CHAPTER 131

The permeability of rubble mound breakwaters. New measurements and new ideas.

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1 ABSTRACT

The results of an extensive series of permeability experiments originally analysed by Shih (1990) are reinterpreted in the light of new experiments. It is proposed that the Forchheimer equation might not fully describe flow at the high Reynolds numbers found in the interior of rubble material. A new series of tests designed to test for deviations from the Forchheimer equation and investigate the effects of material shape are described. While no evidence can be found to indicate a deviation from the Forchheimer equation a dependency of permeability and the surface roughness the material is demonstrated.

2 INTRODUCTION

This work forms part of a study under the MAST project G6 Coastal Structures, funded by the European Community, to develop a package of techniques to model wave action on rubble mound breakwaters. An important parameter necessary for such models is the permeability; HR Wallingford of the UK, Aalborg University of Denmark and Delft Geotechnics of the Netherlands. The major part of the testing has been carried out at HR Wallingford, by Williams, while contributions to the analysis have been made by Burcharth and den Adel.

Traditionally, permeability of the material in rubble mounds has been a difficult

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parameter to measure. Prototype size experiments are not feasible due to the large scale of the material involved. The problem is compounded by uncertainties about the way small scale experiments may be scaled to prototype size. Before the start of this study a large amount of work had already been carried out on permeability at Wallingford. A large range of material sizes (4-60mm) were tested so that the effects of material size could be investigated. Shih (1990) carried out the first analysis of the results and looked for the desired scaling effects. Here further ideas about the interpretation of these experiments are discussed and a further set measurements designed to look at the effects of material shape are described.

Following the analysis by Shih and similar work carried out at Aalborg, it became apparent that there were uncertainties about the interpretation of the data. Burcharth & Christensen (1991) suggested that there might be several distinct flow regimes possible in rubble material and that the transition from a laminar flow pattern to a turbulent flow pattern would be described by a Reynolds number. If this was the case then it would effect attempts to scale permeability measurements made at model scale to prototype size.

Further experiments were required to investigate the transition from laminar to turbulent flow regimes. In order to do this the permeameter at Wallingford was modified to increase the maximum possible flow rate and, thus enable a wider range of Reynolds number, for each single material size, to be investigated.

In addition to looking for flow regime changes, these new tests also provided the opportunity to investigate the effects of material shape on permeability. It was possible to obtain samples of material that had been carefully prepared according to shape. This material had been analysed using an image processing technique, by Latham et al. (1988).

3 FLOW EQUATIONS

The most common interpretation of permeability of a material is as a measure of the pressure gradient required to produce a given flow rate of fluid through the material. This was formulated into an equation by Darcy,

$$u = ki \tag{1}$$

where u is the discharge velocity, and k is the effective permeability

The linear form of Darcy's law makes it easy to deal with, and has proved very successful in the modelling flows through pores materials when the flow rate or Reynolds number is low. However, at higher Reynolds number the inertia of the fluid becomes important and the relationship between i and u can no longer be expressed in this linear form. The size of the rubble material within a breakwater and the flow rates induced by wave action result in flows with typically large Reynolds numbers (>500). This means that to model correctly the breakwater behaviour requires a non-linear expression within the flow equation.

The exact form that such a non-linear flow law should take is at present unclear, for the purposes of these experiments the equation suggested by Forchheimer (and described by Engelund (1953)) shall be used.

$$i = au + bu^2 \tag{2}$$

In this equation the permeability is described by the two coefficients a and b which are intended to be independent of the discharge rate u.

It is generally considered that the two terms on the right hand side of Equ(2) independently describe the laminar and turbulent flow regimes. The first term au describes the laminar flow and is dominant at small values of u. In laminar flow the viscous terms in the Navier stokes equation dominate and hence the coefficient, a should be dependent on the viscosity of the fluid υ. Conversely the second term, bu^2 is dominant at large values of u and is attributed to the turbulent flow regime, indicating that b must be independent of the fluid viscosity.

The Forchheimer equation is the most widely used in the analysis of permeability data, however it may not fully describe the complex flows that occur in the large porous material of a rubble mound. If the Forchheimer equation is our chosen flow equation, the question is now one of determining the form of the coefficients a and b.

4 THE *a* AND *b* COEFFICIENTS

Engelund (1953) suggests that the relevant parameters are the porosity of the porous medium n, and some measure of the grain size D. By demanding that the coefficients remain dimensionally correct he obtained the best fit to his data with the expressions:-

$$a = \alpha \frac{(1-n)^3}{n^2} \frac{\nu}{gD^2} \tag{3}$$

These expressions provide the permeability in terms of only two simply-determined parameters, by the incorporation of the two dimensionless

$$b = \beta \frac{(1-n)}{n^3} \frac{1}{gD} \tag{4}$$

coefficients α and β. Ideally α and β would be constants however even Engelund recognised that they may be dependent on other properties of the material, such as shape or grain size distribution. This means that there is a wide range in the possible values for α and β which must be determined for the material under consideration.

Engelund chose his coefficients to provide the best possible fit to his data. It has been pointed out by Burcharth & Christensen (1991), that Engelund's data set is restricted to results from material with very small particle sizes, and hence low Reynolds number flows. Burcharth goes on to suggest a new formula for the laminar coefficient, *a*, based on arguments of dimensional analysis and geometry of the voids in a porous material. Burcharth's new coefficient takes the form:-

$$a = \alpha_B \frac{(1-n)^2}{n^3} \frac{\nu}{gd^2} \tag{5}$$

This results in the new dimensionless coefficient α_B. It should be noticed that in the range of experimental observation of n=0.3 to n=0.45 the general form of Engelund's laminar coefficient and Burcharth's new coefficient are so similar as to be indistinguishable.

Shih (1990) analysed the large amount of data collected in previous experiments at Wallingford in terms of the Engelund coefficients. This data set contained measurements for a wide range of material sizes and gradations. The α and β values were investigated for any dependence on grain size (D_{15}). For material with a narrow grain size distribution laminar constant α was found to increase with D_{15} in the way illustrated in Fig.(1). The turbulent constant β was found to decease exponentially with D_{15}, as shown in Fig.(1). These results suggested that Engelund's expressions for *a* and *b* could be modified to provide a scaling law for the permeability. The coefficients *a* and *b* for the Forchheimer equation become.

$$a = \left[\alpha_1 + \alpha_2 \left(\frac{g}{\nu^2} \right)^{\frac{2}{3}} D_{15}^2 \right] \frac{(1-n^3)}{n^2} \frac{\nu}{g} \frac{1}{D_{15}^2} \tag{6}$$

where n = porosity α_1 = 1683.71, $\alpha_2 = 3.12 \times 10^{-3}$, υ = kinematic viscosity of water $\simeq 1.14 \times 10^{-6}$ m^2 s^{-1}, g = gravitational acceleration = 9.81 ms^{-2}

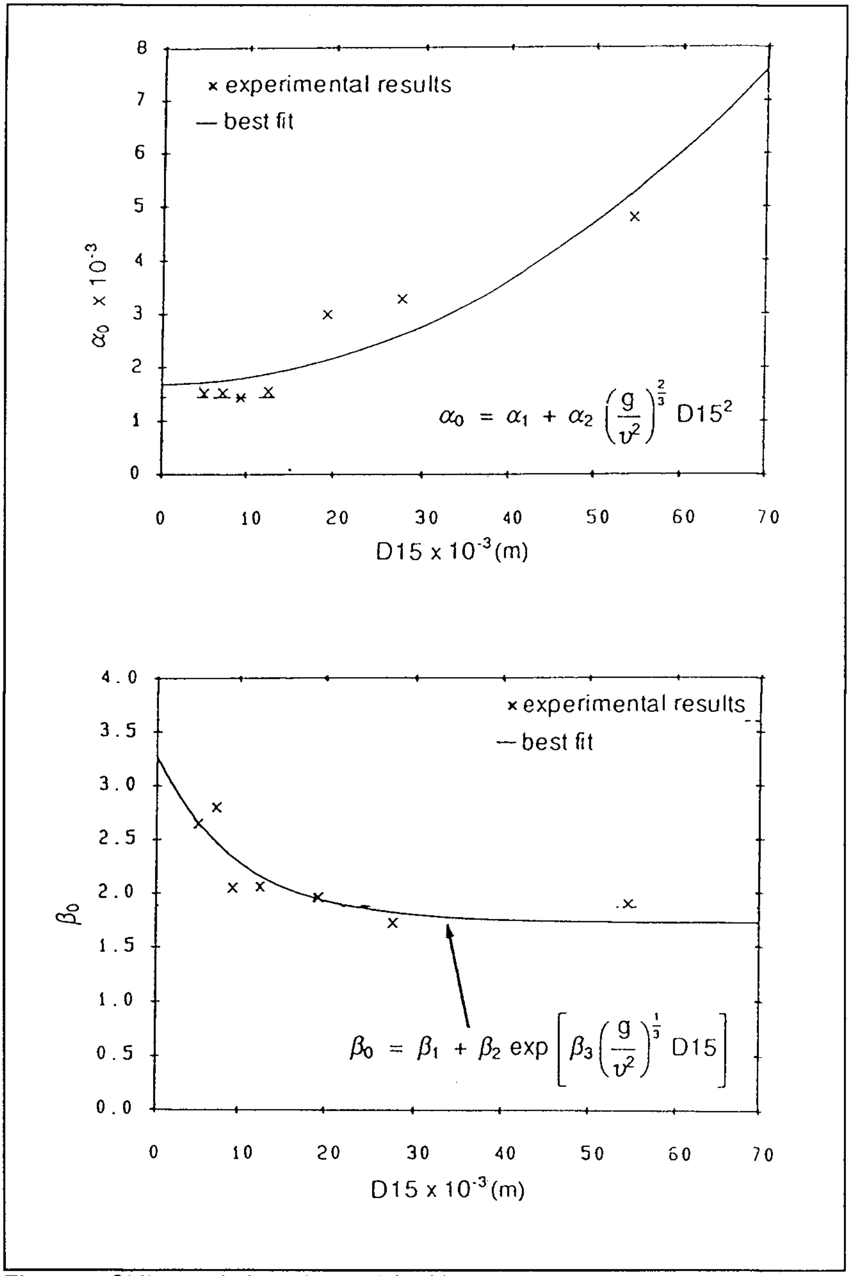

Figure 1 Shih's variation of α and β with D_{15}

$$b = \left\{\beta_1 + \beta_2 \exp\left[\beta_3 \left(\frac{g}{\nu^2}\right)^{\frac{1}{3}} D_{15}\right]\right\} \frac{(1-n)}{n^3} \frac{1}{gD_{15}} \tag{7}$$

A similar analysis was carried out on the data from material with a wider size grading and a formula for an equivalent D_{15} is proposed for use in these formulae.

Following the analysis by Shih and inspection of data collected at Aalborg University, Burcharth & Christensen (1991) suggests that the Forchheimer equation does not accurately model the behaviour of the flow through porous media. He proposes that a plot of i/u against u does not have the linear form

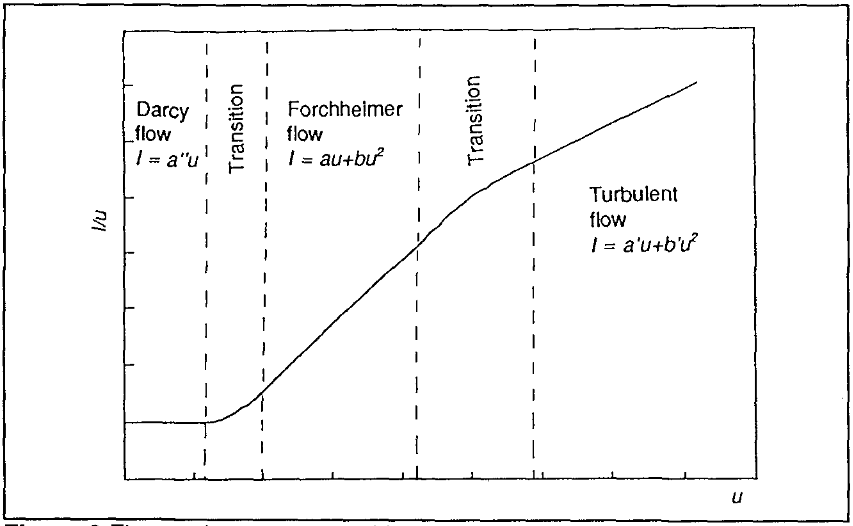

Figure 2 Flow regimes suggested by Burcharth

demanded by the Forchheimer equation, but has the form shown in Fig.(2). The extreme portions of the graph represent laminar and turbulent flow regimes as indicated, while between these two extremes lies a transition region modeled by the Forchheimer equation.

Burcharth interprets the trends in α and β as the result of differing flow regimes in the separate tests. The intercept and gradient of the line in Fig(2) is a measure of α and β respectively. For tests conducted with small material the flow is describe by the Forchheimer equation and yields small values of α and large values of β. Conversely of large material the flow regime is turbulent and has correspondingly large values for α and small values for β.

This would mean that the Forchheimer equation could not be used to extrapolate from small scale tests to prototype material. At this stage this hypothesis could not be confirmed as no single experiment had been conducted over sufficiently wide a range of Reynolds number to allow any deviation from the Forchheimer equation to be observed.

In addition to this interpretation there are further doubts about the validity of the scaling formulae proposed by Shih. The accuracy of the measurements of the α and β coefficients are both functions of material size in just the form required to produce the trends reported by Shih.

5 THE PERMEAMETER EXPERIMENTS

To test Burcharth's hypothesis the permeameter at Wallingford was modified to approximately double its maximum flow rate. This would allow measurements to be made over a larger range of Reynolds number and thus increase the opportunity of finding deviations from the Forchheimer equation.

To test for the effects of shape five types of material were tested and were classified according to shape in the following manner:

TABULAR:	The maximum/minimum dimension was greater than two. Flat and elongate material was included. Selection was by eye.
CUBIC:	The maximum/minimum dimension was less than two and there was at least one pair of parallel faces. Selection was by eye.
FRESH:	The angular material left after the tabular rock had been removed.
SEMI-ROUND:	Fresh material was rounded by abrasion to achieve a 5 to 10% weight loss.
ROUND:	Fresh material was rounded by abrasion to achieve a 20 to 25% weight loss.

The source material was crushed Carboniferous limestone which had been sorted into the above classifications for a previous study on the effects of material shape on stability. The D_{50} of the material in each group $\approx$ 50mm. A full description of the material preparation is given by Bradbury et al. (1988).

The above material was tested in the large permeameter at Wallingford. This permeameter consists of a large cylinder of 0.6m diameter and 1m long, which is mounted with its axis vertical. The material is loaded into the permeameter and water is pumped into the bottom of the cylinder and allowed to flow freely out from the top. The pressure gradient is measured across the material by two pressure tappings 0.5m apart attached to a differential pressure cell. The corresponding flow rate is measured by a magnetic flowmeter mounted in the entry pipe.

6 SHAPE ANALYSIS

The material used in the tests with the modified permeameter had previously been analysis by Latham, at Queen Mary College, University of London, and a full description of the technique is given in Latham et all. (1988).

A sample of each type of stone was placed on a light table and an image of its silhouette obtained with a video camera. The resulting image was then digitised and passed on to a computer. Both a Fourier transform and a fractal technique were used in the analysis of the results.

6.1 Fourier transform analysis

Once the digital form of the stone's silhouette has been obtained on the computer, it is a simple task to obtain the coordinates of the silhouette outline. From these coordinates the centre of the projected area or "centre of gravity" of the silhouette may be calculated and this is then used as a centre reference point. The outline may then be described in polar coordinates (r,θ)

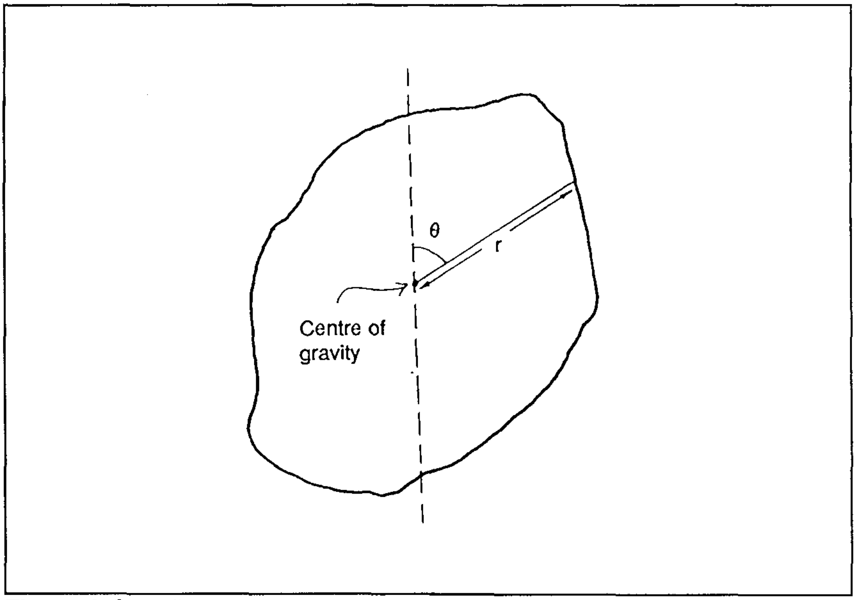

Figure 3 Coordinate system for shape analysis

using the centre of gravity as the origin (see Fig.(3)). A Fourier analysis was carried out on the normalised radius vector of the outline. The outline was be described by a Fourier series in the following manner:

$$r(\theta)=C_0+\sum_{n=1}^{\infty} C_n\cos(n\theta-A_n) \tag{8}$$

where C_n is the amplitude coefficient of the nth harmonic, A_n is the phase angle of the nth harmonic, n is the harmonic order, θ is the polar angle measured from an arbitrary reference line.

The gross shape of the outline is described by the lower harmonics in the series, while the higher harmonics provide information on the degree of surface roughness.

The problem now remains of choosing suitable parameters from the resulting harmonic amplitude coefficients. It is useful to first define a coefficient Q which provides a flexible quantitative index which can be computed over all or a chosen range of harmonics and is defined as:

$$Q=\left(0.5\sum C_n^2\right)^{0.5} \tag{9}$$

The following parameters are suggested by Latham & Poole (1987)

Symbol	Range of n	Name
P_n	1 to ∞	Fourier noncircularity
P_C	1 to 10	Fourier shape factor
P_R	11 to 20	Fourier asperity roughness factor

In their report Latham & Poole (1987) choose to give the results of the shape analysis in terms of P_C and P_R, and these results are presented in Table (1). Along with the mean values the 15, 50 and 85% exceedance values, are also given to provide an indication of the degree of spread within a sample.

6.2 FRACTAL SHAPE DESCRIPTION

A better measure of the convolution or roughness of a stones outline may be provided using the concepts of fractal geometry, described by Mandelbrot (1982). The concept is centred on the way the measured perimeter of the silhouette image changes as a function of the scale of the measuring instrument. In our case the perimeters of the silhouette were measured by stepping around the outline with a "hypothetical" pair of dividers set at a given step length. This process was then repeated many times with differing step

SHAPE	GROSS SHAPE Fourier Shape Contribution Factor P_C				ROUGHNESS Fourier Asperity Roughness P_R				SURFACE TEXTURE Fractal Coefficient F	
	Mean	P_{C50}	P_{C85}	P_{C15}	Log Mean	P_{R50}	P_{R85}	P_{R15}	Mean	F_{40}
TABULAR	2.67	3.03	1.82	4.65	.0165	.0180	.0125	.0325	.014	.017
EQUANT	1.43	1.52	1.00	2.21	.0117	.0124	.0095	.0166	.015	.018
FRESH	1.88	2.08	1.38	3.06	.0138	.0150	.0107	.0216	.016	.019
SEMIROUND	1.89	2.13	1.22	3.19	.0097	.0121	.0080	.0153	.010	.012
VERY ROUND	1.55	1.80	1.05	2.64	.0046	.0053	.0035	.0092	.008	.009

Table 1 Shape analysis of rubble material

lengths so that the variation of measured perimeter with step length could be studied. A Mandelbrot-Richardson plot was then made of the log perimeter measured against the log step length. The gradient of the resulting plot is a measure of the roughness of the outline and is the negative of the fractal shape coefficient *F*. The resulting values of F are given in Table (1).

7 RESULTS

The data from each test was plotted as *i/u* against Reynolds number and a linear regression made to obtain values for *b* from which β was determined. It should be noted that for the size of material used in these tests the flow was always predominately turbulent so no attempt was made to measure the lamina coefficient *a* and α.

Each data set was investigated for variations from the Forchheimer equation as proposed by Burcharth. No variation larger than the typical experimental error was observed for any of the data sets.

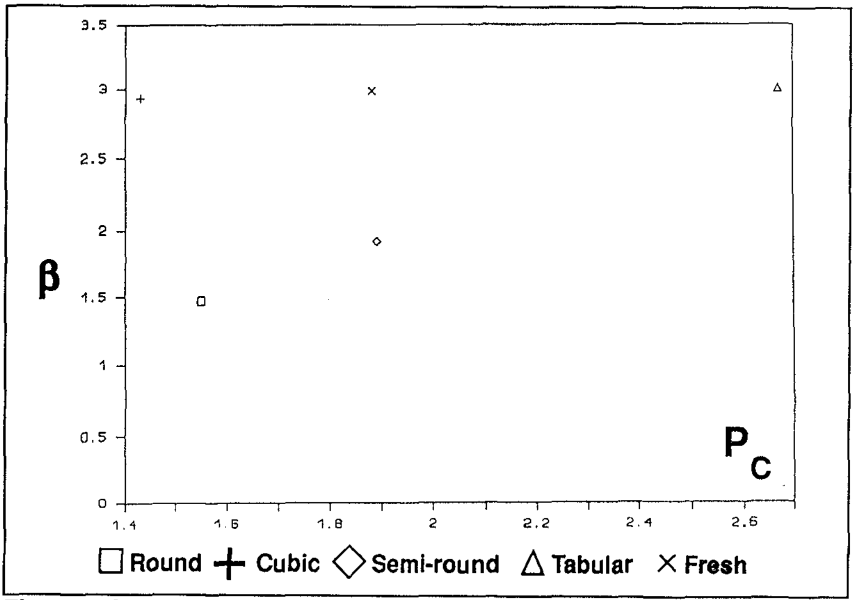

Figure 4 β plotted against the Fourier shape contribution factor P_C

The β values of each test have been plotted against each of the shape parameters described in Section 6, Fig(4,5 & 6). No trend can be seen for the Fourier shape contribution factor P_C. The Fourier asperity roughness and

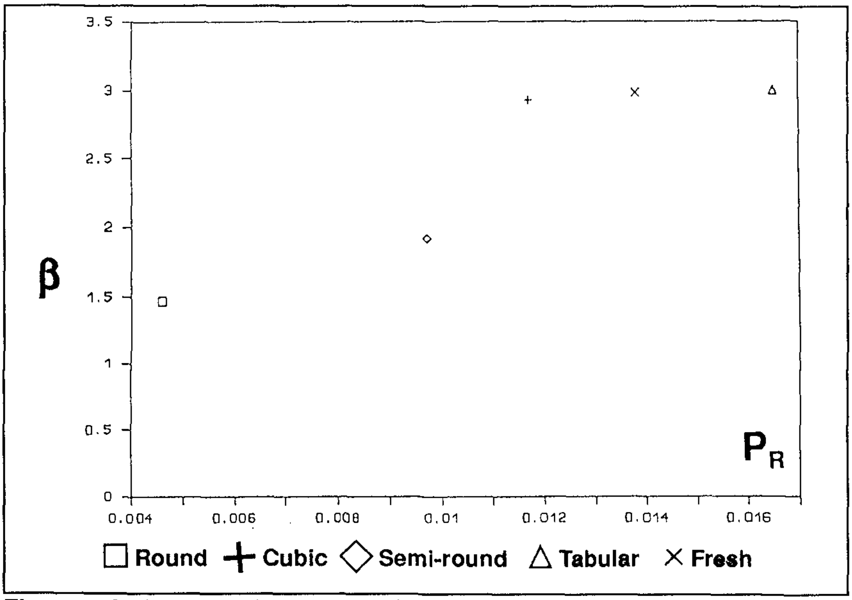

Figure 5 β plotted against the Fourier asperity roughness P_R

the fractal coefficients both show β as a decreasing function. The Fourier asperity roughness and fractal coefficient are a measure of the surface roughness. It may be concluded that the nature of the surface of the test material is more important than the overall shape of the material in the determination of the turbulent coefficient *b*.

It should be stressed that the range of tested material was small (only five groups), so the form of β as a function of the *F* or P_R, can not be determined. Figs (4,5 & 6) show that this material forms only two distinct groups in terms of surface roughness. The equant, tabular and fresh rock all have very similar surface properties, which are essentially those of freshly crushed rock. The preparation of the round and semi-round material produces a smooth surface which behaves in a different way to turbulent flows.

The intention of these tests was to investigate the suitability of the shape analysis parameters, rather than to produce enough data to derive the form of β as a function of these parameters. It is hoped that these experiments will point the way for further tests, which will make use of these shape description techniques.

8 CONCLUSIONS

The Forchheimer equation has been used in the analysis of permeability data

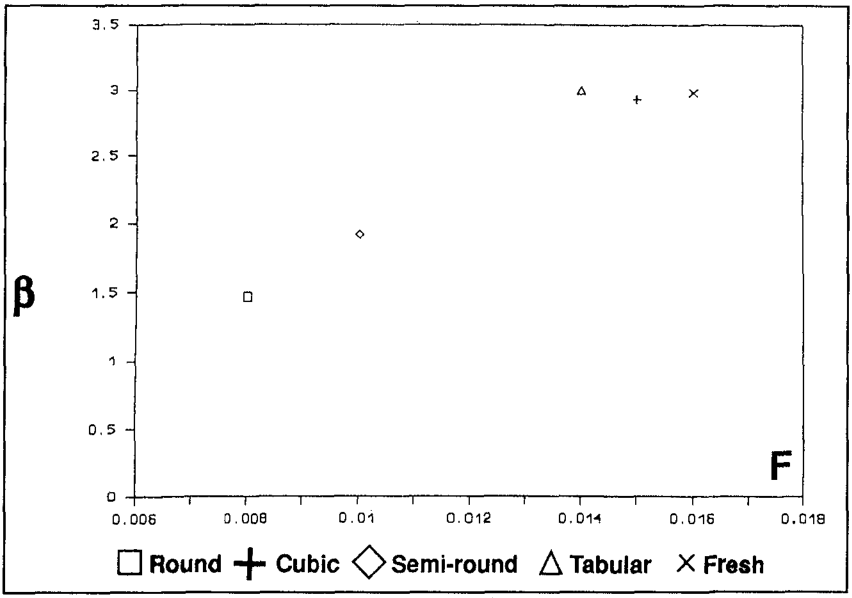

Figure 6 β plotted against the fractal coefficient F

for many years and up to data there has been no significant evidence to suggest that it is not applicable over the whole range of possible flow rates. The data collected in these experiments supports this view, in that within the accuracy of the experiments no deviation from Equ.(2) could be found. The question still remains that the range of Reynolds numbers tested for a single sample might not be large enough to demonstrate a deviation from the Forchheimer equation. If this is the case then some caution must be used in the scaling up of laboratory tests to larger prototype material.

The material tested with the modified permeameter at Wallingford was made up of samples of narrow size grading of around 50mm, which had been hand graded with respect to shape. This material was analysed for shape by the use of a video imaging technique. The shape is described by three parameters, the gross shape of the material is provided by the Fourier shape contribution factor P_C, the surface roughness is described by the Fourier asperity roughness P_R and the fractal coefficient F. The value of β shows no distinguishable trend as a function of the Fourier shape contribution factor P_C, however β is a function of both the asperity roughness, P_R and the fractal F. These results indicate that the turbulent coefficient β is a function of the surface texture of the rubble material, rather that the overall gross shape of the material. The data set from these experiments was small and far more data is required over a wider range of material surface textures before the dependency of β can be determined.

9 ACKNOWLEDGEMENTS

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