CHAPTER 173

APPLICATION OF EQUILIBRIUM BEACH CONCEPTS TO SANDY GREAT LAKES PROFILES

M. Todd Stockberger*
(Associate Member ASCE)
William L. Wood**
(Member ASCE)

ABSTRACT

This study was designed to evaluate the equilibrium beach concept for conditions of "rapidly" varying water level, experienced on the North American Great Lakes. It was determined that the mathematical expression $h(x) = Ax^{2/3}$ is appropriate for describing sandy beach and nearshore profiles of the Great Lakes. In addition, a correlation was found between lake-level change and the shape factor A, that indicates a phase lag in beach and nearshore response to "rapidly" changing water level. Results from this study also raise some questions about the reliability of determining the shape factor A directly from sediment size. Reliability in the determination of A may be related to the stability shape of the profile relative to its equilibrium shape.

INTRODUCTION

During the past two and one half decades many researchers (Bruun, 1962; Edelman, 1968, 1972; Swart, 1974, 1976; Le Mehaute and Soldate, 1980; Kriebel and Dean, 1985; Hands, 1979, 1980, 1984; Weishar and Wood, 1983; Wood and Weishar, 1984) have investigated the response of the beach and nearshore to changes in water level. Concern has

* Research Engineer, Black & Veatch, P.O. Box 8405, Kansas City, MO 64114

** Associate Professor of Ocean Engineering and Director, Great Lakes Coastal Research Laboratory, School of Civil Engineering, Purdue University, West Lafayette, IN 47907

primarily been with rising water level since it is recognized as being a major contributor to the cause of increased shore erosion. While the focus of research has been on increases in water level and the corresponding shore response, there has been little similar research concerning rapidly changing or decreasing water level and the resulting effects on the beach and nearshore.

Recently, efforts have been made to apply the equilibrium beach concept to a variety of coasts, including the Great Lakes (Hands, 1979, 1980, 1984; Dean, 1977; Hughes, 1978). Dean (1977) proposed that nearshore profiles assumed a form consistent with $h(x) = Ax^{2/3}$, where h is the depth, x is the distance offshore, and A is a parameter dependent upon sediment characteristics of the profile being examined. Hands (1984) extended this formulation to the Great Lakes and hypothesized that the effects caused by rising lake level would simply be reversed by falling levels.

It appears reasonable to base a predictive model for Great Lakes beach profile response on the equilibrium beach concept. However, it must be shown that the nearshore profiles of the Great Lakes respond on a time scale similar to that of lake-level change. If the nearshore does not respond on a time scale similar to that of lake-level, serious limitations would be imposed on the ability of the equilibrium beach concept to predict nearshore response.

DATA COLLECTION AND ANALYSIS

The data used for this research consists of two series of nearshore profiles collected along the southeastern shore of Lake Michigan (see inset Figure 1). The first series consists of beach and nearshore profiles taken at 6 locations downdrift of the Michigan City, Indiana Harbor ("MTB" series, Figure 1). A total of 42 profiles were collected during the years 1975 through 1978, 1981, 1983, and 1985. Sediment samples were taken at the time of profiling for each line at water's edge and at the 1, 2, 3, 4, 5, 6, and 7 meter isobaths. The second series consists of beach and nearshore profiles taken at 6 locations updrift of the Michigan City Harbor ("SR" series, Figure 1). A total of 57 profiles were collected during the years 1966 through 1973, 1980, and 1988. Sediment data was also collected for this series for the years 1966 through 1973 (Hawley and Judge, 1969); however, these data were not available for analysis.

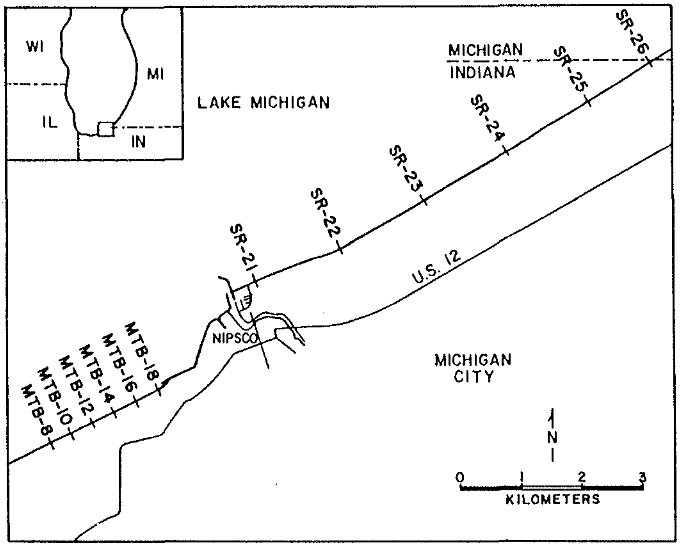

Figure 1 Study Area

Analysis of the Profile Form: $h(x) = Ax^m$

In order to determine whether the equilibrium profile form $h(x) = Ax^m$ is appropriate in describing Lake Michigan profiles, each profile was fit (using the method of least squares) from water's edge to closure depth. This analysis was used to obtain the distribution of the values of the exponent m and coefficient A. If the distributions of both m and A are found to be similar to those found by Dean (1977) and Hughes (1978), it can be concluded that an equation of the form $h(x) = Ax^{2/3}$ is valid for describing the profiles examined. If dissimilar distributions are found, then either a different exponent or a new equation describing the profile form can be established.

Analysis of Coefficient A and Mean Sediment Size

Dean (1977), Hughes (1978), and Moore (1982) showed that a relationship seems to exist between mean sediment size and the shape factor A. Therefore, it is reasonable to analyze the data in such a manner as to determine if this relationship holds true for Lake Michigan profiles. The measured profiles are separated into three sections (Figure 2). The first section consists of that part of the profile extending from water's edge to the point of inflection within the trough of the inner bar. The second section consists of that part of the profile lakeward of the inner bar trough and extending to the point of inflection within the trough of the outer bar. The third section consists of that part of the profile lakeward of the outer bar trough and extends beyond the depth of closure. Although the second slope break occurs at a point coincident with the trough of the outer bar, the outer section of the profile is actually initiated just lakeward of the outer bar. By using the section of the profile which is initiated just past the outer bar, any numerical ambiguity associated with the trough is avoided. The difference in the value of A was calculated for the outer section using both locations for section initiation and the error was found to be on the order of a few tenths of a percent.

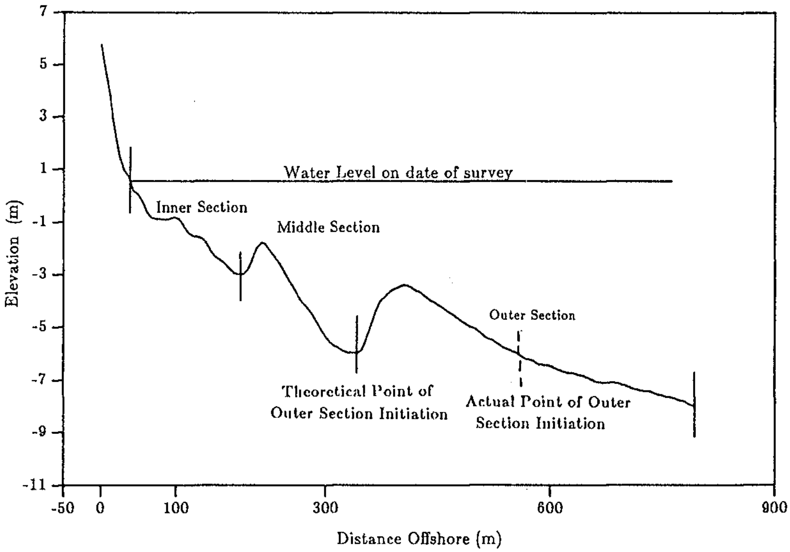

Figure 2 Nearshore Profile Sections

The first, or inner, section of the profile ranges in depth from water's edge to approximately the 1 to 1.5 meter isobath; therefore, the mean grain size at the 1 meter isobath and at water's edge were averaged to characterize the mean grain size of the inner section. The second, or middle, section of the profile ranges from approximately the 1 to 1.5 meter isobath to approximately the 2.5 to 3 meter isobath; therefore, the mean grain size at the 2 and 3 meter isobaths were averaged to characterize the middle section. The third, or outer, section of the profile ranges from approximately the 2.5 to 3 meter isobath to approximately the 7.5 meter isobath. The mean grain size of the 3, 4, 5, 6, and 7 meter isobaths were averaged to characterize the mean grain size of this section. The A value of each section of each profile is plotted against its characteristic mean sediment size. A comparison is then made with the findings of Moore (1982) to determine if his empirically determined curve is valid for the Great Lakes.

The Effect of Water Level Change on Shape Factor A

On the Great Lakes water level changes on a much shorter time scale and a much larger vertical scale than on ocean coasts. The rate of mean still water level change on the Great Lakes is of the order of 10's of centimeters per year. In order to determine whether Great Lakes profiles respond on a time scale similar to that of water level change, the average value of A for all profiles, for each survey period and with each of the two series considered separately, are plotted against time. If the profiles respond on a time scale slower than that of water level, the mean A values will be seen to vary in correspondence with the water level. If the profiles respond on a similar time scale, the mean A values will remain relatively constant.

RESULTS

Nearshore Profile Form

The values of the exponent m and coefficient A were determined for all 99 profiles used in the study. Figure 3(a) shows the distribution of exponent m for all profiles. This distribution compares favorably to the distribution of m values found by combining the results of Dean (1977) and Hughes (1978). The mean m value for Lake Michigan profiles is 0.632 which is in agreement with the values found by Dean, 0.66, and by Hughes, 0.671.

The results of this analysis support Dean's finding that a value of m equal to 2/3 is appropriate in describing nearshore profiles. Figure 3(b) shows the distribution of the coefficient A for all profiles. This distribution also compares well to the distribution found by combining the results of Dean (1977) and Hughes (1978). Owing to the

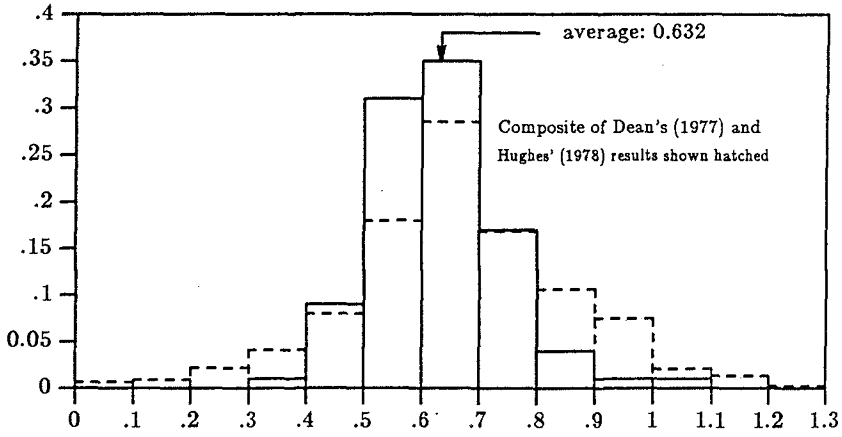

Figure 3. Frequency Distributions of: a) exponent m, and b) coefficient A

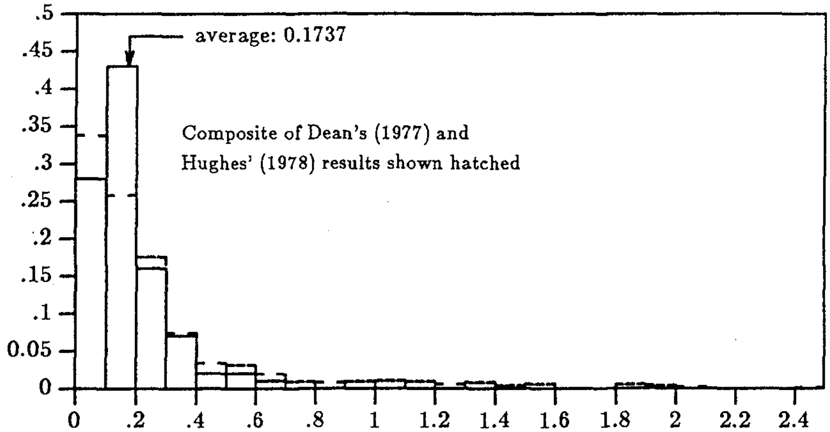

Figure 3. continued

similarity between the distributions of m and A for Lake Michigan profiles and the distributions found by Dean and Hughes, it appears that the nearshore profiles of Lake Michigan can, in general, be described by the equation $h(x) = Ax^{2/3}$.

Variation of Shape Factor A with Mean Grain Size

Figure 4 shows the joint distribution of coefficient A and grain size for the Lake Michigan profiles plotted with a set of curves developed by Moore (1982). Each of the three sections of the nearshore profiles were examined individually to evaluate any relationships that may exist between A and grain size. In general, the majority of data points lie within the range of expected values. The data for the inner sections of the profiles show the most variability in mean sediment size and the best correlation with Moore's curves. Both the middle and outer sections exhibit very little variation in mean grain size, thus, the data points for these sections result in nearly vertical distributions in A (Figure 4).

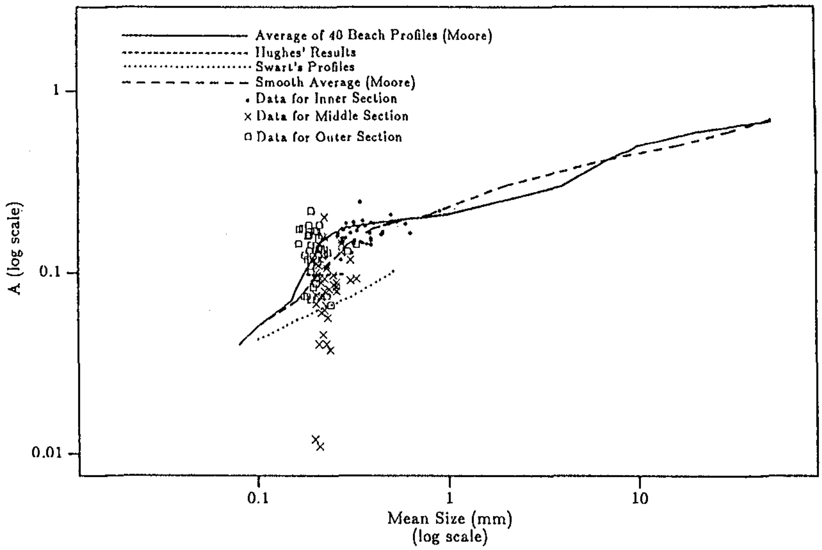

Figure 4. Distribution of A versus Grain Size for the Inner, Middle, Outer Sections of the Profiles.

The data for the inner sections of the nearshore profiles have been plotted separately on the set of curves developed by Moore (Figure 5). The data for the inner section lie primarily between the curve representing Moore's averaged data and the "smooth curve" developed by Moore. In order to better evaluate the relationship between the calculated values of A and the curves developed by Moore (1982), a statistical analysis was performed (Table 1). The results of this analysis show that a correlation between the A data of this study and the curves developed by Moore does not exist. This result raises questions concerning the relationship between sediment size and the shape factor A.

The Effect of Water Level Change on Shape Factor A

In order to evaluate whether the nearshore profiles of the Great Lakes respond on a time scale similar to that of water level change, the average values of A for each year were plotted against time. Figure 6 shows the mean value of A for the inner sections of the SR series plotted against time, with changes in water level superimposed.

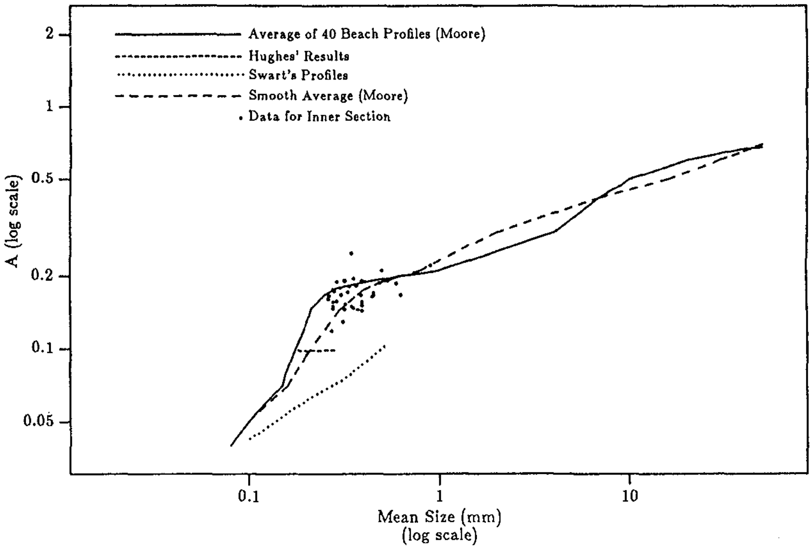

Figure 5. Distribution of A versus Grain Size for the Inner Sections Only

The horizontal hash marks show the respective limits of the coefficient A for plus and minus one standard deviation. The coefficient of determination, r^2, (where r is the correlation coefficient) was calculated for all lines and all sections except for the outer section of the SR lines.

It could be seen from graphical analysis of this outer section that the plot of mean A values with time was essentially a horizontal line, thus showing no correlation with water level change. The coefficient of determination, r^2, represents the amount of variation in the mean values of A that can be predicted or accounted for by water level change. Table 2 shows the values of the correlation coefficient and the coefficient of determination for the analyzed sections.

For Comparison to Moore's Data Curve				
Section	error	rms error	r	r^2
Inner	-0.000 9	0.023	0.32	0.1
Middle	-0.061	0.076	---	---
Outer	0.003	0.050	---	---
For Comparison to the Smooth Curve				
Section	error	rms error	r	r^2
Inner	0.0185	0.030	0.36	0.13
Middle	-0.011	0.050	---	---
Outer	0.0023	0.034	---	---

Table 1. Statistical Results for Sediment Size versus Coefficient A Data Comparison to Moore's Curves

Examination of Table 2 indicates that A value variations within the inner section of the profile correlate well with water level change. However, A value variation within the middle and outer sections of the profiles show no correlation with water level. To better illustrate the correlation between water level change and shape factor A for the inner section, demeaned values of A and demeaned values of the water level elevation are plotted against each other (Figure 7). The parallel trend illustrated in Figure 7 supports the argument that profile change is lagging water level change. The estimated lag time is not easily discernible from the data, but is of the order of years.

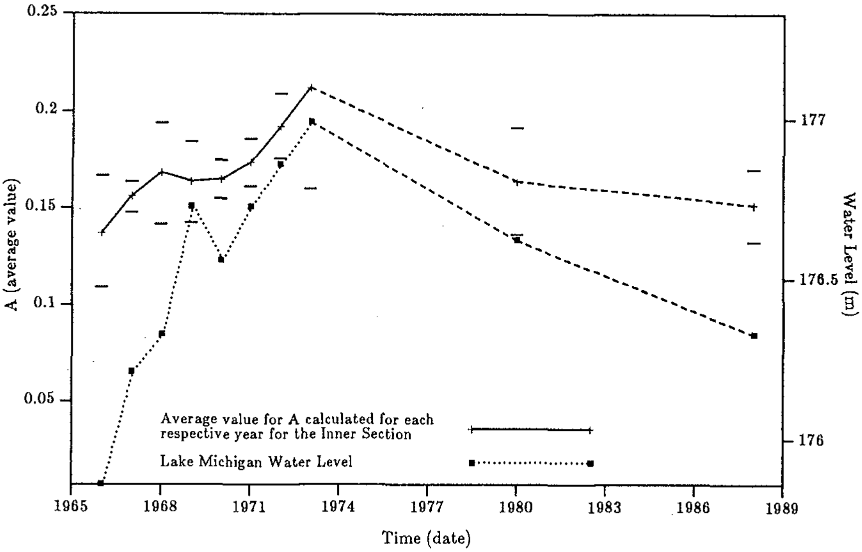

Figure 6. Time Variation of Mean A Values and Water Level for the Inner Sections of the SR Profiles

Section	r	r^2
Results for SR profiles		
Inner	0.865	0.75
Middle	0.325	0.11
Results for MTB profiles		
Inner	0.76	0.58
Middle	0.046	0.00
Outer	-0.438	0.19

Table 2. Correlation Statistics for A Versus Water Level

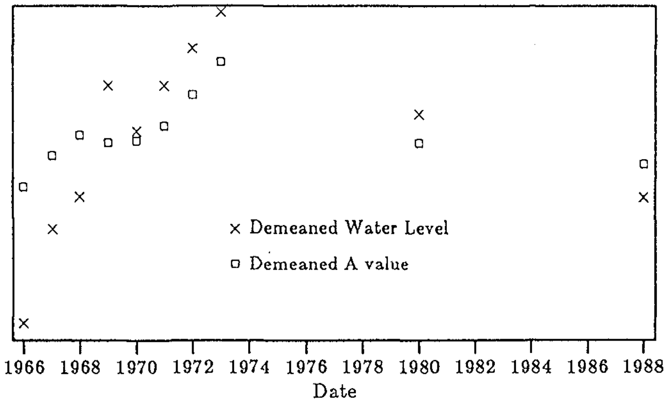

Figure 7. Demeaned A Values and Water Level Plotted Against Time for the Inner Sections of the SR Profiles

DISCUSSION AND CONCLUSIONS

Discussion

The equilibrium beach concept is based essentially on two premises. The first is that the form or shape of the nearshore profile is known, the second is that the nearshore profile responds on a time scale similar to that of water level change. The results of this study have shown that the nearshore profile form found by Dean (1977), $h(x) = Ax^{2/3}$, is appropriate for use in describing the "average" or characteristic nearshore profile found on the Great Lakes. However, the results have also shown that nearshore profiles respond on a much longer time scale than that of mean annual water level change. These results indicate that the equilibrium beach concept should not be used to predict nearshore response to short term water level changes on the Great Lakes. However, use of this concept may be appropriate for prediction of changes in the nearshore due to changes in water level which occur over a relatively long period of time (order of years).
These results are similar to those found by Hands (1980). Hands concluded that response of nearshore profiles on the Great Lakes appeared to be "out-of-phase" or lagging behind that of water level change. In addition, Hands concluded that the lag between profile response and water level change was on the order of a few years.

The effect of falling water level on the nearshore profiles could not be determined due to the slower than expected response of the profiles to water level change. Since questions still exist as to the applicability of the equilibrium beach concept to falling water level, it is strongly suggested that this concept not be applied under conditions of falling water level.

Results of this investigation fail to support the argument that the shape factor A is dependent on mean sediment size. Although some paired data grouped around the curves developed by Moore (1982), there were not significant relationships supported by statistical analysis. This result is contrary to the findings of Dean (1977), Hughes (1978), and Moore (1982).

It is important to note that the parameters m and A are interdependent and that the effect of setting m to a constant is to force the values of A into a much tighter distribution about the mean, thus reducing the amount of information that can be gained by examining the joint distribution. Perhaps the restricted application of the equilibrium profile equation $h(x) = Ax^{2/3}$ is not appropriate. It is recommended that determination of both variables should be carried out in a statistical context.

Conclusions

The following conclusions were reached as a result of this study.

1. The equilibrium profile form determined by Dean (1977), $h(x) = Ax^{2/3}$, is adequate for use in describing the "characteristic" nearshore profiles found on the Great Lakes.

2. The equilibrium beach concept should not be used to predict nearshore profile changes for short periods of time. However, the equilibrium beach concept may be adequate for use in predicting changes to the nearshore due to water level changes which occur over a long period of time.

3. The shape factor A does not appear to be well correlated with mean sediment size.

LIST OF REFERENCES

Bruun, P. (1962). Sea-Level Rise as a Cause of Shore Erosion, *Jour. of the Waterways and Harbors Division*, ASCE, 88(WW1), 117-130.

Edelman, T. (1972). Dune Erosion During Storm Conditions, *Proc. 13th Conf. Coast Eng.*, Vancouver, 1305-1312.

Hands, E.B. (1979). Changes in Rates of Shore Retreat, Lake Michigan, 1967-76, *CERC Technical Paper No. 79-4.*

Hands, E.B. (1980). Prediction of Shore Retreat and Nearshore Profile Adjustment To Rising Water Levels on the Great Lakes, *Technical Paper No. 80-7.*

Hands, E.B. (1984). The Great Lakes as a Test Model for Profile Response to Sea Level Changes, *Misc. Paper CERC-84-14.*

Hawley, E.F., and Judge, C.W. (1969). Characteristics of Lake Michigan Bottom Profiles and Sediments from Lakeside, Michigan to Gary, Indiana, *Proc. 12th Conf. Great Lakes Res.*, Int. Assoc. Great Lakes Res., 198-209.

Hughes S.A., (1978). The Variation of Beach Profiles when Approximated by a Theoretical Curve, M.S. Thesis, Univ. Florida.

Kriebel, D.L., and Dean, R.G. (1985). Numerical Simulation of Time-Dependent Beach and Dune Erosion, *Coastal Eng.*,9, 221-245.

Le Mehaute, B. and Soldate, M. (1980). A Numerical Model for Predicting Shoreline Changes, *CERC Misc. Report No. 80-6.*

Moore, B. (1982). Beach Profile Evolution in Response to Changes in Water Level and Wave Height, M.S. Thesis, Univ. Delaware.

Swart, D.H. (1974). Offshore Sediment Transport and Equilibrium Beach Profiles, Publ. No. 131, Delft Hydraulics Lab., Delft Univ. Technol.

Swart, D.H. (1976). Predictive Equations Regarding Coastal Transports, *Proc. 15th Conf. Coast. Eng.*, Honolulu, 1113-1132.

Weisher, L.L., and Wood, W.L. (1983). An Evaluation of Offshore and Beach Changes on a Tideless Coast, *J. Sed. Petrology*, 54, 847-858.

Wood, W.L., and Weishar, L.L. (1984). Beach Response to Long Period Lake-Level Variation, *Proc. 19th Conf. Coast. Eng.*, ASCE, Houston, 1571-1583.