CHAPTER 3

PRACTICAL COMPARATIVE PERFORMANCE SURVEY OF METHODS USED FOR ESTIMATING DIRECTIONAL WAVE SPECTRA FROM HEAVE-PITCH-ROLL DATA

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Abstract

Twelve methods used for estimating the directional wave spectrum from heave-pitch-roll data are compared on realistic numerical simulations. These methods are based on different modelling approaches including simple and sophisticated ones : Fourier Series decomposition, Fit to unimodal or bimodal parametric models, Variational fitting technique, Maximum Likelihood Methods, Eigenvector Methods, Maximum Entropy Methods and Bayesian approach. The comparison is performed in terms of practical aspects such as estimation error, computation time, implementation difficulty... This study must be regarded as a preliminary step devoted to the choice of optimal methods for operational in situ measurements.

1. Introduction :

The knowledge of directional properties of waves is of greatest interest in ocean and coastal engineering and the directionality of waves has appeared to have a great influence for offshore situations : moored vessels, oil-platforms,... as for nearshore problems : stability of coastal structures, harbour agitation, coastline morphodynamic evolution... A great effort has been devoted in the recent years to the determination of the directional wave spectrum. Several measurement techniques have been proposed for in situ or laboratory applications. They can be divided into several groups depending on the way they proceed :

- the single-point systems that measure at the same location several properties of waves. From this type of sensors one can mention the heave-pitch-roll buoy (Kobune *et al.*, 1985 ; Lygre and Krogstad, 1986 ; Mardsen and Juszko, 1987), the two-component current meter associated with a pressure sensor (Briggs, 1984) and the cloverleaf buoy (Mitsuyasu *et al.*, 1975).
- the gauge arrays that are composed of several sensors set up at various locations. The sensors may be either identical or of various types including for instance current meters and pressure sensors (Hashimoto *et al.*, 1987)
- the remote-sensing systems including active microwave radars (Jackson *et al.*, 1985), aerial stereo-photography techniques...

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Among these systems the heave-pitch-roll buoy is probably the most widely employed for operational use because it is a compact single-point measurement system of moderate cost and easy keeping. In the meantime the sampling of wave statistics is limited to three properties : the sea-surface elevation (heave) and two orthogonal slopes of sea-surface (pitch and roll). Starting with only three measured quantities the estimation of the directional wave spectrum is a difficult inverse problem for which no unique method can be exhibited. To take maximum advantage of these rather limited information, several methods based on various modelling approaches have then be proposed.

The aim of this study is to compare a large number of these methods on numerical representative sea-state simulations. In this study we are interested in evaluating the ability of the methods to estimate the simulated directional spectra (estimation error) and studying practical aspects of their use (implementation problems, computing time). This preliminary work aims to exhibit practical recommendations for the choice of optimal methods that could then be applied in laboratory or in the field.

2. Problem formulation

The directional wave spectrum $S(f,\theta)$ is a function of wave frequency f and direction of propagation θ. The following classical decomposition is used :

$$S(f,\theta) = E(f).D(f,\theta) \tag{1}$$

in which $E(f)$ is the one-sided frequency spectrum that may be estimated by a single record of sea-surface elevation. It is related to the directional spectrum by :

$$E(f) = \int_0^{2\pi} S(f,\theta)\, d\theta \tag{2}$$

$D(f,\theta)$ is the directional spreading function satisfying two important properties :

$$D(f,\theta) \geq 0 \ \text{ over } [\,0\,,2\pi\,] \qquad \text{and} \qquad \int_0^{2\pi} D(f,\theta)\, d\theta = 1 \tag{3}$$

The following pseudo-integral relation may be written between the directional spectrum and the sea-surface elevation η :

$$\eta(x,y,t) = \int_0^{\infty} \int_0^{2\pi} \sqrt{2.S(f,\theta).df.d\theta}\ .\cos\left[2\pi ft - k.(x.\cos\theta + y.\sin\theta) + \varphi\,\right] \tag{4}$$

In the present study wc assume that the buoy is able to measure the sea-surface elavation and two orthogonal slopes on the sea-surface :

$$\begin{aligned} \eta(t) &= X_1(t) \\ \frac{\partial\eta}{\partial x}(t) &= \eta_x(t) = X_2(t) \\ \frac{\partial\eta}{\partial y}(t) &= \eta_y(t) = X_3(t) \end{aligned} \tag{5}$$

By applying spectral analysis procedures it is possible to compute the cross-spectra $G_{ij}(f)$ between each couple (X_i, X_j) of the three measured properties :

$$G_{ij}(f) = 2.\int_{-\infty}^{+\infty} R_{ij}(\tau)\, e^{-i2\pi f\tau}\, d\tau \quad \text{with } R_{ij}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T X_i(t).X_j(t+\tau)\, dt \tag{6}$$

The cross-spectra or spectral cross-correlation coefficients are in the general case complex quantities which are often written in the following form :

$$G_{ij}(f) = C_{ij}(f) - i.Q_{ij}(f) \tag{7}$$

$C_{ij}(f)$ is called "coïncident spectral density function" or "co-spectrum" and $Q_{ij}(f)$ is called "quadrature spectral density function" or "quad-spectrum".

By using (4) and the linear relationship between the elevation and the slopes of sea-surface it may be shown that the cross-correlation coefficients for the heave-pitch-roll buoy take the following expressions :

$$\left.\begin{array}{ll}
C_{11}(f) = \int_0^{2\pi} S(f,\theta)\, d\theta = E(f) & Q_{11}(f) = 0 \\
C_{22}(f) = E(f).k^2\int_0^{2\pi} D(f,\theta).\cos^2(\theta)\ d\theta & Q_{22}(f) = 0 \\
C_{33}(f) = E(f).k^2\int_0^{2\pi} D(f,\theta).\sin^2(\theta)\ d\theta & Q_{33}(f) = 0 \\
C_{12}(f) = 0 & Q_{12}(f) = E(f).k\int_0^{2\pi} D(f,\theta).\cos(\theta)\ d\theta \\
C_{13}(f) = 0 & Q_{13}(f) = E(f).k\int_0^{2\pi} D(f,\theta).\sin(\theta)\ d\theta \\
C_{23}(f) = E(f).k^2\int_0^{2\pi} D(f,\theta).\cos(\theta).\sin(\theta)\ d\theta & Q_{23}(f) = 0
\end{array}\right| \tag{8}$$

From the twelve real cross-correlation coefficients only six are non equal to zero. Furthermore $C_{11}(f)$ does not carry any information about the directional distribution and $C_{11}(f)$, $C_{22}(f)$, $C_{33}(f)$ are tied by the following relation :

$$C_{22}(f) + C_{33}(f) = k^2.C_{11}(f) \tag{9}$$

that we can use for the calculation of the wave number expression :

$$k = \sqrt{\frac{C_{22}(f) + C_{33}(f)}{C_{11}(f)}} \tag{10}$$

As established so far it is possible to compute by spectral analysis only <u>five independent coefficients at each frequency</u> from which one is devoted to the estimation of the frequency spectrum $E(f)$ and the four others may be used for the computation of the directional spreading function $D(f,\theta)$ at this frequency. The problem of directional spectrum estimation is then to determine a continuous function over $[0\ ,\ 2\pi]$ satisfying (3) with only four independent integral properties from (8). It is clear that finding a solution to this difficult inverse problem can not be proceeded in an unique way because of the too feeble number of constraints.

3. Review of methods used for estimating the spreading function

Among the theoretical modelling approaches proposed in the literature to solve the above exposed problem, twelve operational methods have been selected and implemented in the software **PRD-WAS 1.0** (Pitch and Roll Data - Wave Analysis Software) developed at LNH (Benoit, 1991). Only a short description of the theoretical background of each method is reported below. Further information may be found in the mentioned references.

1. Truncated Fourier Series (TFS) : the directional spreading function is expressed as a truncated series whose first four coefficients are easily computed from the spectral cross-correlation coefficients :

$$D(f,\theta) = \frac{1}{\pi}\left(\frac{1}{2} + a_1(f).\cos\theta + b_1(f).\sin\theta + a_2(f).\cos 2\theta + b_2(f).\sin 2\theta\right) \tag{11}$$

2. Weighted truncated Fourier Series (WFS) : Following Longuet-Higgins *et al.*. (1963) a weighting function is used to avoid negative values of the former method :

$$D(f,\theta) = \frac{1}{\pi}\left(\frac{1}{2} + \frac{2}{3}(a_1(f).\cos\theta + b_1(f).\sin\theta\ ) + \frac{1}{6}(a_2(f).\cos 2\theta + b_2(f).\sin 2\theta)\right) \tag{12}$$

3. Fit to Unimodal Gaussian Model (1MF) : A unimodal parametric model of gaussian type (Borgman, 1969) is used whose two unknown parameters α et σ are computed from the first two Fourier coefficients of the spreading function :

$$D(f,\theta) = \frac{1}{\sqrt{2\pi}\,\sigma}\exp\left(-\frac{(\theta-\alpha)^2}{2\sigma^2}\right) \tag{13}$$

4. Fit to Bimodal Mitsuyasu Model (2MF) : A bimodal parametric model obtained from linear combination of two unimodal Mitsuyasu-type models is used:

$$D(f,\theta) = \frac{\lambda}{\Delta(s_1)}\cos^{2s_1}\left(\frac{\theta-\theta_1}{2}\right) + \frac{1-\lambda}{\Delta(s_2)}\cos^{2s_2}\left(\frac{\theta-\theta_2}{2}\right) \tag{14}$$

Its five unknown parameters are calculated from the spectral cross-correlation coefficients by using a method based on the least-square method (Benoit, 1991).

5. Variational Fitting Technique - Long-Hasselmann Method (LHM) : Long and Hasselmann (1979) developed this method by which an initial simple estimate is iteratively modified to minimize a "nastiness" function that takes into account the various conditions on the spreading function.

6. Maximum Likelihood Method (MLM) : By this method the spreading function is regarded as a linear combination of the cross-spectra :

$$D(f,\theta) = \sum_{i=1}^{3}\sum_{j=1}^{3} w_n^*(\theta).w_m(\theta).G_{ij}(f) \tag{15}$$

The weighting coefficients w_n are calculated with the condition of unity gain of the estimator in the absence of noise (Oltman-Shay and Guza, 1984).

7. Iterative Maximum Likelihood Method (IMLM) : The estimator obtained by the former method is not consistent with the data cross-spectra. It is also iteratively modified to let its spectral cross-correlation coefficients become closer to the ones obtained from the data (Oltman-Shay and Guza, 1984).

8. EigenVector Method (EVM) : Mardsen and Juszko (1987) proposed a refinement of the MLM in which the data are partitioned into signal and noise through the calculation of the eigenvalues of the spectral cross-correlation matrix.

9. Iterative EigenVector Method (IEVM) : the same iterative improvement as proposed for the MLM is applied to the Eigenvector Method.

10. Maximum Entropy Method - Version 1 (MEM1) : Lygre and Krogstadt (1986) proposed to find an estimate of the spreading function by maximizing :

$$\chi = -\int_0^{2\pi} \ln (D(f,\theta))\, d\theta \tag{16}$$

under the constraints given by the spectral cross-correlation coefficients.

11. Maximum Entropy Method - Version 2 (MEM2) : The same approach may be considered by using Shannon's definition for entropy (Nwogu *et al.*, 1987) :

$$\chi = -\int_0^{2\pi} D(f,\theta).\ln (D(f,\theta))\, d\theta \tag{17}$$

With the former version the expression of the directional estimate is easily obtained, but with the latter a non-linear system of equations has to be solved.

12. Bayesian Directional Method (BDM) : With this statistical technique used for regression analysis (Hashimoto *et al.*, 1987), no *a priori* assumption is made about the spreading function which is considered as a piecewise-constant function over $[0, 2\pi]$. The unknown values of $D(f,\theta)$ on each of the K segments dividing $[0, 2\pi]$ are obtained by considering the constraints of the spectral cross-correlation coefficients and an additional condition on the smoothness of $D(f,\theta)$.

4. Description of the performed tests

Numerical sea-state simulations are performed by following the single direction per frequency method (Miles, 1989) based on a discretization of (4) :

$$\eta (x,y,t) = \sum_{n=1}^{N} A_n \cos (2\pi.f_n.t - k_n(x.\cos\theta_n + y.\sin\theta_n) + \varphi_n) \tag{18}$$

with :

$$\left.\begin{array}{l} A_n = \sqrt{2\, S(f_n, \theta_n)\, \Delta f_n\, \Delta\theta_n} \\ \varphi_n = 2\pi\, U[0,1] \quad \text{(random phase)} \\ f_n = (n-1)\,\Delta f \ \text{ with } \Delta f = \frac{fc}{N} = \frac{1}{T} \\ \theta_n \text{ are of the form } k.\Delta\theta\text{, but randomly distributed over } [0, 2\pi] \end{array}\right| \tag{19}$$

The frequency spectrum E(f) is a classical JONSWAP spectrum with a significant wave height of 4 m, a peak frequency of 0.1 Hz and a peak-factor $\gamma = 3.3$.

The directional spreading function is of the form :

$$\Pi_{p1,\alpha1,p2,\alpha2,\lambda}(\theta) = \lambda.\Pi_{p1,\alpha1}(\theta) + (1-\lambda).\Pi_{p2,\alpha2}(\theta) \quad \text{with } 0 \le \lambda \le 1 \tag{20}$$

with :

$$\left.\begin{array}{ll} \Pi_{p,\alpha}(\theta) = \frac{1}{\Lambda(p)} \cos^{2p}(\theta-\alpha) & \text{if } \theta \in \left[\alpha-\frac{\pi}{2}\, ;\, \alpha+\frac{\pi}{2}\right] \\ \Pi_{p,\alpha}(\theta) = 0 & \text{if } \theta \in \left[\alpha-\pi\, ;\, \alpha-\frac{\pi}{2}\right] \cup \left[\alpha+\frac{\pi}{2}\, ;\, \alpha+\pi\right] \\ \multicolumn{2}{l}{\Lambda(p) \text{ is a normalizing constant to ensure the properties (3).}} \end{array}\right| \tag{21}$$

A Gaussian real white noise is added to the simulated time series. Its spectral density is constant from zero to the Nyquist frequency fc. The noise level is expressed in terms of a percentage of the RMS (root mean square) amplitude of sea-surface elevation as proposed by Nwogu *et al.* (1987).

The simulated time series have a time step of 0.5 s and a duration of 4096 s (8192 points per signal). This represents a record of around 410 waves with 20 points per wave (at peak frequency).

The spectral estimator is based on the technique of the averaged periodogram on the whole simulated sequence partitioned in segments of 1024 points. An overlapping of 25% between adjacent segments is used and a parabolic data window is applied to the common part of two adjacent segments. The resulting frequency resolution is 0.002 Hz.

Three test cases are performed with the following characteristics for the directional spreading function :

- case 1 : Unimodal Broad Spreading Function (λ=1 | p_1=1 | α_1=120°)

- case 2 : Unimodal Thin Spreading Function (λ=1 | p_1=10 | α_1=70°)

- case 3 : Bimodal Spreading Function (λ=0.5 | p_1=2 | α_1=130° | p_2=6 | α_2=240°)

The first case rather represents a wind-sea with a large spread around a main direction of propagation. In the second case the spread around the main direction of propagation is much lower indicating a rather "old" swell. In the third case we have a crossed-sea with two main directions of propagation. The three directional spreading functions are presented on figure 1.

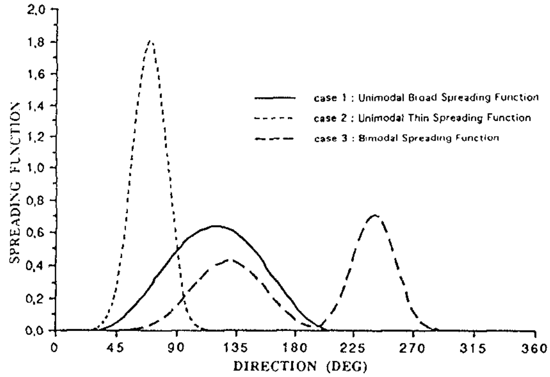


Figure 1 : The three Directional Spreading Functions

To perform the comparison of the methods five criterion are taken into account :

a) **the error of the estimate** $\hat{D}(\theta)$ produced by the method versus the simulated spreading function $D(\theta)$. This error is measured by the Weighted Average Percent Error (WAPE) as proposed by Oltman-Shay and Guza (1984) :

$$\mathrm{WAPE} = \frac{\sum_{\theta} \left|\hat{D}(\theta) - D(\theta)\right|}{\sum_{\theta} D(\theta)} \times 100 \tag{22}$$

b) **the sensitivity to the shape of the spreading function**

c) **the computation time** measured on an IBM 3090 Computer

d) **the noise sensitivity**

e) **the difficulty of implementation**

5. Presentation and discussion of the results

The spreading functions estimated at peak frequency by the various methods on the three cases with a noise level of 20% are reported on figures 2, 3 and 4 respectively. On these figures the target function is represented by a dashed line and the spreading functions estimated at both the frequency surrounding the theoretical peak frequency are represented by continuous lines with different marker symbols. The WAPE and CPU Time are also reported on the same figures. An overview for the comparison of the various methods on the three cases is presented on figure 5 where performance is visualized by points representing WAPE versus CPU Time.

Because on the great volume of results to be presented in such a comparative study only nine methods appear on these figures. Both the Fourier Series methods (TFS and WFS) are not reported because it is now well known that they do not give good results although they are fast and easy to implement. The former often produces negative values which are not acceptable for a spreading function. For the latter the suppression of negative values by a weighting function results in an important smoothing of the curve and strongly under-estimates the peaks of the spreading function. The Iterative Eigenvector Method IEVM is not reported because it has sometimes failed to converge during these tests (in particular on case 2) and produced spurious peaks.

The 1MF method (Fit to Unimodal Gaussian Model) is the fastest one and produces good results on the two first cases where the target spreading function is unimodal, but as execpted do not give a reliable estimate on case 3.

The extension of this approach to Bimodal Model Fitting (2MF) is very satisfactory on the three cases, more especially as one has to kcep in mind that it is tried to determine five model parameters from only four data coefficients. Even if more validation cases are needed for this method currently developed at LNH, the results on these three cases are very promising. Furthermore the computing time may be considered as very short.

The Variational Fitting Technique of Long and Hasselmann gives good results on the unimodal cases because the initial model (based on 1MF method) is already close to the target model, but in the third case when the iterative algorithm has to be activated, the computing time increases rapidly for an estimate that still produces 50% WAPE. At this point of validation this method has not shown definite advantages compared to its rather long and difficult implementation.

The Maximum Likelihood Method produces rather good estimates on all the cases with a very short CPU Time. As it is furthermore rather easy to implement we understand why it is so widely used for operationnal measurement. Its iterative refinement IMLM improves the estimation on case 3, but also shows a trend to split the peak on unimodal cases. Nevertheless it appears better than the MLM on figure 5 even if the CPU Time is clearly higher. During these tests 10 iterations were performed but further tests are required to optimize this value.

The Eingenvector Method does not reveal here a particular interest : the method is fast but on these cases the estimates are not very accurate. This method should be probably more interesting on cases where the noise level is much higher.

The first version of the Maximum Entropy Method (MEM1) is easy to implement and needs very little CPU time, but the estimate is not reliable and split the peak on unimodal cases. On case 3 the directional spreads are under-estimated.

The MEM version 2 (MEM2) is clearly superior and gives very good estimates on the three cases. On the other hand the CPU time is high and the implementation of this method is not simple.

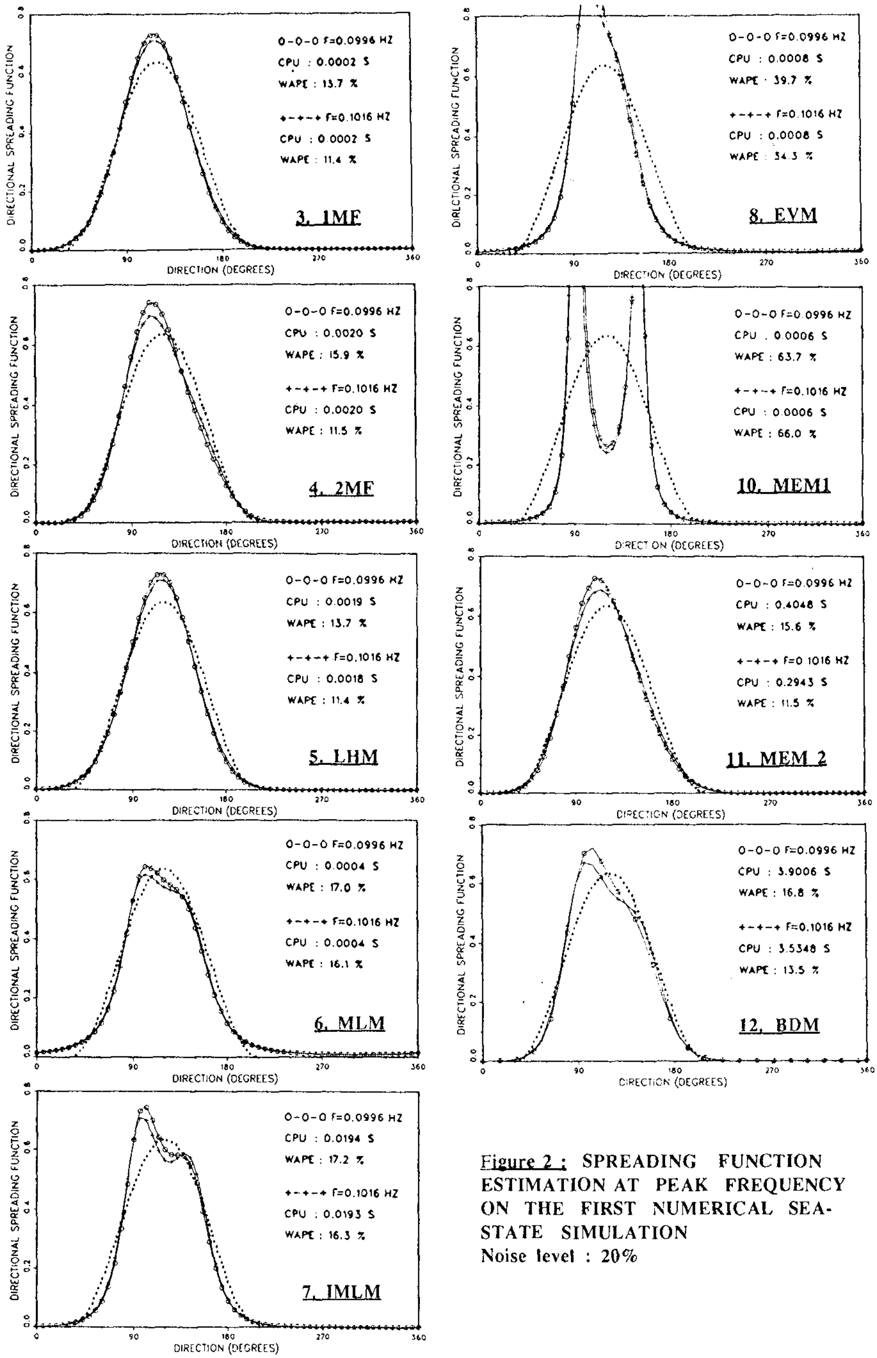


Figure 2 : SPREADING FUNCTION ESTIMATION AT PEAK FREQUENCY ON THE FIRST NUMERICAL SEA-STATE SIMULATION
Noise level : 20%

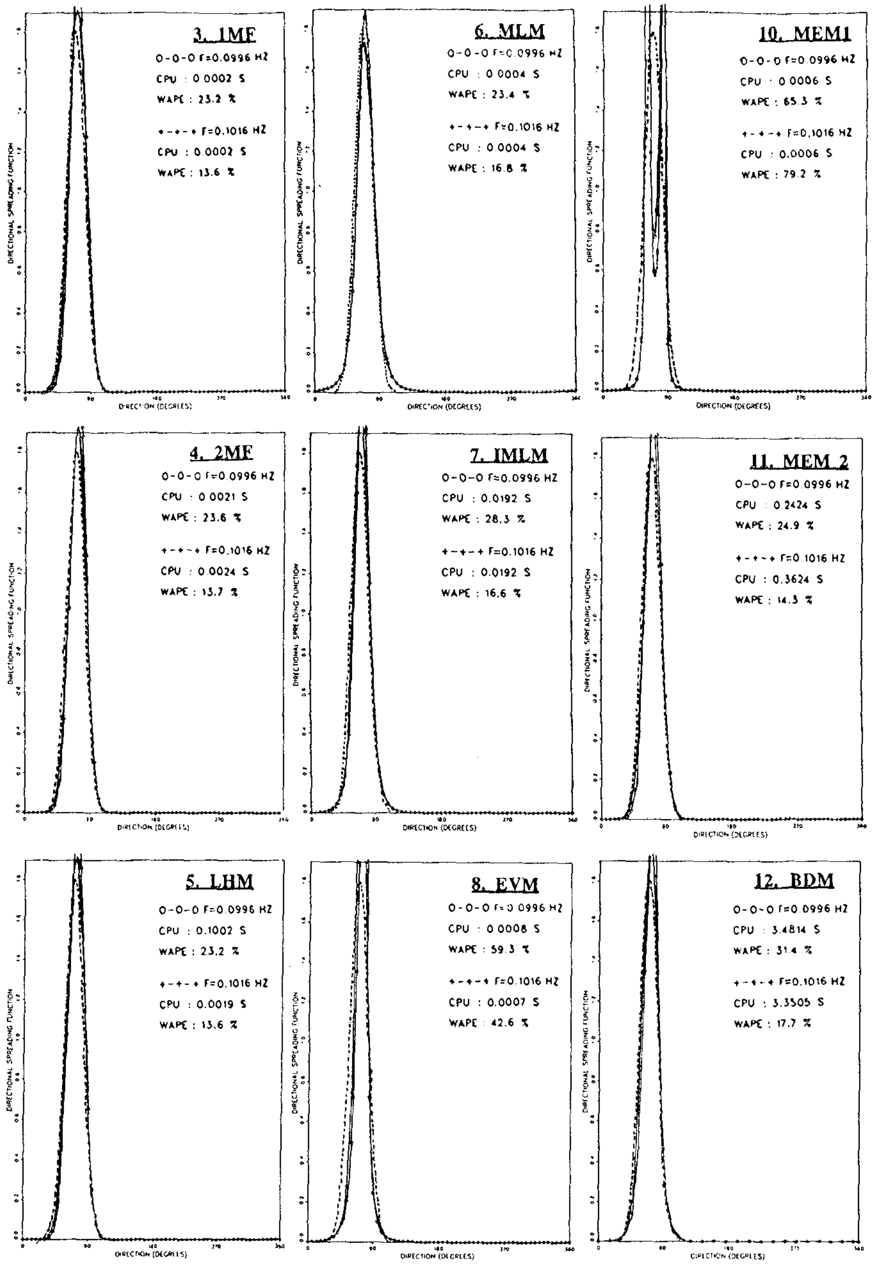
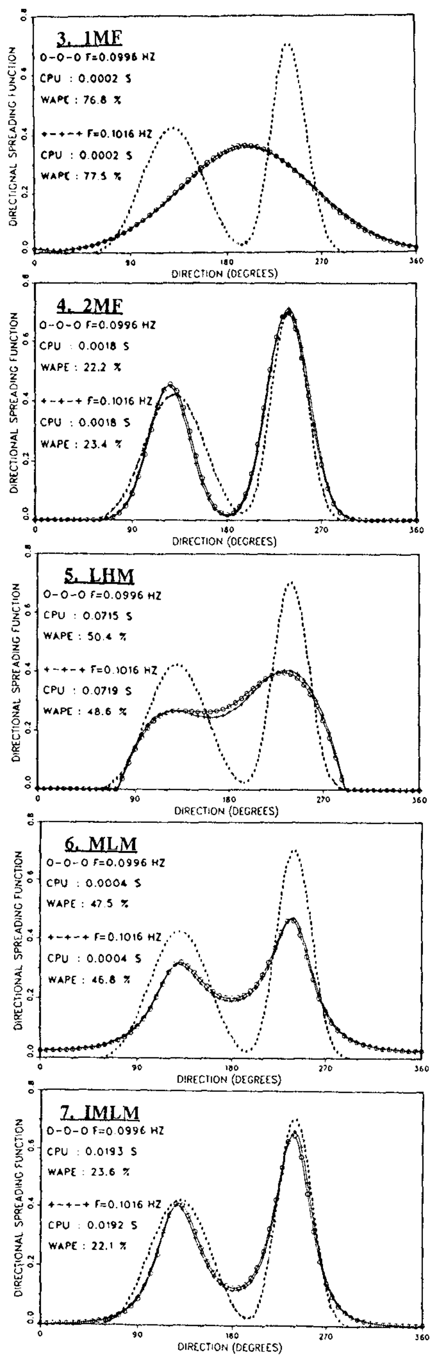


Figure 3 : SPREADING FUNCTION ESTIMATION AT PEAK FREQUENCY ON THE SECOND NUMERICAL SEA-STATE SIMULATION
Noise level : 20%



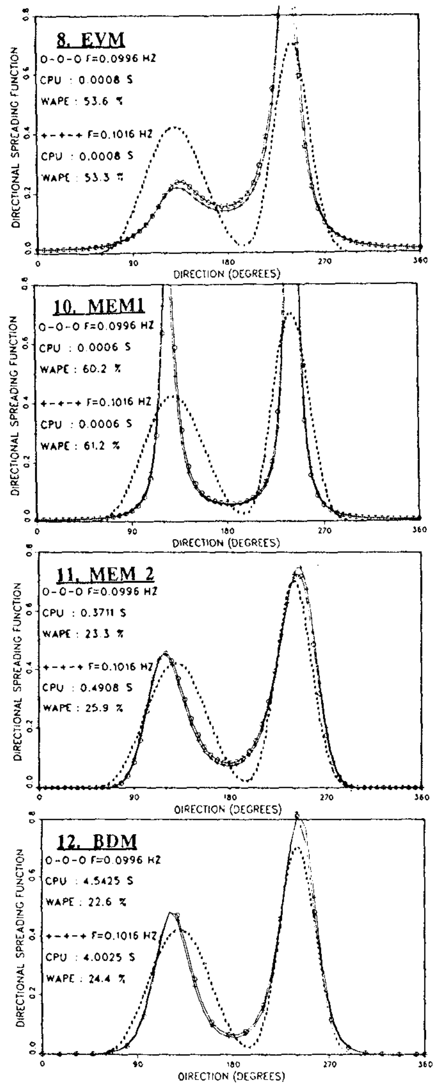


Figure 4 : SPREADING FUNCTION ESTIMATION AT PEAK FREQUENCY ON THE THIRD NUMERICAL SEA-STATE SIMULATION
Noise level : 20%

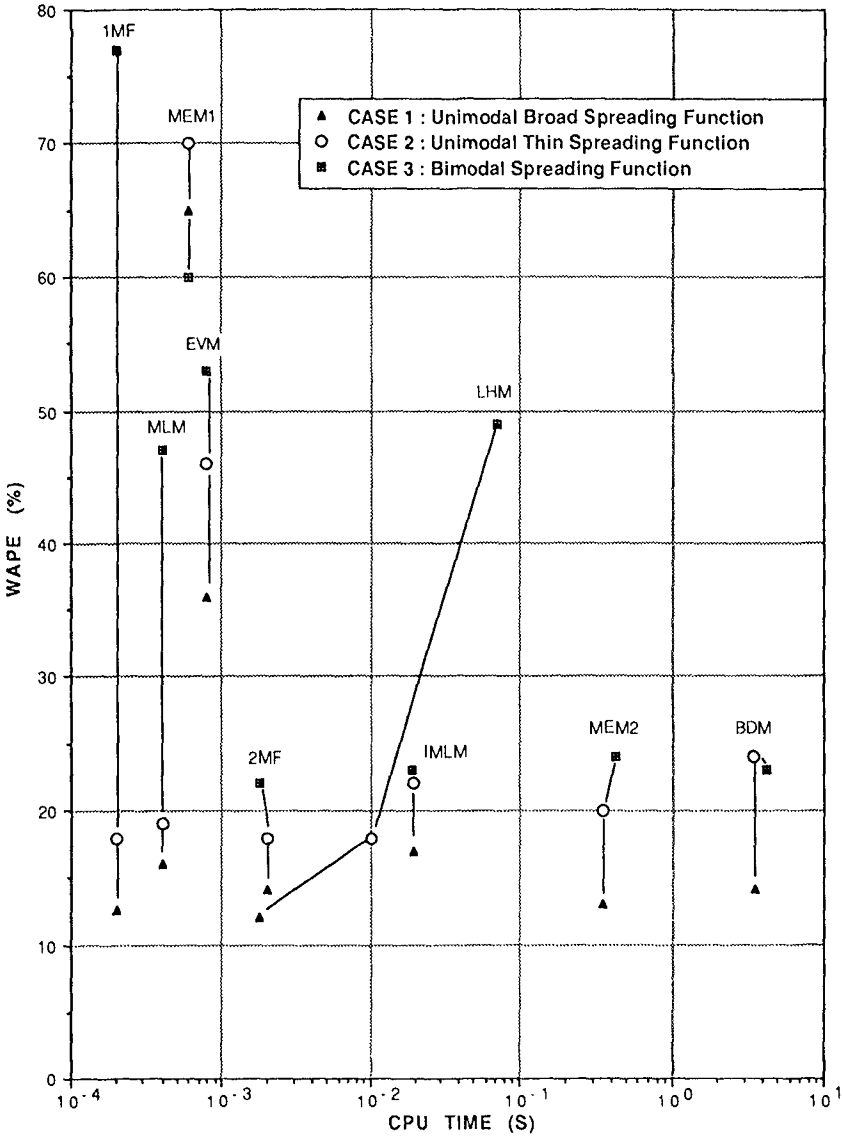


Figure 5 : PERFORMANCE COMPARISON OF THE VARIOUS METHODS ON THE THREE NUMERICAL SEA-STATE SIMULATIONS
Spreading Function estimated at Peak Frequency - Noise level : 20%

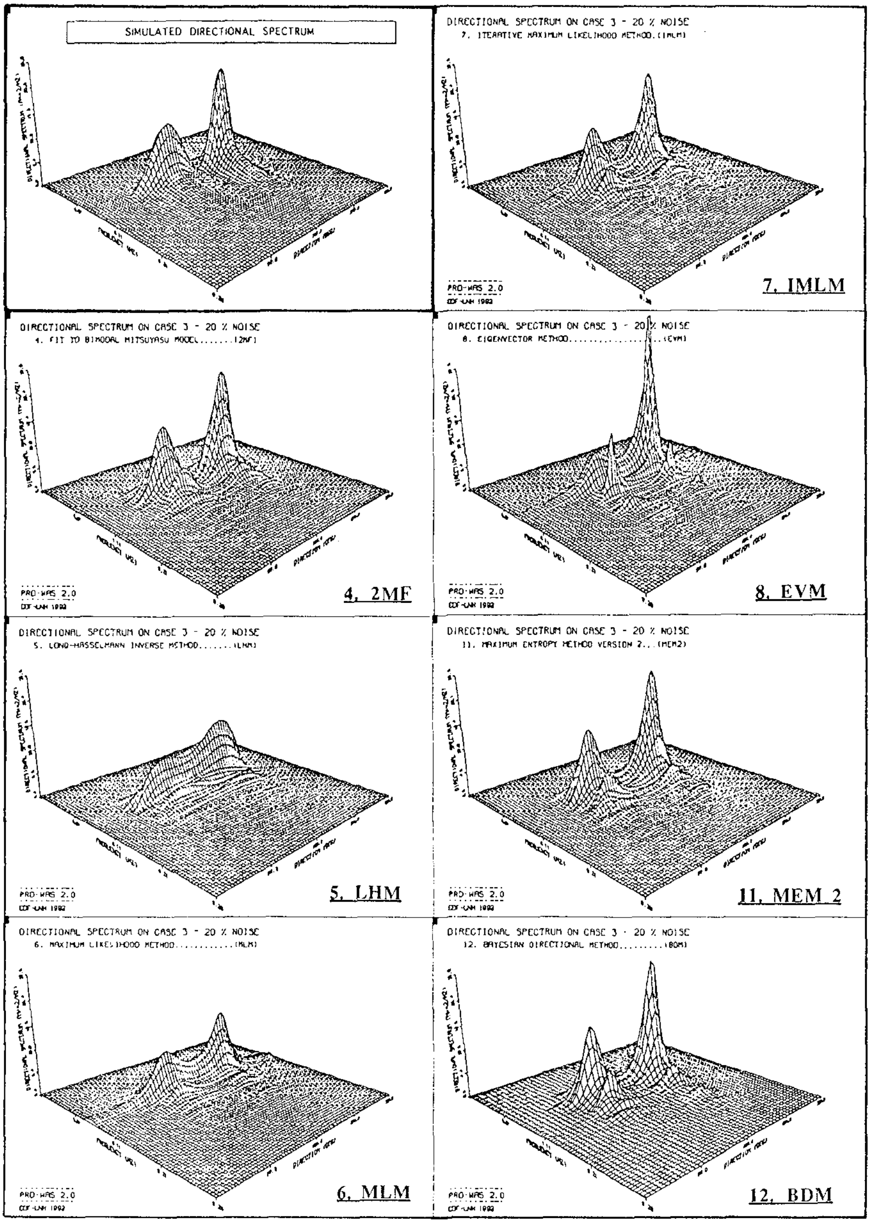


Figure 7 : COMPARISON OF DIRECTIONAL SPECTRUM SHAPES ON THE THIRD NUMERICAL SEA-STATE SIMULATION (Bimodal Spreading Function) - Noise level : 20%

The Bayesian Direction Method also produces good estimates on these cases although there are very few information on input. This is also an extreme application of this sophisticated method and the resulting CPU Time is very high. Following its authors we can not affirm this method will always produce reliable estimates for so little amount of data.

From the analysis of figure 5 four methods giving less than 25% WAPE on the three cases may be identified : 2MF, IMLM, MEM2 and BDM (ordered following increasing CPU Time). These methods are rather stable in the sense that the CPU Time is rather constant and the WAPE do not vary in a great manner from a case to an other. Meanwhile the CPU Time is changing in a ratio of around 10 from a method to the following one.

The effect of noise is sensitive as soon as we try to compute the spreading function too far from the peak frequency. As example the variation of directional spread with noise level is reported on figure 6. Following Nwogu *et al.* (1987) who obtained the same conclusions about noise effect study, we would advise not to compute the spreading function out of the range [0.75 fp, 1.5 fp].

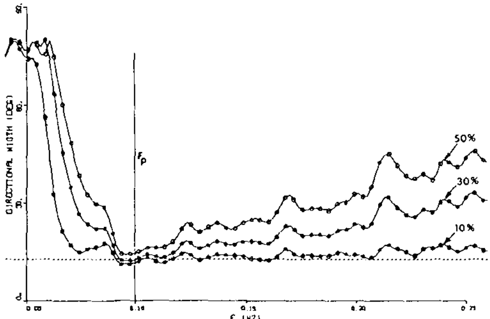


Figure 5 : Effect of Noise Level on Directional Width (Case 2 : Unimodal Thin Spreading Function)

Figure 7 shows 3D-plots of the directional spectra estimated by seven of these methods on case 3. The most accurate estimates are clearly given by the above identified four methods : 2MF, IMLM, MEM2 and BDM. The MLM spectrum may be regarded as acceptable. The LHM and EVM spectra are of relative poor quality.

6. Conclusion

As major conclusions the following points have to be highlightened :

- the determination of the spreading function from heave-pitch-roll data is a critical problem that should be undertaken only around peak frequencies while on the other hand representative parameters such as main direction, directional spread, etc... may be computed over a much larger frequency range.
- Simple methods as Fourier Series Decomposition (TFS, WFS) or Unimodal Fitting (1MF) are not recommended because their estimates are of poor quality.
- the Maximum Likelihood Method (MLM) may be advised because it is fast and rather easy to implement. Its iterative refinment (IMLM) clearly improves the method and appears as an acceptable compromise between error and CPU Time.
- the Maximum Entropy Version version 2 (MEM2) seems to give the most reliable estimate but its computing time is rather high.
- the Bimodal Fitting (2MF) in its form developed at LNH seems very promising but has to be validated on more simulations.
- Sophisticated refinements as Variational Model Fitting (LHM) or Bayesian Directional Method (BDM) are not interesting for the case of the buoy where only few information are available.

Of course these preliminary conclusions have then to be confirmed or altered by applying the methods to laboratory or field data. Nevertheless guidelines for the choice of *a priori* optimal methods may be extracted from this study.

7. Acknowledgements

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