CHAPTER 119

PROFILE CHANGES OF ROCK SLOPES BY IRREGULAR WAVES

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Abstract

Experimental data has been obtained to test the abilities and limitations of existing and new empirical predictive methods for breakwater profile changes under random waves. The thirty data points obtained within this study describe a structure that is neither solely statically nor dynamically stable, and have indicated the difficulties of empirical formulas in accounting for the complex wave and structural interactions affecting breakwater profile development. Data analyses have indicated that current empirical formulas are not able to predict the profile changes very accurately for the data obtained herein. However, qualitative understanding can be obtained based upon the application and adjustment of the formulas to the structure tested herein.

Introduction

Design of conventional and berm breakwaters requires the prediction of structural profile changes caused by irregular waves. These changes can be defined by the damage suffered by a statically stable structure or through the description of the structural profile evolution experienced by a dynamically stable structure. This paper examines experimental profile changes for structures overlapping the dynamic/static stability boundary, and evaluates the predictive capabilities of existing and new empirical formulas tested in comparison to profile change data obtained through specific model tests. The new data set is used to clarify the abilities and limitations of the current design methods, and possibly lend insight into the development of an improved method of design for the structures of interest. An attempt is also made to elucidate similarities and differences existing between rock slopes and sand beaches since different formulas

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proposed for different sizes of cohesionless materials should be synthesized and generalized.

New Static Stability Formula

van der Meer (1987) conducted extensive irregular wave tests and proposed an empirical formula for the static stability of uniform rock slopes, for which only minor profile changes are allowed under design wave conditions. This formula yields a prediction of the damage suffered by a structure under a given wave climate, where damage is qualitatively described as the displacement of armor stones. This damage level, S, is defined as the cross-sectional eroded area, A, normalized by the square of the nominal armor unit diameter based on a median stone mass, D_{n50}, as $S=A/D_{n50}^2$. van der Meer's formula can be arranged to express the structural damage level, S, as a function of $\cot\alpha$ = cotangent of the structural slope angle α, N = number of individual waves, P = empirical permeability coefficient, ξ_m = surf similarity parameter based on the significant wave height, H_s, and the mean wave period, T_m, and N_s = stability number defined as $N_s = H_s/[(s\text{-}1)D_{n50}]$ where s = specific density of the armor unit. This stability number may be used to classify structures as either statically or dynamically stable, where N_s = 1 - 4 was the range van der Meer prescribed for his static stability formula. It is noted that this classification system contains a transition from static to dynamic stability, where dynamic stability was defined with $N_s \geq 3$. It is evident, therefore, that for certain structural types such as berm breakwaters it can be difficult to select design methods, specifically whether to design the structure as dynamically or statically stable. This formula for static stability assumes that S is proportional to $N^{0.5}$ and $S > 0$ as long as $N_s > 0$. This implies that the damage level does not approach an equilibrium value, S_e, with the increase of the duration, NT_m, of the wave action. This is in disagreement with the evolution of the berm breakwater which is initially dynamic, but may eventually reach a stage of static stability similar to sand beaches approaching equilibrium for given wave conditions. Moreover, the formula does not account for the threshold condition of initiation of armor movement.

Kaku *et al.* (1991) found that van der Meer's static stability formula provided reasonable estimates of the stability number for specified damage levels, but failed to yield predicted damage levels with an adequate level of confidence. Therefore, using the data sets of van der Meer (1988), and Ryu and Sawaragi (1986), Kaku *et al.* (1991) proposed a new empirical formula to estimate the damage level of a statically stable structure under design conditions. This data set included 665 data points with the stability number, N_s, ranging between 0.9 - 3.9. By assuming that the damage level approaches an equilibrium value, S_e, asymptotically with time, the damage level, S, is expressed as (van der Meer, 1988)

$$S = S_e(1 - e^{-KN}) \tag{1}$$

where S_e = equilibrium damage level and K = empirical parameter related to the number of waves required to establish an approximately equilibrium slope profile. For example, $S = 0.95S_e$ at $N = 3/K$. A similar equation was employed by Kriebel and Dean (1985) to express time-dependent beach and dune erosion.

The data sets used in the development of the formula suggest that K be given by

$$K \approx \frac{a_1}{S_e^{b_1} + c_1} \tag{2}$$

where $a_1 \approx 0.01$, $b_1 \approx 0.8$, and $c_1 \approx 7$ are empirical constants. The similarity between the stability number, N_s, and the Shields parameter used for sediment transport allowed Kaku *et al.* to include the initiation of armor movement in their formula. A critical stability number, N_c, was defined by modifying van der Meer's formula for very small damage levels, with the structure assumed to be totally stable below this critical value. This critical stability number was given as

$$N_c = C\frac{6.2P^{0.18}}{\sqrt{\xi_m}} \qquad \xi_m < \xi_c \tag{3a}$$

$$N_c = C\xi_m^P \frac{\sqrt{\cot\alpha}}{P^{0.13}} \qquad \xi_m > \xi_c \tag{3b}$$

with $C \approx 0.4$, the surf similarity parameter ξ_m, and the critical value ξ_c, given as

$$\xi_m = \frac{\tan\alpha}{\sqrt{\dfrac{2\pi H_s}{gT_m^2}}} \qquad \xi_c = [6.2P^{0.31}\sqrt{\tan\alpha}]^{\frac{1}{P+0.5}} \tag{4}$$

The equilibrium damage level was then expressed as

$$S_e = a_2(N_s - N_c)^{b_2} \qquad N_s > N_c \tag{5a}$$

$$S_e = 0 \qquad N_s \le N_c \tag{5b}$$

with $a_2 \approx 12$ and $b_2 \approx 1.3$.

van der Meer's formula and the new stability formula are compared with the data sets used to develop these formulas in Figures 1a and 1b, where the comparison is the measured damage level against the empirical damage level. The predicted and measured damage levels can vary by a factor of two for these data sets with significant scatter evident for higher damage levels, which indicates a structure approaching a more dynamic classification. Consequently, larger errors would be expected for other data sets.

Experiment

The experiment was organized to obtain detailed data on macro-scale armor unit

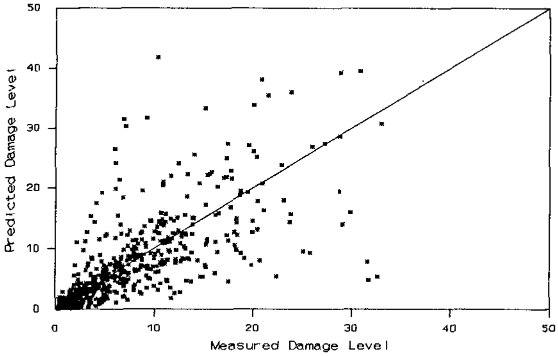

Figure 1a: Comparison with van der Meer's Formula (1987)

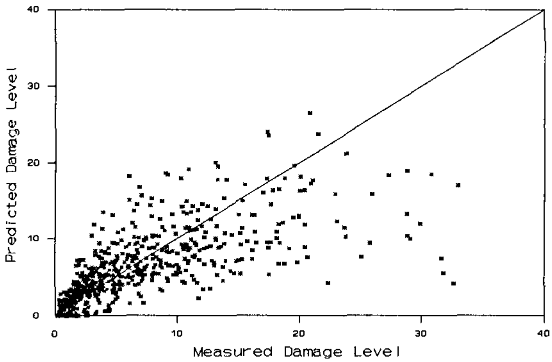

Figure 1b: Comparison with New Formula

movement in the laboratory by measuring altered breakwater profiles under random wave action. A concise summary of the experimental results and analyses from Smith (1991) is presented in the following. The experiment was conducted in a wave tank using a 20 cm thick layer of gravel, D_{n50} = 1.8 cm and a specific density of s = 2.7, placed on top of a 1:3 glued gravel slope (Figure 2a). Evaluation of the armor unit distribution showed an essentially uniform gravel distribution, where the gradation ratio, D_{85}/D_{15}, was approximately equal to 1.25. The wave generation was conducted using a piston-type paddle driven by a hydraulic system, and made use of a TMA spectrum as explained by Kobayashi *et al.* (1990). The random signal typically repeated itself after approximately 200 waves. Six irregular wave signals were prepared by varying H_s and T_m.

Free surface displacement measurements taken at three locations seaward of the slope were separated into incident and reflected wave trains, where the setup of the gages is shown in Figure 2a. A surface profiler, shown in Figures 2a and 2b, was constructed to measure the structural profile automatically using three vertically mounted survey probes, located relative to the structure as shown in Figure 2b. The horizontal and vertical motion of the profiler was controlled automatically and produced structural changes relative to the initial profile. It is also noted that the lateral variability of the profile response was negligible and profile change was essentially two-dimensional.

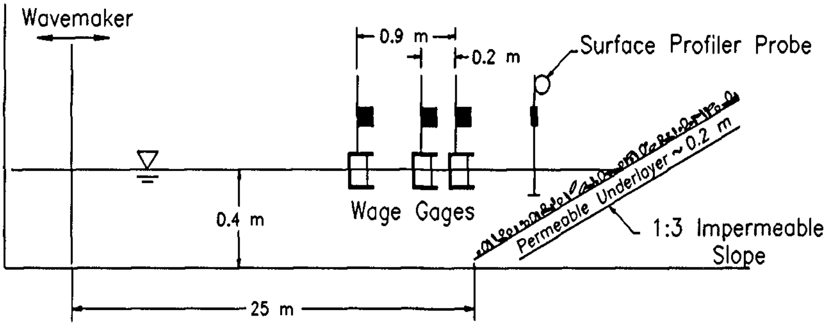

Figure 2a: Experimental Setup

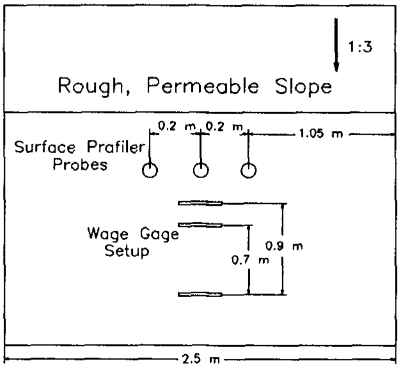

Figure 2b: Plan View of Wave Gage and Surface Profiler Locations

Six tests were performed, with each test consisting of six structural profile measurements at $N = 0$ and $N = 200$, 400, 600, 800, and 1000, where N is the number of individual waves from the beginning of each test. Each of the six tests hence consisted of five runs with the measured incident and reflected waves as well as the slope profile relative to the initial profile for each run. The data from the thirty runs are summarized in Table 1. Figure 3 depicts a typical result obtained for the wave measurements. Figure 3 shows the incident versus reflected wave spectrum, where wave reflection from the structure was minimal. It was also observed that the incident wave train proved reproducible and that the reflected waves during each test were observed to remain essentially the same, thus being insensitive to profile changes. The measured average reflection coefficient, *r*, as defined by Kobayashi *et al.* (1990), is listed for each run in Table 1.

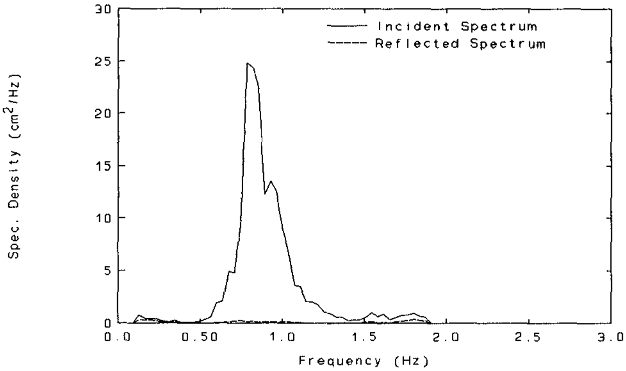

Figure 3: Incident and Reflected Spectrum

The slope geometry of the breakwater structure was obtained for each of the thirty test runs. Figure 4 depicts a typical measured altered profile, where the initial slope is included in an effort to indicate the degree of profile change. Qualitative assessment of the data showed erosion of the profile near the still water level (SWL) at profile elevation zero, with areas of deposition located both above and below the erosional area, corresponding to the transition between accretional and erosional profiles for sandy beaches. The increase of S with the increase of N followed the assumed exponential form as given by Eq. 1 fairly well, where significant alteration of the profile occurred during the first few hundred waves with much less profile change associated with the later stages of each test run. This implies that the profile change is sensitive to the degree of profile deviation from the equilibrium profile. It is also noted that the effects of the wave period proved important as shown in Table 1. In summary, the test runs with N_s = 2.69 - 3.67 and S = 6.3 - 38.1 were close to the upper limit of applicability of the static stability formulas.

Comparison with Static Stability Formulas

The static stability formula proposed herein and that of van der Meer (1987) are compared with the thirty new data points obtained from the present experiment. Before the comparison was completed the validity of the application of the formulas was ensured using the parameter ranges described by van der Meer (1988), including the stability number, wave steepness, surf similarity parameter, and initial profile slope. It was found that the values for the present experiment were within the ranges established by van der Meer (1988) for his static stability formula.

Table 1: Governing Parameters for 30 Test Runs

Run #	*N*	$H_s(cm)$	$T_m(sec)$	N_s	ξ_m	*S*	*r*	*cot α*
L1A	199	10.07	1.11	3.28	1.40	12.6	0.17	3.13
L1B	402	10.17	1.10	3.31	1.38	15.0	0.18	
L1C	601	10.22	1.10	3.33	1.38	18.1	0.18	
L1D	800	10.25	1.10	3.33	1.37	16.9	0.18	
L1E	995	10.25	1.11	3.34	1.39	20.5	0.19	
S1A	195	9.65	1.13	3.14	1.41	6.3	0.17	3.23
S1B	392	9.75	1.12	3.18	1.39	9.1	0.18	
S1C	593	9.78	1.12	3.19	1.39	10.1	0.18	
S1D	792	9.78	1.12	3.19	1.39	13.7	0.18	
S1E	996	9.76	1.11	3.18	1.38	13.1	0.19	
L2A	194	9.83	1.34	3.20	1.60	10.3	0.23	3.33
L2B	380	10.02	1.37	3.26	1.62	16.3	0.24	
L2C	567	10.09	1.38	3.29	1.63	19.7	0.25	
L2D	756	10.13	1.38	3.30	1.63	22.4	0.26	
L2E	948	10.15	1.37	3.31	1.61	23.5	0.26	
S2A	190	8.26	1.37	2.69	1.79	10.9	0.23	3.33
S2B	386	8.68	1.35	2.83	1.81	15.5	0.22	
S2C	576	8.75	1.36	2.85	1.72	18.3	0.24	
S2D	767	8.79	1.36	2.86	1.72	18.7	0.24	
S2E	959	8.82	1.36	2.87	1.72	20.0	0.24	
L3A	202	11.14	1.61	3.63	1.81	18.8	0.34	3.33
L3B	401	11.20	1.62	3.65	1.82	34.5	0.39	
L3C	600	11.21	1.63	3.65	1.83	33.5	0.37	
L3D	794	11.25	1.64	3.67	1.83	37.3	0.36	
L3E	988	11.26	1.65	3.67	1.84	38.1	0.35	
S3A	215	8.53	1.51	2.77	1.81	11.8	0.27	3.57
S3B	421	8.56	1.54	2.79	1.84	14.5	0.27	
S3C	624	8.60	1.56	2.80	1.86	17.4	0.28	
S3D	829	8.61	1.57	2.81	1.87	18.8	0.28	
S3E	1036	8.63	1.57	2.81	1.87	19.8	0.29	

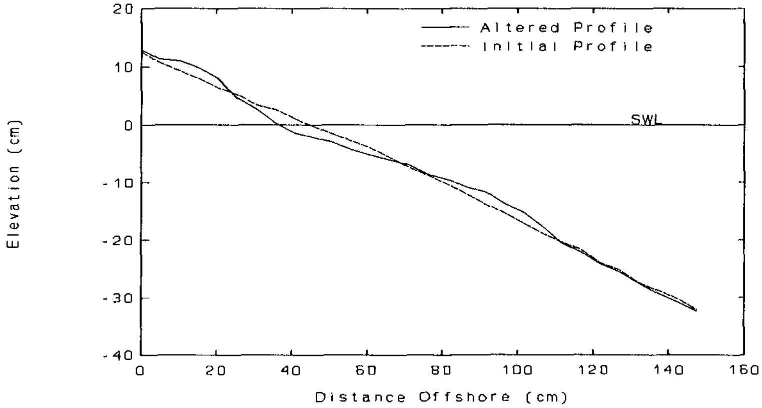

Figure 4: Typical Measured Altered Breakwater Profile

Figures 5a and 5b graphically depict the accuracy of van der Meer's formula, where different permeability coefficients are used in an effort to improve the agreement. The permeability of a structure has been shown to affect the structural response, where in general impermeable structures suffer greater erosion that do permeable structures subject to the same wave conditions. Examining the structure studied herein, a permeability coefficient P = 0.4 - 0.5 seemed appropriate based on van der Meer's recommendations. However, calculations have been conducted using a range of permeability coefficients. Figure 5a indicates that the damage level is being underestimated by roughly a factor of two, using the permeability coefficient, P = 0.4, with the error increasing for the higher levels of damage. Decreasing P to a value of 0.1 caused the predicted damage level to increase as shown in Figure 5b. It can be seen from Figures 5a and 5b that the damage prediction improves for the lower permeability coefficient. However, the lower permeability coefficients are unrealistic for the structure used in the experiment. These observations seem to be in agreement with the data analyses presented by Kaku *et al.* (1991), where it was shown that the formula predicted poorly for higher damage levels. Interpretation of these results indicate that this formula might be sufficient when applied to structures with low stability numbers, but is not accurate enough for structures approaching dynamic stability.

Figures 6a and 6b depict the capability of the new formula in predicting the damage levels, using various empirical coefficients in an attempt to improve the agreement. Figure 6a represents the new formula, using the empirical coefficients as fitted to the data sets of van der Meer (1988), and Ryu and Sawaragi (1986), and displays results similar to those shown in Figure 5a. It is noted that the permeability coefficient was again adjusted for this new formula, however, it provided relatively insignificant improvement in the formula's prediction. Hence P = 0.4 is assumed for the new formula. Two alternative approaches were undertaken toward the improvement of the agreement for Kaku *et al.*'s formula. First, it was observed that the equilibrium damage level, predicted by Eq. 5, was only slightly higher than most of the damage levels

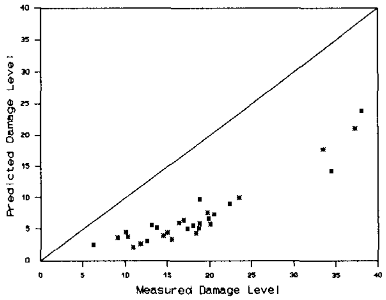

Figure 5a: Comparison with van der Meer's Formula(P = 0.4)

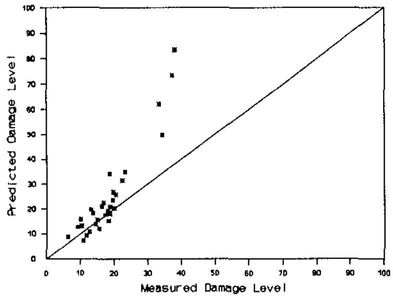

Figure 5b: Comparison with van der Meer's Formula(P=0.1)

suffered by the structure following 1000 waves. It was not anticipated that the structure would reach equilibrium over this amount of time, therefore, the value of the equilibrium damage level was increased in an effort to obtain higher damage level estimates. This increased equilibrium damage level caused a decrease of the exponential growth factor, K, as predicted by Eq. 2. The lower value of K in Eq. 1 offset the increase of the equilibrium damage level and little improvement was obtained from this adjustment.

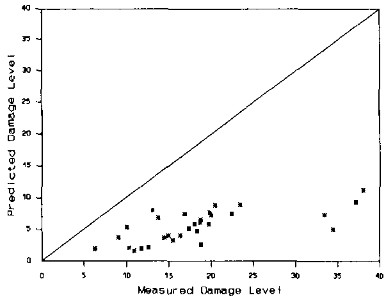

Figure 6a: Comparison with New Formula (a_1 = 0.01 & b_2 = 1.3)

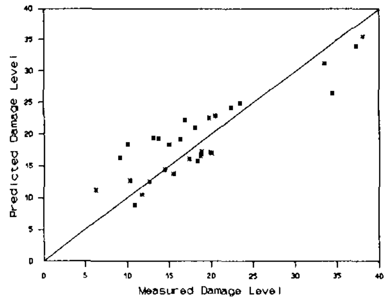

Figure 6b: Comparison with New Formula (a_1 = 0.08 & b_2 = 1.5)

Secondly, it was observed that approximately half of the damage was occurring during the first 200 waves, thus indicating that this profile would possibly approach equilibrium more rapidly than anticipated. Kaku *et al.*'s formula, however, predicts a much smoother rise towards equilibrium, with the initial damage level increase much less than that observed during the experiment. Adjustment of the exponential growth parameter a_1 for K in Eq. 2 caused the damage levels to be predicted more accurately, while the increase of the equilibrium damage level by adjusting the parameter b_2 in Eq. 5 also allowed for improved prediction of the higher damage levels. These two empirical parameters, a_1 and b_2, were fitted to the present data set. Figure 6b depicts the improved results of the new formula, where $a_1 = 0.08$ and $b_2 = 1.5$ are the parameters fit to the new data set. This adjustment has improved the performance of Kaku *et al.*'s method for the data described herein. However, any conclusive results will require more data in order to properly calibrate the parameters. Nevertheless, better agreement is achieved with the adjusted formula of Kaku *et al.* in comparison with van der Meer's formula. Also, it is important to point out the adjustment of van der Meer's formula required unrealistic permeability coefficients. The framework of Kaku *et al.*'s formula may be more realistic, for it adjusts the time scale of profile development, which is not fully defined by any previous work.

In summary, the damage levels predicted by the empirical formulas of van der Meer (1987) and Kaku *et al.* (1991) failed to accurately predict the damage suffered by the structure in this experiment. However, it is expected that the formula due to Kaku *et al.* is more adjustable to various conditions. Moreover, the necessity of the adjustments of the empirical parameters suggest that these simple formulas are not sufficiently accurate partly because they do not account for detailed irregular wave and structural characteristics.

Comparison with Dynamic Stability Formulas

In addition to the previous static stability approach to profile evolution prediction, the range of N_s = 2.69 - 3.67 associated with the new data set may allow for the evaluation of dynamic profile prediction methods. Dynamic stability allows for significant alteration of the structure from the initial profile configuration, thus requiring the prediction of the actual evolved profile shape. van der Meer and Pilarczyk (1986) studied the dependency of the evolved profile on various wave and structural parameters, and developed an empirical formula that predicts the locations of abrupt profile changes on an assumed profile shape. This formula connects these points with straight lines and curves, which are described by power functions similar to those used to express the equilibrium profile for sandy beaches. Their formula is applicable to structures with N_s between 3 and 200 with arbitrary initial profile configurations, and the new data set corresponds to the lower limit of applicability for this dynamic stability formula.

A typical result of van der Meer and Pilarczyk's formula is shown in Figure 7, where the predicted profile is plotted with the measured altered structural slope. It is evident that the formula overpredicts the response of the breakwater. It is noted that this result was anticipated due to the low stability associated with the data set in relationship to the data used to develop the dynamic stability formula. It can be seen that the accretion of the berm was greatly overpredicted, and that a large area of erosion was predicted below the SWL. In general, the formula predicts accretion above the SWL and erosion below, which is in contrast to the data which depicts more of an S-shape, where erosion is restricted to the area around the SWL with areas of accretion both above and below the SWL as shown in Figure 4. The damage levels associated with the predicted profiles were also calculated in order to quantify the predictive capabilities of the formula when applied to the structure tested herein. It was found that the damage level obtained for the predicted profiles is overestimated by roughly a factor of two as shown in Figure 8, where application of this method might be used to define an upper limit of damage for a static/dynamic structure.

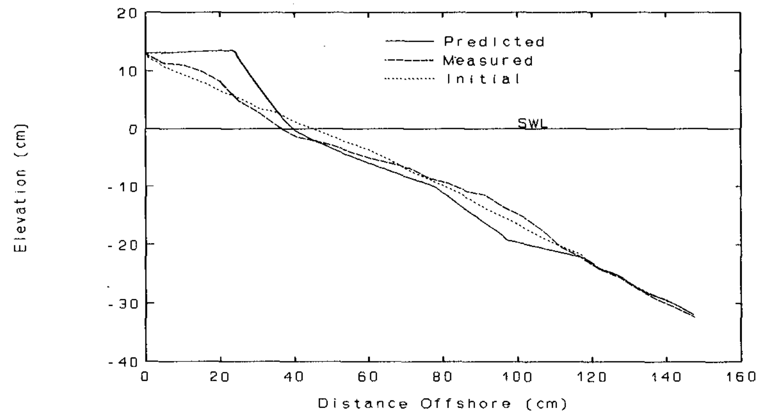

Figure 7: Predicted Dynamic Profile(van der Meer and Pilarczyk, 1986)

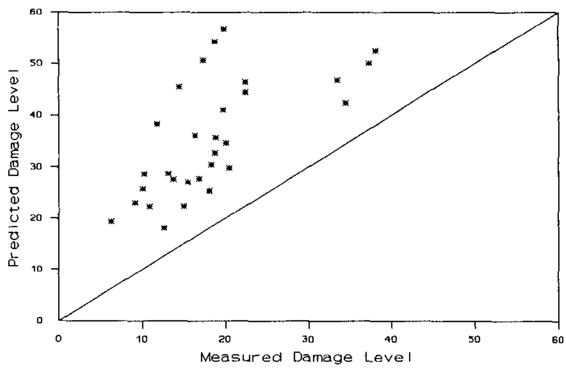

Figure 8: Comparison with Formula of van der Meer and Pilarczyk (1986)

van der Meer (1992) reanalyzed the data set used previously to develop the dynamic stability formula of van der Meer and Pilarczyk (1986), with emphasis placed on berm breakwaters which pass through stages of dynamic stability and eventually develop an essentially stable profile. The data set contained structures with N_s between 3 and 500, but a new formula was develop for the data set concentrating on structures with N_s < 6. The modified method notes the probability of a more rapid increase in the damage level than that predicted for statically stable structures and may be applied for $N = 250$ - 10,000 with alterations to the profile taking place even after significant durations. The effects of the initial slope are discussed by van der Meer (1992) where it is noted that for $N_s < 10$ the initial slope largely affects the final profile, much as is the case with statically stable structures. This lead to the inclusion of a definition of an equivalent slope profile to be used with the modified formula, which would prove important for a typical berm breakwater profile. The present experiment, however, used a uniform slope of approximately 1:3, and the qualitative assessment of the profile development as discussed by van der Meer (1992) agrees with the observed profile evolution within the experiment.

The modified formula has been applied to the new data set described herein, where comparisons are made between the measured and predicted slope profiles and corresponding damage levels. A typical result of the new formula is shown in Figure 9. Figure 9 depicts that the predicted slope underestimates the berm accretion both in volume and in the height to which the berm is developed, also the erosion around the SWL and the accretion below the SWL are poorly predicted. van der Meer (1992) encountered similar problems with the prediction of the size of the structural crest when the formula was applied to the data of Ahrens and Heimbaugh (1989), and suggests that the limited data did not allow for calibration of the parameter related to this structural feature. However, the trend of the profile seems to agree with the present data, because the formula predicts accretional areas both above and below the SWL, and an erosional area near the SWL. Despite the appropriate profile shape, profile development is insufficient due to the relatively small alteration of the initial profile as predicted by the formula. This observation was also noted quantitatively when calculating the damage levels of the predicted profiles, where the damage was found to be underpredicted by roughly a factor of two as shown in Figure 10. In analyzing the results it is anticipated that an improved prediction of the berm crest would allow better agreement to be realized since this would in turn upgrade the erosive portion about the SWL, thus improving overall results for the entire predicted profile.

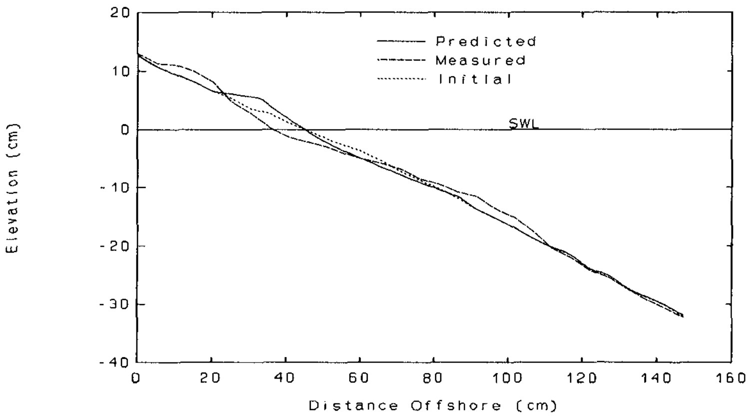

Figure 9: Predicted Dynamic Profile(van der Meer, 1992)

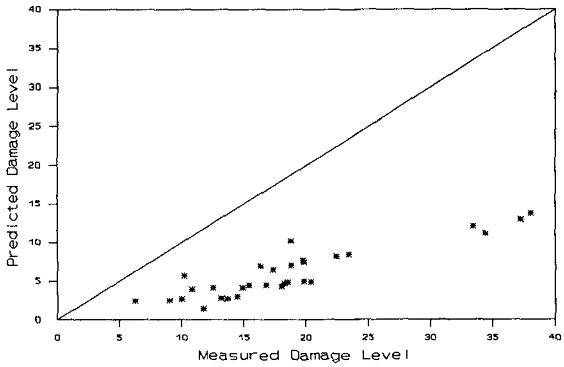

Figure 10: Comparison with Formula of van der Meer (1992)

Conclusions

It has been shown that for the new data set with a structural classification that is neither strictly statically or dynamically stable, current design methods are insufficient in predicting profile response with any consistent accuracy. The static stability formulas of van der Meer (1987) and Kaku *et al.* (1991) failed to accurately predict the damage suffered by the structure, where the formulas underestimated the damage. This result is not surprising in that the formulas were developed with data that included numerous structures with low damage levels. The flexibility of the static stability formulas was tested for the new data set with the adjustment of the empirical permeability coefficient and the empirical parameters contained within the formula due to Kaku *et al.* (1991). van der Meer's formula improved with the decreasing permeability coefficient, however, the results were dependent on what appears to be an unrealistic value for the permeability coefficient. Kaku *et al.*'s formula showed improved accuracy with the adjustment of the time scale for profile development, which occurred at a higher rate for the more dynamic structure studied herein. It appears, therefore, that the formula due to Kaku *et al.* is more versatile and allows for adjustments to be made dependent upon the structural type of interest. More data will be required to properly calibrate this formula. It seems that the previous investigators tried to describe too varying a range of structural types with simple formulas, where it is not plausible given the wide variety of influences to which different structural types are subjected.

Secondly, the dynamic stability formulas of van der Meer (1992), and van der Meer and Pilarczyk (1986) were evaluated using the new data set. The data tested the ability of the formulas to predict the actual profile response and damage levels suffered. van der Meer (1992) revised the stability formula for structures close to the static stability boundary. The revised formula provided good qualitative results in that the profile shape was predicted relatively well despite a repeated underprediction of the amount of actual profile response. It is anticipated that this formula is a step in the right direction, but requires additional data in order to produce more accurate results. van der Meer and Pilarczyk's (1986) dynamic stability formula repeatedly overpredicted profile response. In summary, the existing dynamic formulas seem to be insufficient for the specific structures studied within this study.

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