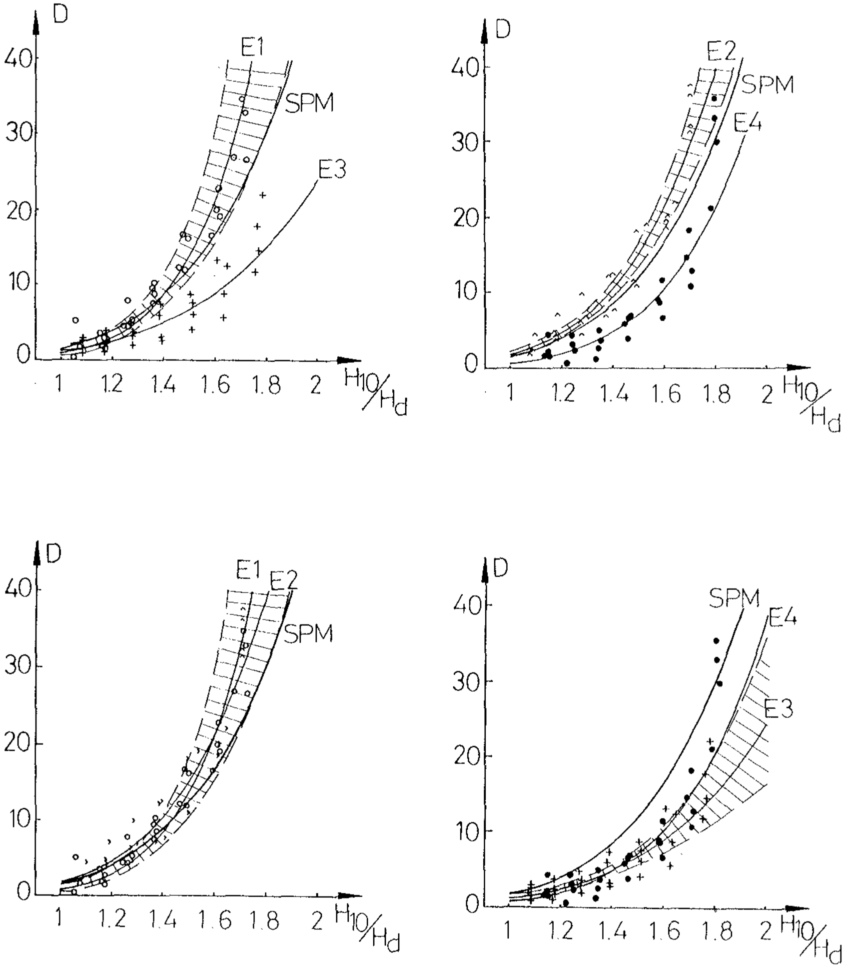

Fig. 2. Comparison of Damage to Small Rocks: a) Narrow & b) Broad Spectra; c) High & d) Low α.

In the past, characteristics of wave groups have only been correlated with spectral peakedness, with mean run lengths, or with other related groupiness parameters. These groupiness parameters have not proven to be completely satisfactory for determining the stability of the armor layers. However, our experimental data demonstrate that wave envelopes may be correlated with stability. Two wave trains incident at the toe of a structure, having exactly the same spectral shape and Rice groupiness parameter, may have significantly different wave envelope functions. By analyzing the wave envelope function as a measure of wave groupiness, it is possible to determine the one that will cause the more damage.

The envelope exceedance coefficient, α, may be related to a groupiness factor, not as defined by Rice, but by $GF=\sigma[H^2(t)]/8m_0$. This definition may not be calculated by the SIWEH as proposed by Funke and Mansard (1979). If the GF were calculated by the SIWEH, it would be biased and would depend on the spectral shape in a way similar to the Rice definition. Our definition for a GF was found to be highly correlated with the envelope exceedance coefficient α computed from analyses of 1000 DSA random simulations for the two spectral shapes used in the experiments (viz., $\gamma=1$ & $=10$). For both of these values of γ, a linear relationship given by $GF=(9+\alpha)/10$ was found to correlate with the simulated data at the 90% confidence level. The mean value of α is $E[\alpha]=1$ by definition and the standard deviation depends on the spectral shape; e.g., $\sigma[\alpha]\approx 0.45$ if $\gamma=10$ & $\sigma[\alpha]\approx 0.30$ if $\gamma=1$.

CONCLUSIONS

The envelope exceedance coefficient, α, defined by Eq. 15 and correlated with the groupiness factor, $GF=\sigma[H^2(t)]$, is a groupiness parameter which controls the stability. Random wave trains in wave flumes may have exactly the same spectral shape, but cause different levels of damage to the armor. The groupiness parameters α or GF may be able to resolve as much as 50% of the variability of the mean damage for a given design sea state and storm duration.

The spectral shape and the number of waves alone are not sufficient to describe the characteristics of wave groups and amount of armor erosion. Wave groups are more appropriately described by the parameters given for the envelope exceedance coefficient, α, or for the groupiness factor defined by $GF=\sigma[H^2(t)]/8m_0$. The spectral peakedness parameter and the mean run lengths are incomplete parameters for analyzing armor stability.

The influence of wave groupiness on armor stability may be of the same order of magnitude as the storm duration. The statistical variability of the observations, and the limited number of cases analyzed in these experiments do not permit a more precise description of the effects of wave groups on the stability of rubble mound breakwaters. However, for a preliminary design, it appears to be reasonable to modify existing formulae for damage functions by a factor of $(\alpha)^{1/2}$. The envelope exceedance coefficient, α, in random seas demonstrated a log-normal distribution having $E[\alpha]=1$ and $\sigma[\alpha]$ ranging from $[0.30<\alpha<0.45]$ depending on the spectral shape.

ACKNOWLEDGEMENTS

We gratefully acknowledge the financial support provided by the following agencies: The Dirección General de Investigación Científica y Técnica, under Grant PB88-0353; The Office of Naval Research under the University Research Initiative (ONR-URI) Contract No. N00014-86-K-0687; and The Oregon Sea Grant and the National Oceanic and Atmospheric Administration, Office of Sea Grant, Dept. of Commerce, under grant no. NA85AA-D-SG095 (project no. R/CE-21) and from appropriations made by the Oregon State Legislature.

REFERENCES

Battjes, J.A., and Vledder, V. (1984): "Verification of Kimura's Theory for Wave Group Statistics," *Proceedings*, 19th ICCE, 1984, Houston, TX, 642-648

Bruun, P. (ed.), (1985): *Design and Construction of Mounds for Breakwaters and Coastal Protection*, Elsevier Science Publ., Amsterdam, The Netherlands.

Burchart, H.F. (1979): "The Effect of Wave Grouping on On-Shore Structures," *Coastal Engineering*, No. 2, 189-199.

Funke, E.R. and Mansard, E.P.D. (1979): "On the Synthesis of Realistic Sea States in a Laboratory Flume," *HLR Report LTR-HY 66*, National Research Council of Canada, Ottawa.

Goda, Y. (1983): "Analysis of Wave Grouping and Spectra of Long-Traveled Swell," *Report of the Port and Harbour Research Institute*, Vol. 22, No. 1, 1983.

Goda, Y,, and Suzuki, y. (1976): "Estimation of Incident and Reflected Waves in Random Wave Experiments," Proceedings, 15th ICCE, 1976, Honolulu, Hawaii,828-845.

Hudspeth, R.T., and Medina, J.R. (1988): "Wave Groups Analyses by the Hilbert Transform," Proceedings, 21st ICCE, 1988, Torremolinos, Spain, 884-898.

Johnson, R.R., Mansard, E.P.D., and Ploeg, J. (1978): "Effects of Wave Grouping on Breakwater Stability," *Proceedings*, 16th ICCE, 1978, Hamburg, Germany, 2228-2243.

Mase, M., and Iwagaki, Y. (1986): "Wave Group Analysis from Statistical Viewpoint," *Proceedings*, Ocean Structural Dynamics Symposium '86, Corvallis, OR, 145-157.

Medina, J.R. and Hudspeth, R.T. (1987): "Sea States Defined by Wave Height and Period Functions," *Proceedings,* IAHR Seminar on Wave Analysis and Generation in Laboratory Basins, 22nd IAHR Congress, Lausanne, Switzerland, 249-259.

Medina, J.R. and Hudspeth, R.T. (1990): "A Review of the Analyses of Wave Groups," Coastal Engineering, (in press).

Medina, J.R., and McDougal, W.G. (1990): "Deterministic and Probabilistic Design of Breakwater Armor Layers" (Discussion), Journal of Waterway, Port, Coastal and Ocean Engineering, 116(4), 508-510.

Rice, S.O. (1954): "Mathematical Analysis of Random Noise," *Bell System Technical Journal*, Vol. 23, 1944, and Vol. 24, 1945. (Reprinted in *Selected Papers on Noise and Stochastic Processes*, N. Wax, Ed., Dover Publications, Inc., New York, NY, 1954, 123-244.)

Rye, H. (1982): *Ocean Wave Groups*, Dept. Marine Technology, Norwegian Institute of Technology, Report UR-82-18.

Shore Protection Manual (1984). Coastal Engineering Research Center, Department of the Army, Waterways Experiment Station, Vicksburg, Miss.

Van der Meer, J.W. (1988): "Deterministic and Probabilistic Design of Breakwater Armor Layer," Journal of Waterway, Port, Coastal and Ocean Engineering, 114(1), 66-80.

Van der Meer, J.W., and Pilarczyk, K.W. (1988): "Stability of Breakwater Armor Layers. Deterministic and Probabilistic Design," Delft Hydraulics Communication No. 378, 12,18,and 19.