CHAPTER 5

COMPUTATION OF BREAKING WAVES WITH A PANEL METHOD

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Abstract

Some basic properties and computational results are presented of a solution method for potential flow problems, with nonlinear waves at the free surface. The results show that stable and accurate results can be obtained for nonlinear wave propagation problems, up to the stage of breaking waves, though no numerical smoothing is applied. Also the interaction with constructions on the bottom can be computed well.

For efficient usage of the available CPU-time, a suitable condition for the time increments in nonlinear computations is given.

1. Introduction

In this paper we will consider the description of propagating and breaking waves with a potential model for the fluid flow.

For the description of potential flow problems in two-dimensional and three-dimensional domains, we have developed an accurate panel method. One of the boundaries of the domain is the free surface, on which nonlinear boundary conditions are imposed for the description of the solution in time.

For accurate descriptions of highly nonlinear wave evolution in complex three-dimensional regions, a model should be used that includes the behaviour of

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the solution in vertical direction. This can be achieved by discretizing the whole fluid domain. Such an approach makes computations on extreme problcms like the development of breaking waves possible.

For incompressible potential flow problems, the field equation reduces to Laplace's equation for the potential. This is an elliptic equation, so that solving the problem on the boundaries of the domain is sufficient. Green's third identity provides a boundary integral relation for the solution on the boundary.
Boundary element methods and panel methods are based on a discretization of Green's third identity in the physical domain. That is why they can be used for modelling the fluid flow in domains with arbitrary boundary shapes.

Time dependence comes into the problem by the time-dependent boundary conditions. These give expressions for the evolution of the free surface position and the velocity potential in time.

Many computations on breaking waves have been discussed in literature (see Peregrine, 1990). However, due to restrictions of the numerical methods, in most cases the chosen initial solution is not physical. For example, an exact solution of a periodic high wave is imposed on a geometry with a smaller depth. In such situations, the numerical results are useful for studying the local solution near the tip of the wave. However, the global solution is not valid in physical situations.

Computations on highly nonlinear wave problems in three-dimensional domains are still very rare. One example can be found in the work by Xü & Yue (1992). They computed the evolution of a breaking wave with a three-dimensional method. A breaking wave was generated by prescribing a non-physical pressure distribution at the surface. However, for practical applications, a method for computing wave evolution due to interactions with a bottom profile is more useful.

Romate has developed a higher order panel method for the computation of three-dimensional potential flow problems with a free surface. As far as the results were presented in his thesis (see Romate, 1989), the method was very well capable of computing the evolution of linear wave solutions and weakly nonlinear waves. However, stable computations of highly nonlinear waves were not possible yet.

On the basis of Romate's work, we have further developed the method for the evolution of highly nonlinear waves. By the improvements, stable results can be obtained for highly nonlinear wave problems up to the stage of the development of breaking waves. For an extensive analysis of algorithms used in the method, the reader is referred to Romate (1989) and Broeze (1993a, 1993b).

In this paper we will briefly describe the method of computation. Furthermore we will show some numerical results on three-dimensional wave propagation problems, including the development of a breaking wave.

2. Numerical solution method

The fluid motion is modelled with a higher order panel method. This method is based on the assumption of a potential flow, i.e. the velocity can be derived from a potential ϕ:

$$\underline{v} = \nabla\phi \tag{1}$$

Due to incompressibility, the velocity potential satisfies Laplace's equation:

$$\nabla^2\phi = 0 \tag{2}$$

In our panel method, the field equation problem is solved by using a boundary integral equation formulation, where Green's third identity is applied:

$$\frac{1}{2}\phi(\underline{x}) = \int_{\partial\Omega} [\phi(\underline{\xi})\,G_n(\underline{x},\underline{\xi}) - \phi_n(\underline{\xi})\,G(\underline{x},\underline{\xi})]\,\mathrm{d}S \quad (\underline{x} \text{ on } \partial\Omega) \tag{3}$$

For the discretization of this equation, the boundary of the domain is divided into a number of smooth panels, with one collocation point near the centre of each panel. Values in the collocation points of a number of adjacent panels are used to determine tangential derivatives.

Green's identity is solved in the physical domain. Up to quadratic variations of the velocity potential and linear variations of its normal derivative are assumed in the discretization of the boundary integral equation. Also contributions due to curvature of the panels are included in the expressions for the influence coefficients. This provides very accurate solutions of the field equation.

Time dependence comes into the problem by the time-dependent boundary conditions.

In our Lagrangian method, at the free surface S_f we have the kinematic condition, expressing that a fluid particle remains at the free surface:

$$\frac{\mathrm{D}x_f}{\mathrm{D}t} \equiv \underline{v} = \nabla\phi \quad \text{at } S_f \tag{4}$$

and the dynamic boundary condition, expressing zero pressure:

$$\frac{\mathrm{D}\phi}{\mathrm{D}t} = -gz + \frac{1}{2}(\nabla\phi)^2 \quad \text{at } S_f \tag{5}$$

where g is the gravitational acceleration, and z is the vertical coordinate.

On the bottom S_f, on fixed constructions and on symmetry boundaries, the no-flux condition should be satisfied:

$$\phi_n = 0 \quad \text{at } S_b \tag{6}$$

where $\underline{n}$ is the outward directed normal with respect to the fluid domain.

On a moving structure S_s, or a wave maker, the normal velocity is prescribed according to the motion velocity $\underline{V}$ of the structure:

$$\phi_n = \underline{V} \cdot \underline{n} \quad \text{at } S_s \tag{7}$$

In order to represent large physical domains with a small computational domain, we can truncate the domain with artificial boundaries S_a. Radiation boundary conditions, that only allow waves travelling out of the domain with minimal reflections, are needed on these boundaries. A simple example is Higdon's (1987) first order condition:

$$\frac{\partial \phi}{\partial t} = -\frac{c}{\cos\alpha}\frac{\partial \phi}{\partial n} \quad \text{on } S_a \tag{8}$$

where $\underline{n}$ is the normal with respect to the vertical boundary, positive in outward direction. This condition perfectly radiates waves that travel at phase speed c and at an angle α with the normal $\underline{n}$. For second order conditions (that can perfectly radiate waves from different directions) the reader is referred to Romate (1992) and Broeze & Romate (1992).

The above described boundary conditions provide values for the time derivatives of the potential and the positions of the collocation points as a function of spatial derivatives of the potential and the vertical coordinate.

Various methods can be used for integrating the problem in time. We have considered the classical fourth order Runge-Kutta method, a third order Taylor method and a 2-stage 2-derivative Runge-Kutta method (that uses first and second order time derivatives on the original and one intermediate time level). We have concluded that the latter method is most favourable for our problem, because it provides accurate results in relatively short CPU times.

An adaptive grid evolution technique was used to obtain desirable grid distributions in time. This is especially important in 3-D computations, where a strongly deformed grid on the lateral boundaries may have negative influences on the accuracy of the solution.

Our method is very suitable for computations of the interaction of highly nonlinear waves with constructions, because there are no restrictions on the shape of the boundaries of the domain. Another favourable property is the stability of the method, so that no artificial dissipation or smoothing terms need to be added (which may have a large influence on the solution during breaking).

Time-step restrictions

One important aspect that we want to discuss here is the chosen time increment in the numerical computations.

We found that stability restriction based on linear theory (see e.g. Dommermuth et al., 1988) are not suitable in computations on highly nonlinear wave propagation problems.

Another frequently applied condition is that the highest order terms in the Taylor time integration method should be below a given value (see e.g. Nakayama, 1990). Such condition is rather arbitrary, and lacks a theoretical basis.

We have derived a more appropriate condition for the full nonlinear problem.

From a perturbation analysis of the solution around the nonlinear solution, the following evolution equations for small disturbances (ε,ζ) in the potential ϕ and elevation η can be derived:

$$\frac{d}{dt}\begin{bmatrix}\varepsilon\\ \zeta\end{bmatrix}=\begin{bmatrix}-i\phi_s k & -g\\ |k|\cos\gamma & 0\end{bmatrix}\begin{bmatrix}\varepsilon\\ \zeta\end{bmatrix} \tag{9}$$

where s represents a tangential coordinate to the surface, k is the wave number of the disturbance and γ is the local angle of inclination of the free surface.

For stability of the numerical method for nonlinear problems, the eigenvalues of the matrix in eq.(9) should be in the stability region of the domain:

$$\frac{i}{2}\left(-\phi_s k \pm\sqrt{(\phi_s k)^2+4g\,|k|\cos\gamma}\right)\Delta t \in R_{\text{stab.}} \qquad \forall k \in [0,\pi/\Delta x] \tag{10}$$

This is a straightforward condition for the time increment. From numerical test computations we found that it is very appropriate: satisfying this condition provides a stable evolution of the solution, whereas instabilities occur if it is violated. For an extensive description of the derivation of condition (10), the reader is referred to Broeze (1993a, 1993b).

Eq. (10) can reduce the required amount of CPU time in highly nonlinear computations with large velocities and fine grids. A maximal time increment can be chosen then.

3. Computational results on periodic wave propagation problems

The panel method provides stable and very accurate results for linear (see e.g. Broeze and Romate, 1992), mildly nonlinear and highly nonlinear wave problems (see Broeze, 1993a, 1993b and Broeze et al., 1993). Computations can be performed on highly nonlinear steady propagating waves near the maximum wave height without instabilities, with errors in the elevation of only a few percents after a large number of wave periods.

Fig. 1 shows results of computations on a typical highly nonlinear propagating plane wave problem. The wave height is 5m on 10m water (over 80% of the maximum), with wave length 60m and Eulerian period 6.5s. This figure shows the results of the computations after 0, 1, 2 and 4 Lagrangian periods. The errors in the elevation are within 2% of the wave height.

The results illustrate that the method is well-capable of accurately describing highly nonlinear propagation.

In order to analyze the accuracy of the method for three-dimensional problems, we have computed the same wave solution, where the propagation direction of the wave is at an angle $\pi/6$ with one of the grid lines.

Fig. 2 shows the numerical boundary shape from 0 to 6s at intervals 2s. Again the results illustrate that no large growing errors occur in the solution.

4. The interaction of a solitary wave with a mild slope

A solitary wave is generated on a numerical wave flume with depth 5m. A weak slope (1:10) truncates the domain in horizontal direction. The solitary wave has a height 3.5m (84% of the maximum) and a phase velocity of 9.0m/s. Initially the fluid is at rest. We computed the interaction of the solitary wave and the slope with a two-dimensional variant of our panel method.

The shape of the domain boundaries after 2s, 4s and 6s is given in Fig.3. This figure shows how the wave starts deforming on the slope. When the wave approaches the end of the slope, it starts to break, and a plunging breaker develops. The shape of the jet from t=6.6s to 7.3s is given in Fig.4. A finer grid would be needed to further continue the computations.

It is noted that in our method, no collocation points occur at the intersection. Also no special condition is applied at the intersection of the slope with the free surface.

In order to analyze the quality of the numerical results of these computations, we have considered a number of theoretical models for the description of breaking waves.

A few theoretical models exist for the evolution of the tip of a breaking wave and the surface region below the jet (see e.g. Longuet-Higgins (1980) and New (1983). These models were developed for breaking waves on deep water. They cannot be applied to our situation, because of the influence of the slope on the solution.

Peregrine (1990) suggested to check the motion of the tip of the wave (it should be in free fall).
Fig.5 shows the evolution of the coordinates of the tip of the wave during the final stage of computation. Obviously, the free-fall model holds very well from t=6.95s (when the jet has developed) to 7.3s.

5. Results on breaking wave computations with the three-dimensional method

We have used the three-dimensional panel method to compute the interaction of the highly nonlinear solitary wave with a smooth construction the bottom. Fig.6 shows the bottom profile in these computation.
Fig.7 shows the grid on surface and on the lateral boundaries of the domain when the wave front has started to overturn. Obviously, the method is capable very well to compute the evolution of the solution so far. The well-arranged grid on the lateral boundaries illustrates the suitability of our adaptive grid motion algorithm. The forming of the jet is more clearly illustrated in Fig. 9.

In order to show the height of the construction in comparison with the domain dept, in Fig. 8 the bottom profile and the surface are depicted.

The computations on this wave cannot be further continued due to the small number of panels near the tip of the wave. A large number of extra panels would be needed to increase the grid density near the jet. However, this cannot be achieved due to memory restrictions on our supercomputer.

6. Conclusions

In this paper we have seen that highly nonlinear waves can be very well described with an accurate panel method. Also real three-dimensional effects can be computed. Efficient solution methods for the spatial problem, for the time integration and for the grid motion are of crucial importance for the success of the method.

The results show that even extreme problems like the development of breaking waves due to a construction on the bottom can be modelled. We think that such models offer new possibilities for studies on nonlinear wave propagation problems in complex three-dimensional domains.

Acknowledgements

The investigations were supported by the Netherlands Technology Foundations (STW). Supercomputer usage was financed by the Dutch National Computing Facilities Foundation (NCF). Professor P.J. Zandbergen is greatly acknowledged for his support and advise.

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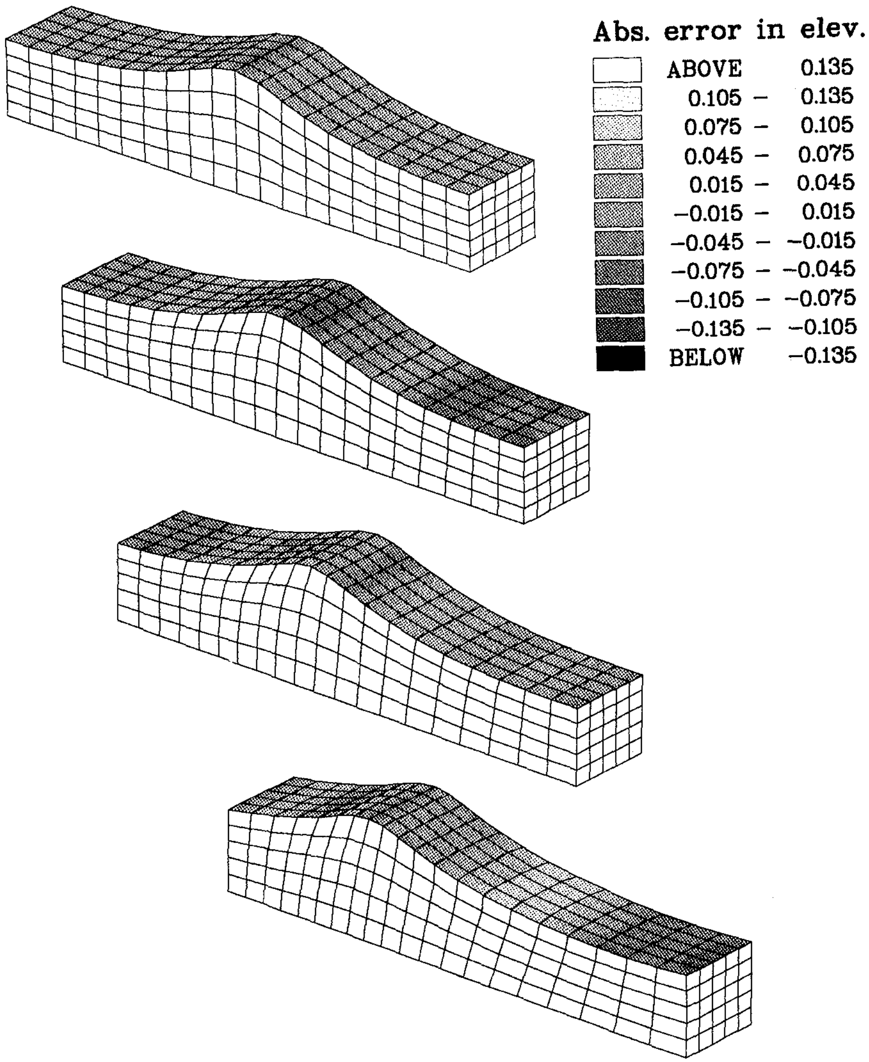

Fig. 1. Numerical results from computations on highly nonlinear wave problem. Shape of boundary and errors in elevation after 0s, 7s, 14s and 28s.

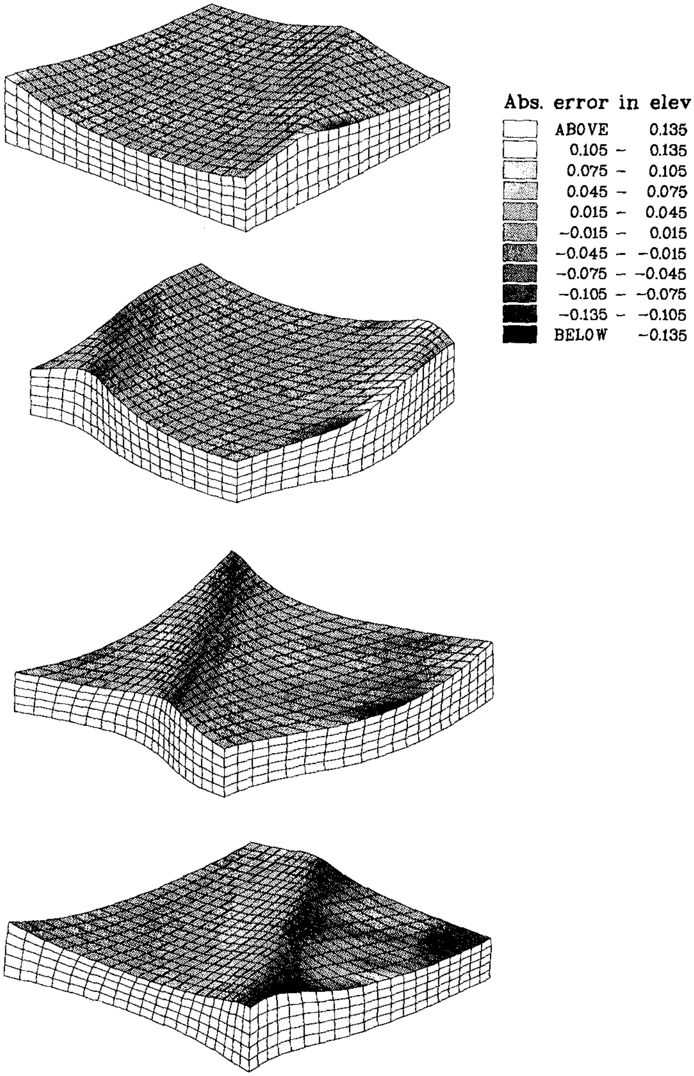

Fig. 2. Numerical results from computations on highly nonlinear wave, propagating at angle $\pi/6$ with one grid direction.
Shape of boundary and errors in elevation after 0s, 2s, 4s and 6s.

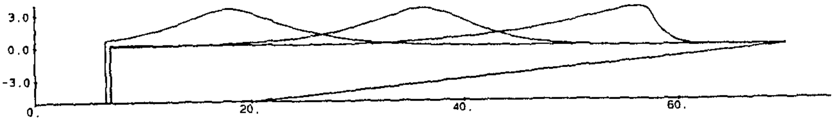

Fig. 3. Shape of the domain boundaries in the 2-D computation of the interaction of a solitary wave with a slope at t=2s, 4s and 6s.

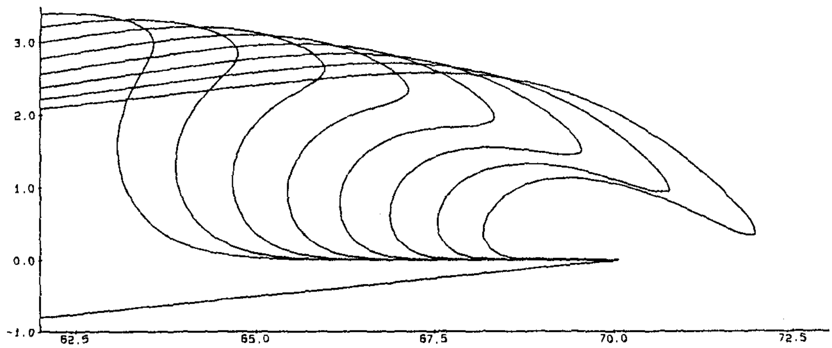

Fig. 4. Jet of the breaking wave in two dimensions from t=6.6s. to 7.3s (every 0.1s).

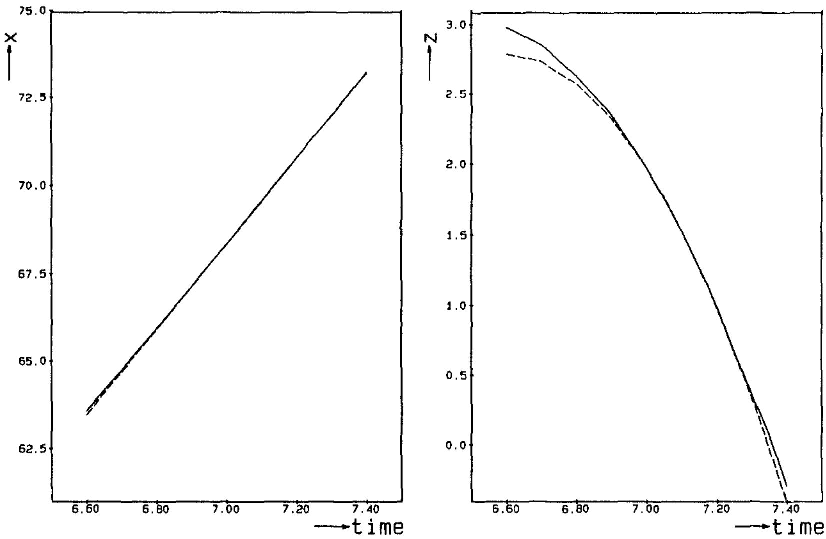

Fig. 5. Evolution of the position of the tip of the wave jet from 6.6s to 7.4s (exact data for particle in free-fall are dotted.

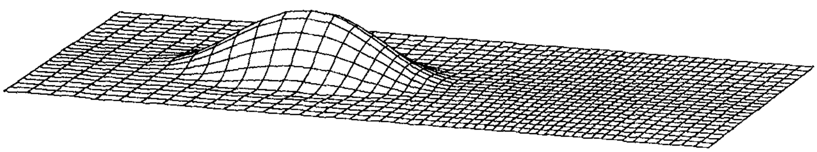

Fig. 6. Shape of bottom profile in 3-D computation on interaction of solitary wave with construction.

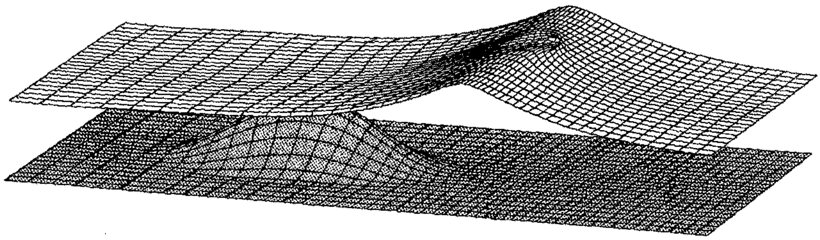

Fig. 7. Shape of the grid on surface and lateral boundaries in computation of a breaking wave in three dimensional configuration.

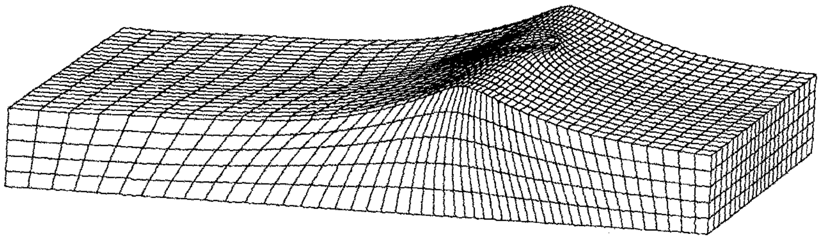

Fig. 8. Surface and bottom profile in computation of breaking wave.

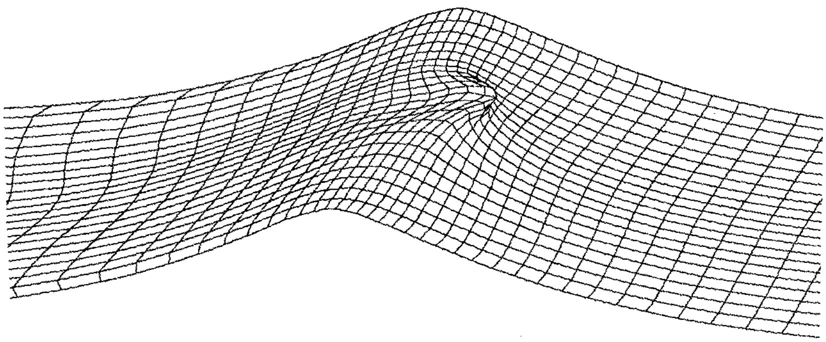

Fig. 9. Surface profile near jet of wave.