CHAPTER 67

Occurrence Distribution of Maximum Wave Height including Wave Grouping Effect

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Abstract

The effect of wave grouping on the occurrence frequency distribution of maximum wave height was investigated on the basis of field observation data. The observed maximum wave height distribution fitted the distribution derived from Weibull distribution and was closely related with not only the wave grouping but also wave non-linearity. In the case of large degrees of wave grouping and wave non-linearity, it was found that the maximum wave height was greater than that estimated by the Rayleigh distribution.

1. Introduction

Since the wave grouping may influence several important issues of coastal engineering, many researchers have been studying the characteristics of wave grouping. The first treatment of wave grouping was the analysis of statistical properties of run length (Goda, 1970; Kimura, 1980; Longuet-Higgins, 1984). The characteristics of run length of field data have been also studied (Goda, 1983). Besides the statistics of run length, the square of water surface elevation has been commonly used for wave grouping analysis (Funke and Mansard, 1979). Recently the concept of the modulational instability has been used for describing the wave

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grouping process (Mase and Iwagaki, 1986) and the spatial change of wave group properties due to wave group propagation has been investigated (Ukai, 1990). Also, the relationship between wave grouping and occurrence frequency distribution of wave height has been studied by Mase(1989).

According to Mase's study, it was suggested that the frequency distribution of wave height becomes wider as the degree of wave grouping is higher. This result means that the wave grouping effect has to be considered when the maximum wave height is adopted as the design wave height because the maximum wave height becomes higher, when the occurrence frequency distribution becomes wider. This will be an important problem, especially for the design of deep water structures.

In this study, we aim to investigate the relationship between wave grouping and occurrence frequency distribution of wave height on the basis of field observation data, and to study the effect of wave grouping on the occurrence frequency distribution of the maximum wave height.

2. Field observations

Field observations were conducted at the port of Kashiwazaki-Kariwa nuclear power plant from January through March in 1989. This power plant was constructed in both Kashiwazaki city and Kariwa town, so we call it shortly Kashiwazaki-Kariwa. Figure 1 shows the location of observation site which is located in Niigata Prefecture in Japan and faces to the Sea of Japan. Along the WNW direction from this site, there is Noto peninsula and in the direction of N there is Sado island. Therefore the incident wave direction to this site is restricted to a range from WNW-direction to N-direction. Figure 2 shows the port of Kashiwazaki-Kariwa nuclear power plant. This port has an area of about 3 km wide along the shoreline and about 1.3 km long normal to shoreline. The wave height and direction were measured at the water depth of 13m by using ultrasonic wave gage and electromagnetic current meter. The locations of observation points are shown in this figure. More than 200 time series of wave data, 20 minutes each, were recorded every two hour. During this observation, the observed maximum significant wave had height of 3.45m and period of 8.3 s. However, waves higher than 2m were observed only a few times. Therefore wave data which were obtained in January and March in 1987, as part of a long term study by Tokyo Electric Power Co. Inc. were also analyzed. In this period, waves had relatively large height and maximum significant wave height was 5.13m with period of 9.9 s. In this long term observation, ultrasonic wave gage was laid 1.5 km far from the shoreline.

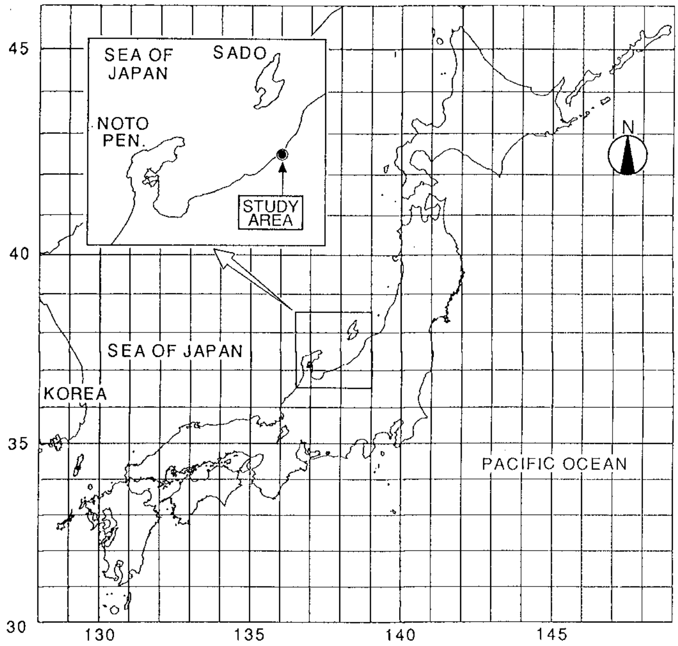

Figure 1. The location of the investigation site.

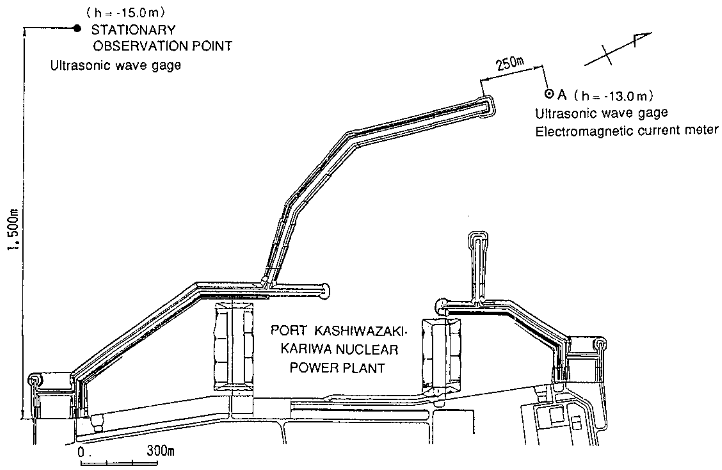

Figure 2. Location of observation points
(Port of Kashiwazaki-Kariwa Nuclear Power Plant).

3. Wave Grouping

The wave grouping properties were discussed by using the groupiness factor, *GF*. The relationship between *GF* and the wave dispersion *kh*, non-linearity *ka*, and Ursell's parameter *Ur* were investigated, where *k* is the wave number, *h* the water depth and *a* the wave amplitude. The wave amplitude means half of significant wave height in this study.

Figure 3 shows the results of wave grouping analysis. Values of *GF* are plotted against the relative water depth, *kh*. The figures are classified for different ranges of the wave non-linearity, *ka*. From these figures, the so-called "shallow water effect", which is the phenomenon that *GF* becomes smaller as *kh* becomes smaller, can be confirmed. It is also confirmed that the non-linearity effect exists. In the case of large *kh*, *GF* becomes large as *ka* becomes large and in the case of small *kh*, *GF* and its scatter become smaller as *ka* becomes large.

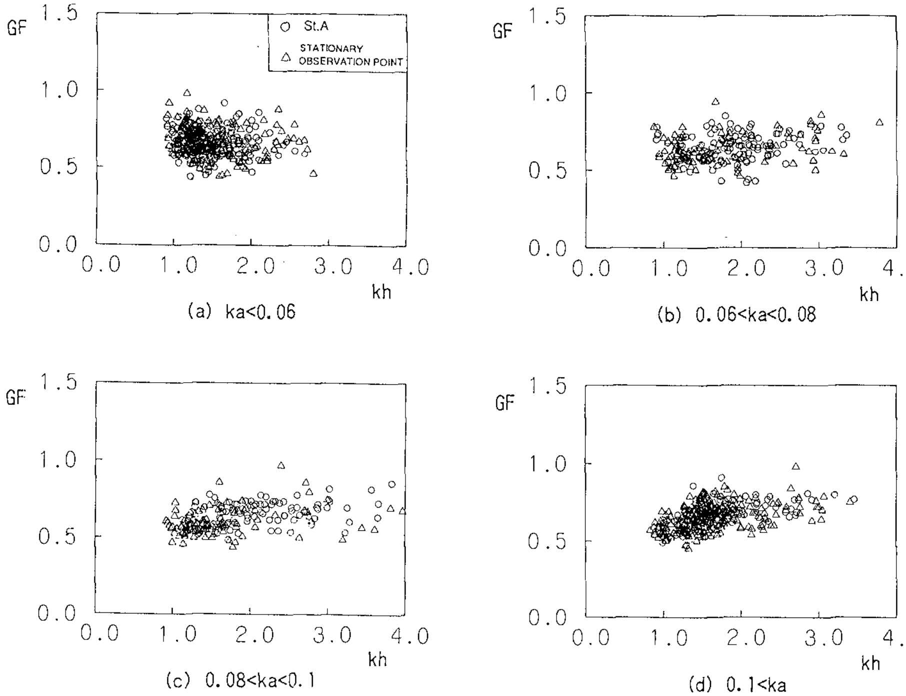

Figure 3. The relationship between relative water depth *kh* and groupiness factor *GF*.

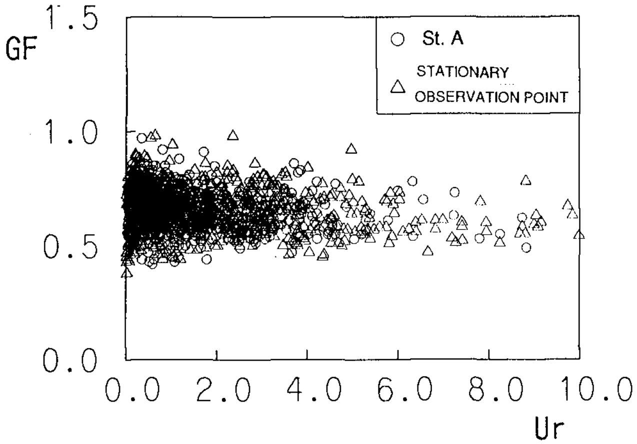

Figure 4. The relationship between Ursell's parameter *Ur* and groupiness factor *GF*.

Next, the relationship between *GF* and Ursell's parameter was investigated. Values of *GF* are plotted against Ursell's parameter in figure 4. This figure shows that *GF* becomes smaller and its distribution becomes narrower as the Ursell's parameter becomes larger. From these analyses of wave grouping, it was found that non-linearity of waves affects the wave grouping.

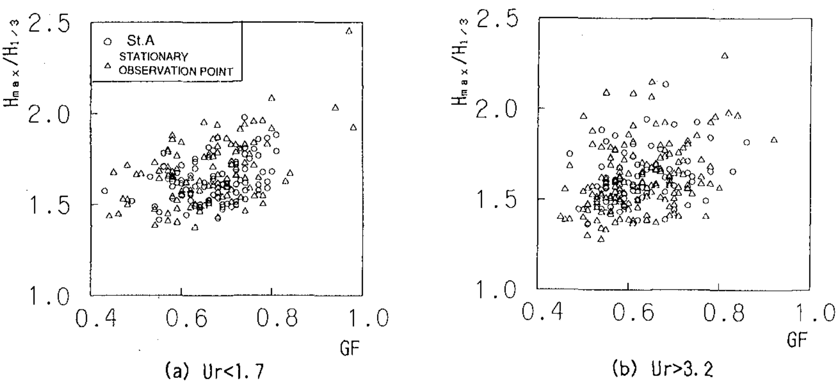

Figure 5. The relationship between groupiness factor *GF* and maximum wave height.

In this study, our main interest was in the occurrence distribution of maximum wave height. Therefore the relationship between maximum wave height and GF was investigated. Figure 5 shows this relationship. The ordinate is $H_{max}/H_{1/3}$ and the abscissa is GF in this figure. It was found that weak correlation is available between GF and maximum wave height. In the case of large Ursell's parameter, distribution of maximum wave height is much broader. However, it was found that the lower limit of maximum wave height is almost the same in both, but the upper limit of maximum wave height is larger in the case of large Ursell's parameter. This results suggested that in the case of large Ursell's parameter higher maximum wave height occurs even if GF has the same value. Hence, also the occurrence frequency of maximum wave height is affected by the non-linearity of waves.

4. Wave height distribution

Following the study of Mase(1989), the Weibull distribution was adopted to the observed wave heights. The Weibull distribution for ocean waves was given by Kimura(1981) and it's representation is

$$p(x) = \frac{m}{2\phi} x^{m-1} \exp\left(-\frac{x^m}{2\phi}\right) \tag{1}$$

where, ϕ is the normalized factor, which is changed by a reference wave height, and x is the normalized wave height. ϕ and x were expressed as follows.

$$\phi = \frac{1}{2}\left[\Gamma\left(\frac{m+1}{m}\right)\right]^{-m} \tag{2}$$

$$x = \frac{H}{\overline{H}} \tag{3}$$

in which $\Gamma(*)$ is the gamma function. The probability density is changed by the shape factor m. When m is small the distribution is wide and when m is large the distribution is narrow. The reference wave height employed here is the mean wave height $\overline{H}$.

The shape factor m of the observed waves was estimated by the maximum likelihood procedure proposed by Cohen (1960), which was also used by Mase (1989).

$$m = \frac{N_W}{\frac{1}{2\phi}\sum_{i=1}^{N_W} x_i^m \ln x_i - \sum_{i=1}^{N_W} \ln x_i} \tag{4}$$

where x_i means each wave height in a wave train normalized with the mean wave height and N_W is the number of waves.

The observed wave height distributions were shown in the form of histograms expressed in terms of probability density. Figure 6 presents an example of wave height distribution of an observed wave train. The fitted Weibull distribution is drawn as smooth curves in the figures. Figure 6(1) presents a very narrow distribution case and (4) is a very wide one. Figure

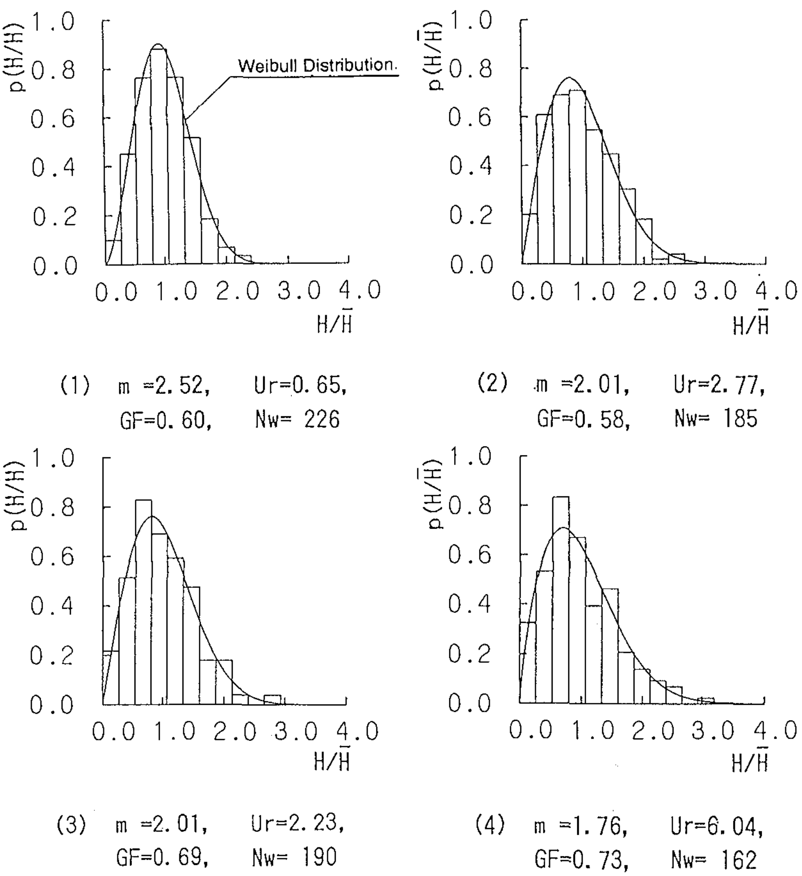

Figure 6. The occurrence distribution of wave height with Weibull distribution.

6(2) and (3) have the same shape factor. In these figures, the number of waves are 150 through 200. Considering the reliability of a small sample, the agreement between the observation results and fitted curve by means of Weibull distribution is good.

Since the application of Weibull distribution to observed wave height distribution was confirmed, the relationship between *GF* and shape factor *m* was investigated. Since *GF* is defined as the coefficient of variation of smoothed instantaneous wave energy history, distributions of wave height become broad as *GF* becomes large. Thus, it seems that a negative correlation should be available between the shape factor and *GF*. Mase(1989) proposed this relationship empirically.

$$m = 3.44 - 1.99GF \tag{5}$$

Figure 7(1) is the result of comparison between all observed data and the above empirical relation. It was confirmed that Mase's relationship seems to describe the mean of the observation data.

After careful analysis of the relationship between *m* and *GF*, it was found that the relationship between *m* and *GF* depends on Ursell's parameter. Figures 7(2), (3) and (4) show the case that *Ur* is less than unity, the case that *Ur* is from 1 to 3, and that *Ur* is greater than 3, respectively. For the same value of *GF*, the greater the Ursell's parameter is, the smaller *m* becomes. That is, in the case of larger Ursell's parameter, the occurrence frequency distributions of wave height become wider and therefore the maximum wave heights become larger, even if *GF* is the same. It should be mentioned that the wave height distributions become wider due to wave non-linearity.

According to these figures, even for the case of small Ursell's parameter, when *GF* is above 0.7 the shape factor *m* is less than 2.0. The Rayleigh distribution, which is often selected as the wave height distribution, has shape factor of 2. Therefore, the Rayleigh distribution underestimates the occurrence frequency distribution of higher wave heights, when *GF* is greater than 0.7.

5. Maximum wave height

Since the Weibull distribution successfully described the observed occurrence distribution, comparison was made between the occurrence distribution of maximum wave height derived from Weibull distribution and the observed one. Maximum wave height distribution on the basis of Weibull distribution can be expressed by equation (6). This equation is derived using the same manner proposed by Longuet-Higgins (1952), with the assumption of independence of each wave from others.

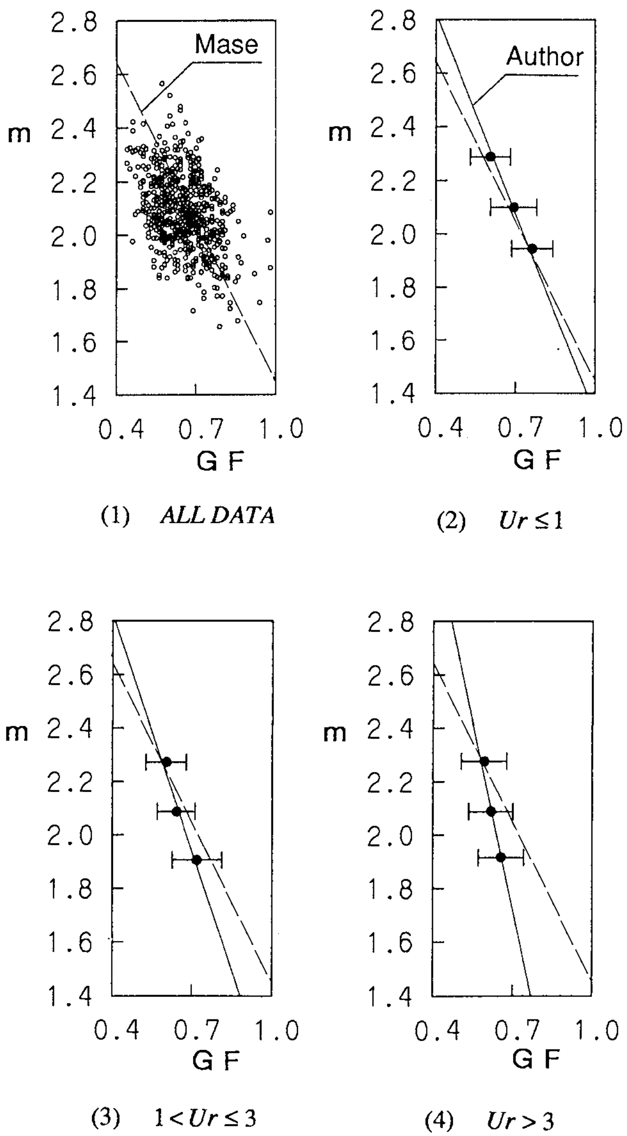

Figure 7. The relationship between groupiness factor GF and shape factor m of Weibull distribution.

$$p*(X) = mN_w\beta(\beta X)^{m-1}\left[1-\exp\left\{-(\beta X)^m\right\}\right]^{N_w-1}\exp\left\{-(\beta X)^m\right\} \tag{6}$$

$$\beta = (-\ln q)^{1/m} + \Gamma(1/m, -\ln q)/(qm) \tag{7}$$

$$X = H_{\max}/H_q \tag{8}$$

In these equations, $\Gamma(*,*)$ is the incomplete gamma function of the second kind, H_q is a reference wave height and it is taken as the mean value of the highest qN_W waves, where q is less than or equal unity. When q is 1/3, H_q means significant wave height and q is 1, H_q means the mean wave height.

The results of the comparison between the observed maximum wave height and those predicted from Weibull distribution were described for four ranges of the shape factor. The lower part of figure 8 shows occurrence distribution of wave height with the Weibull distribution.

It was found that Weibull distribution fits these histograms very well. The upper part of the figure shows the distribution of the maximum wave height normalized by the significant wave height. In these figures, the three solid curves indicate the distribution derived from Weibull distribution with minimum, mean and maximum number of waves in the wave records. The occurrence frequency distribution of maximum wave height derived from Weibull distribution for the mean number of waves showed good agreement with histograms of the observed maximum waves except for the case of small value of the shape factor. When the shape factor is small, *GF* is large and the correlation between waves in a wave train is generally high. So, the agreement between the derived and observed distribution is not so good in this case.

Figure 9 shows the relationship between $H_{max}/H_{1/3}$ and m. The mean values of observed data are plotted by dots and their positive and negative standard deviations are descried by lines. In this figure, the expected value of $H_{max}/H_{1/3}$ is drawn by dashed line. This expected value was calculated for 196 waves which is the mean number of waves in the observed records. Excluding the case that the shape factor is less than 2, calculation results described the observed data well. However, even in the case that the shape factor is less than two, the deviation between calculated results and observed results is small. Then, it may be concluded that the maximum wave height can be estimated by using the probability density derived from Weibull distribution. The shape factor of Weibull distribution may be estimated from *GF* and *Ur*.

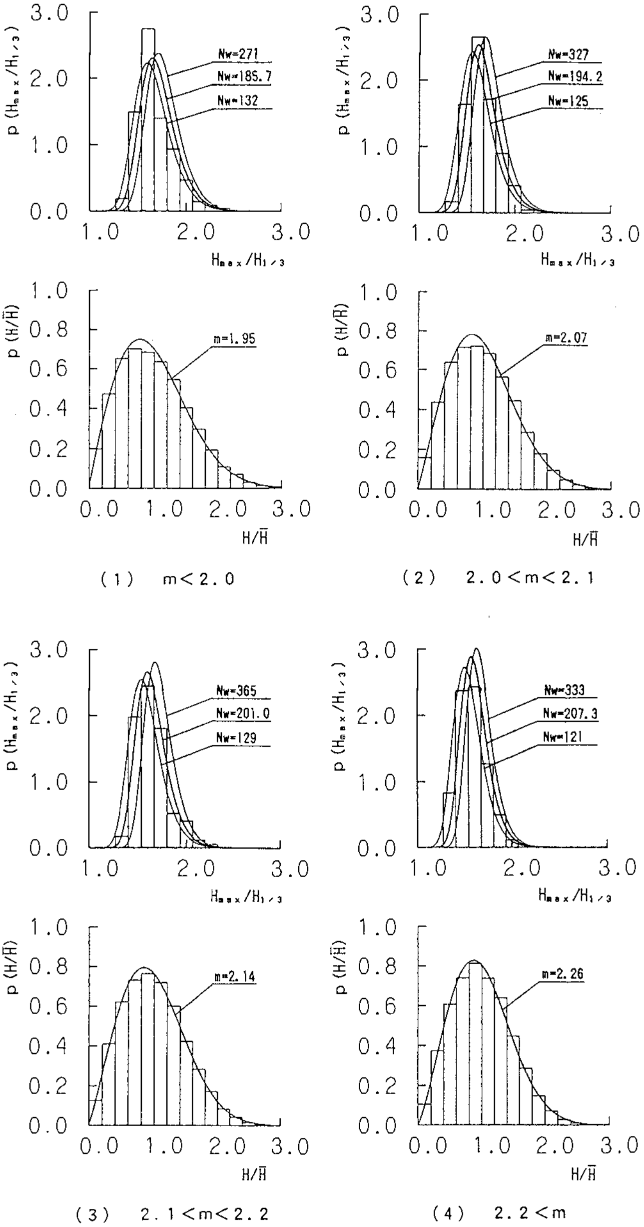

Figure 8. Comparison between the observed maximum wave heights and those predicted from Weibull distribution.

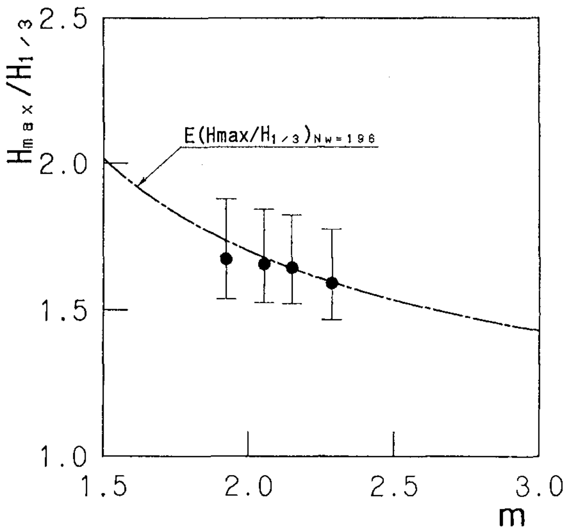

Figure 9. The relationship between shape factor *m* of Weibull distribution and maximum wave height.

6. Conclusions

The effect of wave grouping on the occurrence frequency distribution of maximum wave height was investigated through the field observations.

The conclusions of this study are summarized as follows.

(1) Weibull distribution fits the observed wave height distribution very well.

(2) The wave height distribution is affected by the wave groupiness.

(3) The relationship between *GF* and the shape factor *m* of wave height distribution is affected by the Ursell's parameter.

(4) Maximum wave height distribution derived from Weibull distribution fits the observed maximum wave height.

(5) The maximum wave height can be estimated by the shape factor *m*. Then the maximum wave height is connected with the wave groupiness by using the shape factor with the Ursell's parameter.

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