CHAPTER 50

INTERACTION of NONLINEAR WAVE and CURRENT

S. Supharatid[1], H. Tanaka[2], and N. Shuto[3]

Abstract

A nonlinear wave-current model is developed to obtain the velocity and shear stress. The model employing an "empirical velocity deviation" which introduces the modification of the time-mean velocity caused by waves is proposed. With an application of a time-invarient linear eddy viscosity and making use of the truncated Fourier series, a boundary value problem is formulated and is solved numerically. Comparisons among many cases of measured and predicted results show reasonable agreements.

1 Introduction

Knowledge on the combined wave and current in the boundary layer flows has been investigated theoretically and experimentally by many reasearchers. However, the modifications of the fundamental characteristics of the waves and current resulted from their interactions have been rarely mentioned.

At present, there are some numerical models which incoporate currents into their formulations. Dalrymple(1974) proposed a numerical perturbation while Teles da Silva and Peregrine(1988) used a boundary integral method to simulate the flow fields under waves with a linear shear current. Kishida and Sobey(1988) modified the Stokes theory to include the effect of a linear shear current. These wave-current model are still needed to be verified with experimental data or field measurements.

Recently, Tanaka(1989) extended his one-layer model to account for the nonlinear wave by a modifying Dean's stream function.(Dean, 1965) This model, however, does not succeed to predict the time-mean velocity near the water surface.

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1 PTT Exploration and Production Co., Ltd., Bangkok, Thailand.
2 Assoc. Prof., Asian Institute of Technology, Bangkok, Thailand.
3 Prof., Disaster Control Research Center, Faculty of Engineering, Tohoku Univ., Sendai, Japan.

The present work was made by the incorporation of a modified current profile instead of the conventional log-current profile. The usual boundary value problem, with the application of the eddy viscosity approach and Fourier wave theory, is formulated in term of the stream function. The predicted velocity and bottom shear stress are compared with auther's experimental data.

2 Boundary value problem

Prior to the formulation, the following assumptions should be made.

1) Waves propagate without change in form.
2) Wave height is known.
3) Current is uniform in the flow direction.
4) Flow is incompressible and fully turbulent.

Consider a two-dimensional periodic wave train propagating on a steady uniform current over a horizontal bottom(see Fig. 1). Equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

where (u, w) are $(\partial\psi/\partial z, -\partial\psi/\partial x)$ for the stream function

The boundary conditions to be satisfied in a moving reference frame with the wave celerity, c are as follows:

Non-slip condition at the bottom,

$$\frac{\partial\psi}{\partial z} = 0, \quad z = z_0 \tag{2}$$

No flow through the bottom,

$$\psi(x,z) = 0, \quad z = z_0 \tag{3}$$

No flow through the water surface(KFSBC),

$$\psi(x,z) = Q, \quad z = \eta(x) \tag{4}$$

Dynamic free surface boundary condition(DFSBC),

$$\frac{1}{2}\left(\frac{\partial\psi}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial\psi}{\partial z}\right)^2 + gz = R, \quad z = \eta(x) \tag{5}$$

Periodic lateral boundary condition with respect to wave length, L

$$\psi(x+L,z) = \psi(x,z) \tag{6}$$

The mass conservation requires the invarient mean water level, on taking the x-co-ordinate with the origin at the wave crest,

$$\int_0^{L/2} \eta dx = D \qquad (7)$$

The wave height, H is assumed known and is defined as the vertical difference between wave crest and wave trough.

$$\eta(0) - \eta(L/2) = H \qquad (8)$$

where η = water surface elevation measured from the bottom
Q, R = constants
D = water depth
zo = bottom roughness height
H, L = wave height and wave length.

3 Stream function formulation

The stream function for a combined wave-current motion is expressed as

$$\psi = \psi_C + \psi_{WIR} + \psi_{WR} \qquad (9)$$

where ψ_C, ψ_{WIR}, and ψ_{WR} correspond to the stream functions for a steady current, irrotation(WIR) and rotation(WR) waves respectively.

3.1 Stream function for a steady current

The stream function for a steady current, ψ_c is obtained from the normal momentum equation with the use of a time-invarient linear eddy viscosity, $\tau_c/\rho = \kappa u_{*wc} z \partial u_c/\partial z$. However, with the introduction of an experimentally determined "velocity deviation, Δu_{cd}" as the water surface boundary condition, the final solution can be written in term of the time-mean velocity, u_c as

$$u_c = \frac{u_{*c}|u_{*c}|}{\kappa u_{*wc}} \ln\frac{z}{z_0} \pm \Delta u_{cd} \frac{z-z_0}{D-z_0} \qquad (10)$$

where u_{*c} = friction velocity for the steady current
u_{*wc} = friction velocity for the combined flow
κ = 0.41.

Symbols + and - are for waves with the following and opposing currents, respectively. Here, Δu_{cd} (see Fig. 2) is expressed empirically in term of the relative strength of waves to current, Uo/Uc by

$$\frac{\Delta u_{cd}}{|U_c|} = -\frac{1}{5}\frac{U_0}{U_c} \qquad (11)$$

where Uo is the velocity amplitude at the bottom of the wave-induced motion(linear theory). Uc is the depth-averaged current velocity,. symbol | | means the absolute value.

It is clearly seen from Eq. (10) that the neglection of the second term on the right hand side reduces to a familiar log-current profile similar to that of the steady current alone.

3.2 Stream function for waves

Equations (12) and (13) represent the stream function for irrotational and rotational waves, respectively. For j = 1, they reduce to the fundamental forms of the first-order solution obtained by Fenton(1980) and Grant & Madsen(1979).

$$\psi_{WIR} = B_0(z-z_0) + \sum_{j=1}^{n} \frac{B_j \sinh\{jk(z-z_0)\}\cos(jkx)}{\cosh(jkD)} \qquad (12)$$

$$\psi_{WR} = \frac{K_0 k}{\omega}\sum_{j=1}^{n} \frac{B_j}{R_{j0}\cosh(jkD)} F_j \qquad (13)$$

where Bo, B1,......Bn = unknown constants

$K_0 = \kappa u_{*wc}$, k = Wave number, ω = Angular frequency of wave

$R_{jo} = 2(\mathrm{Ker}^2\xi_{j0} + \mathrm{Kei}^2\xi_{j0})$ $\quad \xi_{j0} = 2(j\omega z_0/K_0)^{1/2}$

$\xi_j = 2(j\omega z/K_0)^{1/2}$

$$\begin{aligned} F_j = & \ \xi_j(\mathrm{Ker}'\xi_j \mathrm{Kei}\xi_{j0} - \mathrm{Kei}'\xi_j \mathrm{Ker}\xi_{j0})\cos(jkx) \\ & - \xi_j(\mathrm{Ker}'\xi_j \mathrm{Ker}\xi_{j0} + \mathrm{Kei}'\xi_j \mathrm{Kei}\xi_{j0})\sin(jkx) \\ & + \xi_{j0}(\mathrm{Kei}'\xi_{j0} \mathrm{Ker}\xi_{j0} - \mathrm{Ker}'\xi_{j0} \mathrm{Kei}\xi_{j0})\cos(jkx) \\ & + \xi_{j0}(\mathrm{Kei}'\xi_{j0} \mathrm{Kei}\xi_{j0} + \mathrm{Ker}'\xi_{j0} \mathrm{Ker}\xi_{j0})\sin(jkx). \end{aligned}$$

Kei, Ker = Kelvin functions of zero order.

Subscript "j" denotes the order of the finite Fourier expansions. Subscript "o" denotes values evaluated at the bottom except Bo which is one of unknown constants, Bj. Symbol " ' " means the 1st derivative of Ker and Kei with respect to ξ.

4 Solution method

It is convenient to make variables dimensionless with respect to the mean water depth, D, and gravitational accelcration, g as shown Table 1.

The stream functions introduced above satisfy the equation of continuity, non-slip condition at the bottom, no vertical velocity at the bottom, and the periodicity with the wave length, L. Therefore, equations to be solved are kinematic and dynamic conditions at the free surface Eqs. (4) and (5), invaricnt condition for the mean water level Eq. (7), and the wave height Eq. (8).

Strictly speaking, the dynamic boundary condition at the free surface(Eq. (5)) is only applicable for an inviscid flow but not to the present case. This is because the flow under consideration inevitably yields the energy dissipation casued by viscous and turbulent motions. However, most of the energy dissipation occurs within the very thin region near the bottom, and outside this region the flow nearly behaves like an inviscid flow, making Eq. (5) applicable to the present case.

Now, we have (2n+4) equations for (2n+7) unknowns(u_{*wc}, Bo, c, k, Q, R, η_j(j=0, n), and Bj(j=0, n). Thercfore, three more equations are introduced as follows.

The value of u*wc has to be solved by iteration on an assumption of constant stress layer approximation near the bottom, i.e.

$$\tau_{wc} = \rho u^2_{*wc} = \rho K_0 z_0 \left.\frac{\partial^2 \psi}{\partial z^2}\right|_{z=z_0} \quad (14)$$

where τ_{wc} = maximum bottom shear stress for the combined flow.

The first definition of wave celerity, c is related to the time-mean velocity, $\bar{u}$ and the Eulerian velocity, UE as

$$U_E = c + \bar{u} \quad (15)$$

where

$$\bar{u} = \frac{1}{L}\int_0^L \frac{\partial \psi}{\partial z} dx = B_0 + \partial \psi_c / \partial z \quad (16)$$

The z-dependent terms in Eq. (15) with Eq. (16) inserted should be balanced with each other. This yields

$$c = -B_0 \quad (17)$$

Now, the problem formulation provides (2n+7) non-linear equations for (2n+7) unknowns. The solutions are obtained by using the Newton-Raphson method. The solution flowchart is shown in Fig. 3

5 Results.

5.1 Time-mean velocity over a rough bed

The computed results are compared with the measurements conducted by authers in term of the time-mean velocity in Fig. 4. Modifications caused by waves are dependent on the flow direction, i.e. decrease and increase of the time-mean velocity near the free surface for waves with following and opposing currents. Introducing the "velocity deviation", Δu_{cd} results in relatively good agreement above a height of about 100 mm.

In fact, discrepancies below this height may be caused by violent generation and transportation of vortices excited by triangular roughnesses on the bottom. With the use of the Nikuradse roughness to represent the bottom, the present model does not allow to predict such a 3-D phenomenon.

5.2 Ensemble-averaged velocity over a rough bed

Profiles of the ensemble-averaged velocity at phases of acceleration and deceleration are shown in Fig. 5(a)(following flow) and 5(b)(opposing flow). Agreements are reasonably well except in the vicinity of the bottom and the overshooting zone. However, the height of overshooting is well predicted. This near bottom phenomena, in fact, suggests that any model employing the closure assumption of a time-invarient eddy viscosity will suffer the same effect.

To improve the solution near the overshooting zone, it is necessary to adopt more realistic models such as the $K-\varepsilon$ model (Supharatid et. al., 1992). This certainly requires a considerably time-consuming computation.

5.3 Bottom shear stress over a smooth bed

The bottom shear stress for a smooth bed was measured directly using a hot-film sensor. Comparisons with the present theory are made in Fig. 6 for both waves with following and opposing currents. In the figure, two thin lines denote values of $<\tau_b>+\sigma_\tau$ and $<\tau_b>-\sigma_\tau$ of the measured data where σ_τ is the standard deviation.

Throughout the wave cycle, the model generally gives satisfactory agreement, except for the results near the phase of wave trough in the case of waves with the opposing current. The distinct phase advance of the bottom shear stress is observed and predicted rather well by the model.

6 Conclusions

A model for nonlinear wave interacting with the steady current is developd to obtain the velocity as well as the shear stress. The effect of waves on the current is introduced in term of the "velocity deviation". The time-mean velocity, as a result, is given by a summation of logarithmic and linear profiles.

Good agreements are generally found between the predicted results and measured data. Velocity profiles are prdicted rather well except in the vicinity of the "overshooting zone" where a time-invarient eddy viscosity assumed by the model is not applicable. Time histories of the bottom shear stress are also predicted reasonably well.

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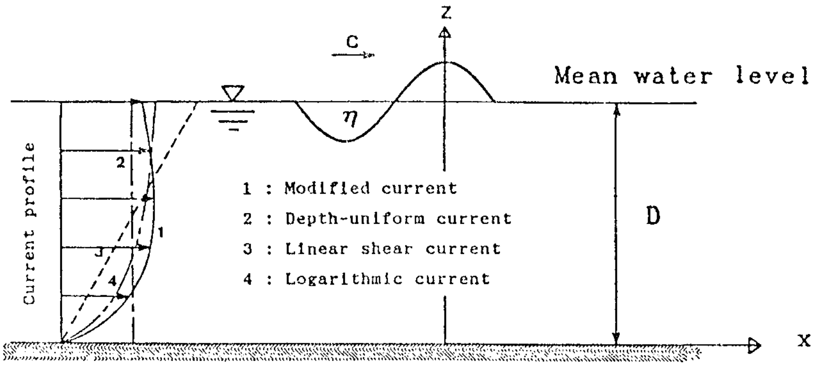


Fig. 1 Definition sketch

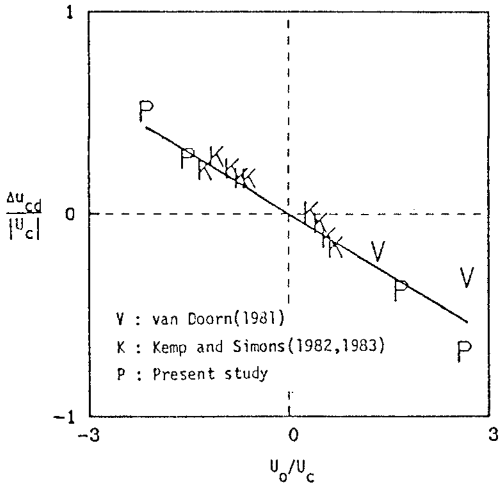


Fig. 2 Velocity deviation

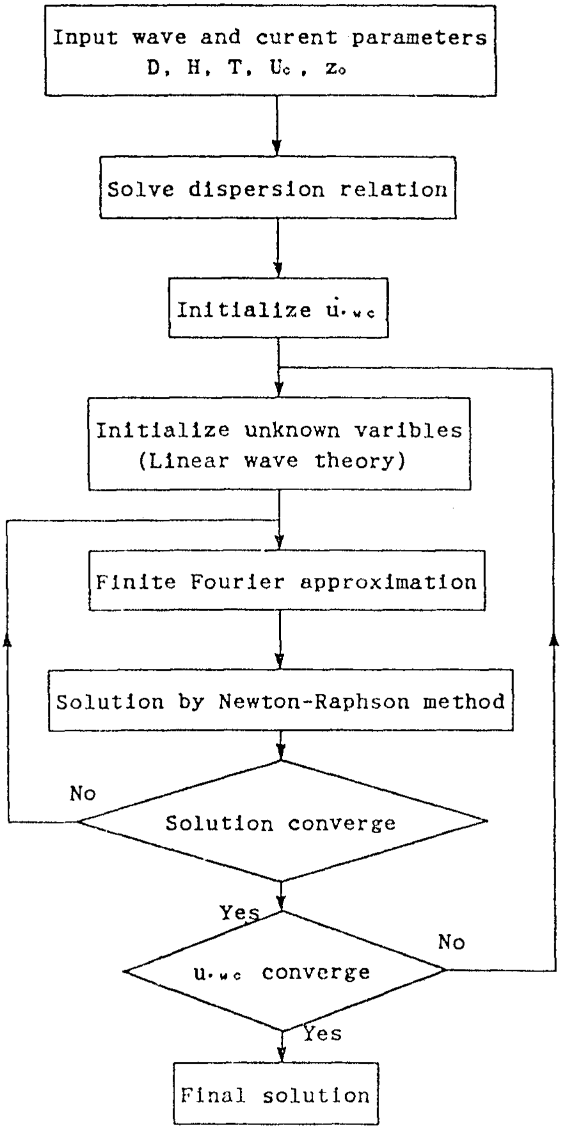
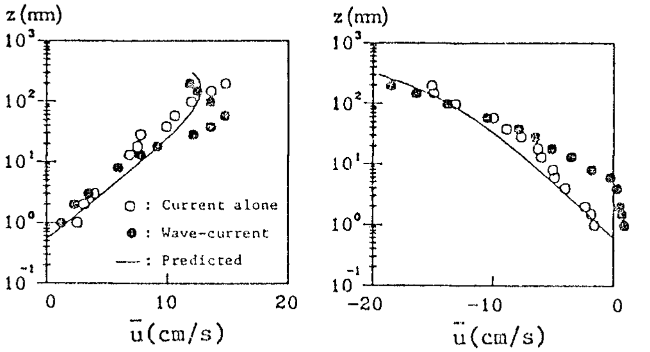


Fig. 3 Solution Flow chart

Table 1 Dimensionless variables

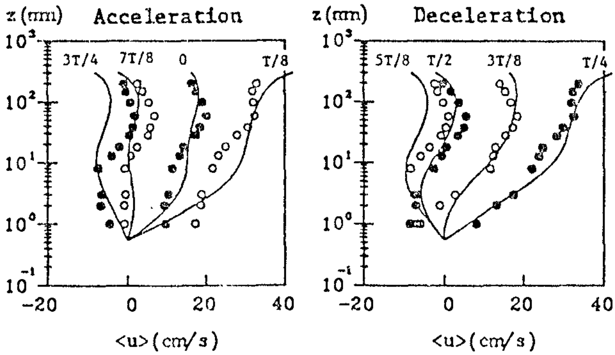
Dimensional variables	Dimensionless variables
x	x/D
z	z/D
η	η/D
ψ	$\psi/(gD^3)^{1/2}$
Q	$Q/(gD^3)^{1/2}$
R	R/gD
k	kD
c	$c/(gD)^{1/2}$
u	$u/(gD)^{1/2}$
w	$w/(gD)^{1/2}$



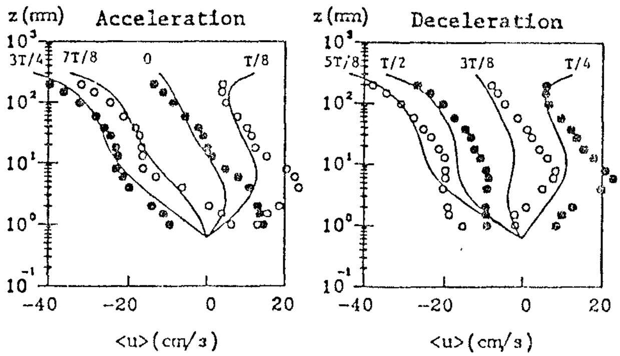
(a) Following flow (D=0.30 m, H=9.4 cm, T=1.3 sec, Uc=11.9 cm/s)

(b) Opposing flow (D=0.30 m, H=9.7 cm, T=1.3 sec, Uc=-14.7 cm/s)

Fig. 4 Time-mean velocity(rough bed)

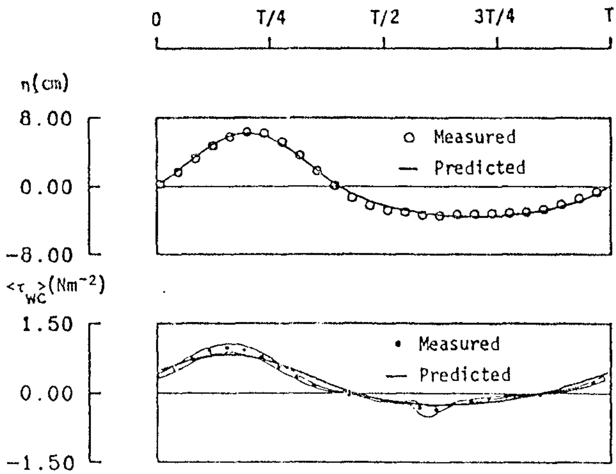


(a) Following flow
(D=0.30 m, H=9.4 cm, T=1.3 sec, Uc=11.9 cm/s)

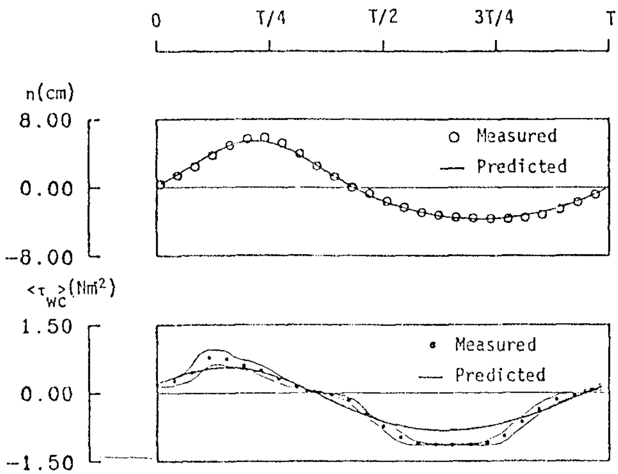


(b) Opposing flow
(D=0.30 m, H=9.7 cm, T=1.3 sec, Uc=-14.7 cm/s)

Fig. 5 Ensemble-averaged horizontal velocity(rough bed)



(a) Following flow
(D=0.30 m, H=9.4 cm, T=1.3 sec, Uc=14.0 cm/s)



(b) Opposing flow
(D=0.30 m, H=9.5 cm, T=1.3 sec, Uc=-14.9 cm/s)

Fig. 6 Time series of the bottom shear stress(smooth bed)