CHAPTER 132

NEW METHODS TO EVALUATE WAVE RUN-UP HEIGHT AND WAVE OVERTOPPING RATE

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Abstract

Recently, waterfront development and the mean sea level rise due to greenhouse effect have been noticed, and it has become important to develop a practical method to calculate wave run-up height or wave overtopping rate.

This paper presents new methods for calculating the wave run-up height on and the wave overtopping rate over a seawall located on a complicated bottom profile of sea coast. The proposed methods were tested both in the laboratory and in the field. The predicted results coincide well with the available data.

Introduction

Waterfront development has extended rapidly over recent years. Governmental agencies are also seriously concerned about the mean sea level rise due to greenhouse effect. The mean sea level is expected to be risen some 20cm to 110cm by the year of 2030. Under such circumstances, precise evaluation of the wave run-up height or of wave overtopping rate is of extreme importance for the future planning of coastal preservation.

Many conventional methods to calculate the wave run-up height and the wave overtopping rate exist, but the scope of applicability of these methods is limited (refer to Herbich,1991). For example, Saville's nomograph (1957) for calculating the wave run-up height can not evaluate the difference of bottom profiles from the breaking point to the extent of maximum wave run-up. Goda's nomographs (1970a) for calculating the wave overtopping rate are graphs for the condition of vertical seawalls. Battjes' calculating method (1974) for the overtopping rates over sloped structures does not consider the influence

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of bottom profiles. Thus this method is applicable only for non-breaking waves. Therefore it is important to develop methods that can be applied to a wide range of conditions such as complicated bottom profiles and arbitrary sloped seawalls.

Kobayashi et al.(1989) developed a numerical model for the calculation of the wave overtopping rate. In their study, shallow water long wave theory was used and a finite difference method was applied. However the effects of wave breaking on wave overtopping were not taken into account in the above model. Mizuguchi et al.(1988) considered the effects of wave breaking on wave run-up height by using a set of an energy equation including a term of energy dissipation by wave breaking and a time-averaged integral momentum equation. Although the wave run-up height can be obtained by using the above method, but the wave overtopping rate can not be obtained. These numerical models should be keenly promoted in order to develop suitable calculating methods.

On the other hand, it is also important to develop methods that can be easily applied to complicated coastal profiles on site using a personal computer. Therefore in this paper characteristics of wave run-up profile were investigated and new methods consisting of experimental equations were proposed.

New Method for the Evaluation of the Wave Run-up Height

(1) For Breaking Waves

It is assumed that the influence of the complicated coastal profile on the wave run-up height can be evaluated by introducing a hypothetical single slope angle α proposed by Nakamura et al.(1972) as follows:

$$\alpha = \tan^{-1}(R + h_b)^2 / 2A \qquad (1)$$

where R is the wave run-up height, h_b is the breaking water depth and A is the shaded area from the depth at the breaking point to the extent of maximum wave run-up, as shown in Figure 1.

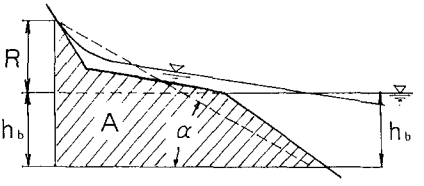


Figure 1. Hypothetical single slope angle (Nakamura et al.,1972).

By considering the balance between the potential energy of maximum wave run-up and the kinetic energy of waves on a shoreline, the wave run-up height can be expressed as follows:

$$R = (1-k)\, Us^2 / 2g \tag{2}$$

where, k is a coefficient of energy loss caused by sea-bed friction, Us is the fluid velocity at the shoreline and g is the acceleration of gravity. According to Iwagaki et al.(1966), Us can be expressed by Eq.(3).

$$Us = C\sqrt{g\overline{\eta_s}}\cos\alpha \tag{3}$$

where, C is a coefficient dependent on the sea-bed topography in surf zone and $\overline{\eta_s}$ is the wave set-up at the shoreline. Substitution of Eq. (3) into Eq.(2) gives

$$R = 0.5(1-k)C^2\,\overline{\eta_s}(\cos\alpha)^2 \tag{4}$$

If Eq.(4) is valid, the value of $R/\overline{\eta_s}$ should be constant in case of the coasts with the same coefficients k, C. In the laboratory experiment against the same sea-bed, the wave set-up was measured at a shallow point near the shoreline by a servo-type waterlevel gauge. It was assumed that this measured value of the wave set-up corresponds to the value of $\overline{\eta_s}$ and the relationship between the value of $R/\overline{\eta_s}$ and the deep water wave steepness was investigated. Figure 2 shows the result of this investigation. From this figure, it is remarkable that Eq.(4) is applicable with the conditions of the experiment. Equation (4) indicates that the wave set-up is important for the evaluation of the wave run-up height for breaking waves.

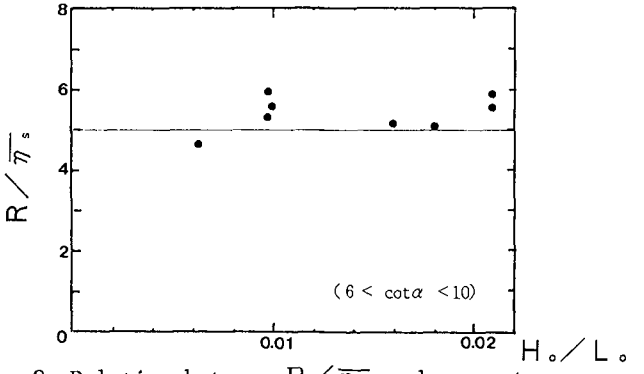


Figure 2. Relation between $R/\overline{\eta_s}$ and wave steepness.

Yamamoto (1988) obtained the following empirical equation:

$$\overline{\eta_s} = Hs/[2.4(\tan\alpha)^{0.3}] = 0.8(\tan\alpha)^{0.6} H_b \tag{5}$$

where Hs is the wave height at the shoreline and H_b is the breaking wave height. Sunamura (1983) obtained the following equation for H_b:

$$H_b = (\tan\alpha)^{0.2}\,(Ho/Lo)^{-1/4} Ho \tag{6}$$

where H_o and L_o are the wave height and the wavelength in deep water respectively. From Eqs.(4), (5) and (6), the following equation can be obtained:

$$R = 0.4(1-k)C^2(\cos\alpha)^2(\tan\alpha)^{0.8}(H_o/L_o)^{-1/4}H_o \qquad (7)$$

The coefficient $(1-k)C^2$ is formulated by comparison of Eq.(7) with data of Nakamura et al.(1972) against several types of seawalls located on complicated bottom profiles as follows:

$$(1-k)C^2 = 3.125(\tan\alpha)^{-0.2} \quad [1/3 \geqq \tan\alpha \geqq 1/50] \qquad (8)$$

Now, the wave run-up height R and the hypothetical slope $\tan\alpha$ can be determined by using Eqs.(1), (7) and (8). Since the relationship between R and $\tan\alpha$ is nonlinear, an iterative scheme is used.

First, the breaking water depth is evaluated by using the nomograph for non-overtopping by Goda (1970b). The reduction of the breaking water depth due to wave overtopping is ignored in order to simplify the treatment. The comparison of the breaking water depths and the breaking wave heights obtained from wave overtopping tests in this study with those for non-overtopping by Goda (1970b) is presented in Figure 3. This figure shows that the breaking water depth during wave overtopping is smaller than that for non-overtopping.

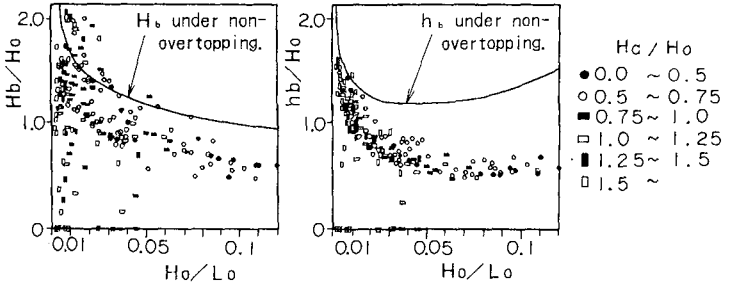


Figure 3. Breaking wave height and breaking water depth during wave overtopping.

Second, the wave run-up height is assumed and an approximate value of $\tan\alpha$ is obtained by using Eq.(1).

Third, the wave run-up height is calculated by Eq.(7) with the evaluated $\tan\alpha$. The procedure is repeated until the difference between successive solutions of the wave run-up height R and $\tan\alpha$ is less than some prescribed tolerance.

The wave run-up heights calculated by using Eqs.(1), (7) and (8) agree well with those obtained by using the nomograph presented by Nakamura et al.(1972) as shown in Figure 4. The comparison of calculated values with field data is shown in Figure 5.

Usually, the assumption that the influence of the seabed profile on the wave run-up height is small for non-breaking wave is acceptable. Thus the following equation of Sainflou (1928) was used to evaluate the wave run-up height.

$$R = [1.0 + \pi (H/L) \coth(2\pi h/L)] Ho \qquad (9)$$

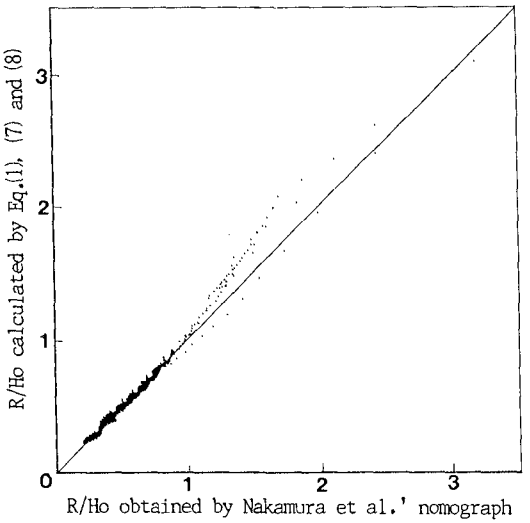


Figure 4. Comparison of R/Ho calculated by using Eq.(7) with R/Ho obtained by nomograph presented by Nakamura et al.

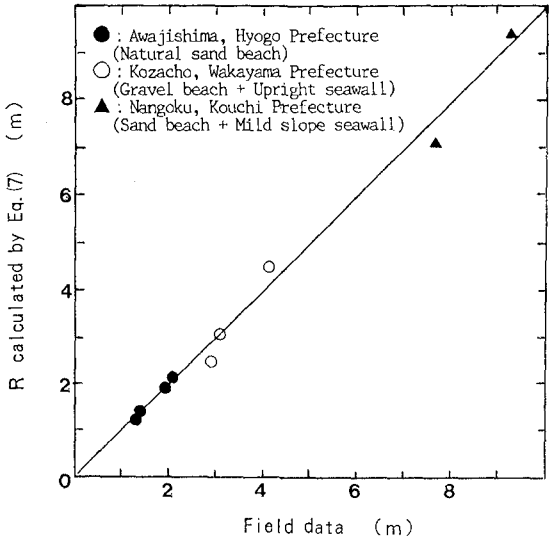


Figure 5. Comparison of R calculated by using Eq.(7) with field data.

New Method for the Evaluation of the Wave Overtopping Rate

(1) For Breaking Waves

The actual shape of the wave run-up profile is presented in Figure 6(a). Takada(1977) assumed that it could be approximated by the one presented in Figure 6(b) and studied the wave overtopping rate over one wave period T. He found that this value is proportional to the shaded area A in Figure 6(b). That is,

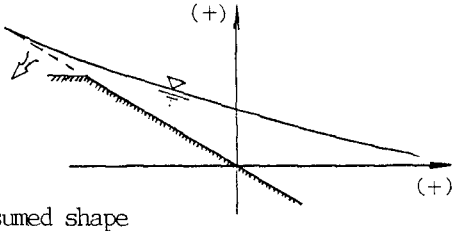
$$q \propto A \tag{10}$$

where q is the wave overtopping rate over one wave period ($m^3/m/T$) and A is a hypothetical area above the seawall crown in a wave run-up profile shown in Figure 6(b) and obtained by the following equation.

$$A=(R-Hc)\times[(Xo/R)-\cot\alpha\,](R-Hc)/2 \tag{11}$$

where Hc is the freeboard above SWL and Xo is the horizontal length of the shape of the wave run-up profile.

(a) Actual shape



(b) Assumed shape

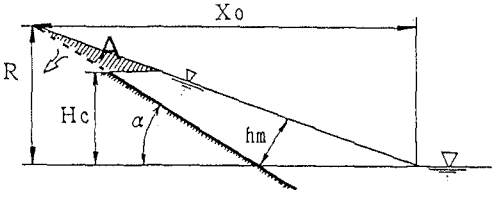


Figure 6. Actual shape and assumed shape of wave run-up profile.

From Eqs.(10) and (11), the overtopping rate q can be predicted by the following equation:

$$q=c\,[(Xo/R)-\cot\alpha\,](R-Hc)^2/2 \tag{12}$$

where c is a overtopping coefficient which can be determined from experiment.

In Figure 6, the upper part of the actual shape of the wave run-up is thinner than that of the assumed shape, and the value of Xo in the actual shape is longer than that of the assumed shape. Thus if the

value of Xo of the actual shape is used, the resultant evaluation of the area A will be extremely exaggerated. Therefore the value of Xo of assumed shape is used. It is calculated by using the following equation obtained from geometrical relationship.

$$Xo/R=\cot[\alpha-\tan^{-1}(hm/R/\sin\alpha)] \tag{13}$$

where hm is the maximum thickness of the water tongue shown in Figure 6(b).

The expression for hm can be found by the following procedure: By using the long wave theory, the wave run-up profile on the uniform slope can be obtained as follows:

$$\eta=\frac{H_a}{2}J_0\left(\frac{\omega}{2ig}\sigma\right)\sin\left(\frac{\omega}{2ig}\lambda\right)-\frac{2gH_a^2}{\sigma^2}J_1^2\left(\frac{\omega}{2ig}\sigma\right)\cos^2\left(\frac{\omega}{2ig}\lambda\right) \tag{14}$$

where η is the water surface elevation above SWL, H_a is the wave height at the point where there is no energy loss by the breaking waves and the average water level $\overline{\eta}$ nearly equals 0, ω is the angular frequency ($=2\pi/T$), J_0 and J_1 are the Bessel functions of the zeroth and first order respectively, $\sigma=4\sqrt{g(ix+\eta)}$, $\lambda=2(u-git)$, i is the bottom slope ($=\tan\alpha$), x is the spatial coordinate directed from shore towards offshore, u is the horizontal component of the water particle velocity and t is the time.

Next the simplified form of the energy equation proposed by Izumiya and Horikawa (1983) is solved in order to obtain the relationship between the wave height H_a and the breaking wave height H_b.

$$\frac{d\gamma}{dx}+\frac{5\gamma}{4x}-\underbrace{\frac{1}{400i}\frac{\gamma^2}{x}}_{\text{Bottom friction term}}-\underbrace{\frac{9}{80i}\frac{\gamma^3}{x}\left(1-\frac{0.072}{\gamma^2}\right)^{1/2}}_{\text{Breaking wave term}}=0 \tag{15}$$

The coordinates used here is the same as that in Figure 6(a) and γ is the wave height - water depth ratio. By performing a Taylor expansion on the breaking wave term in Eq.(15) with respect to $0.072/\gamma^2$ and omitting small terms at the position where the variable x is reasonably large, the remained term of the energy loss is the only that by the breaking waves. The integration of this simplified energy equation under the condition of $\gamma=\gamma_b$ at the breaking point $x=x_b$ results in Eq.(16).

$$\gamma=1/\left[\left(\frac{1}{\gamma_b^2}-\frac{9}{100i}\right)\left(\frac{x}{x_b}\right)^{5/2}+\frac{9}{100i}\right]^{1/2} \tag{16}$$

Experimentally it can be assumed that the position where the average water level $\overline{\eta}$ nearly equals 0 is expressed by $x_a\fallingdotseq 0.8x_b$. Therefore $H_a(\fallingdotseq\gamma\times 0.8x_b i)$ is obtained by Eq.(17).

$$H_a=(0.8H_b/\gamma_b)/\left[0.572\left(\frac{1}{\gamma_b^2}-\frac{9}{100i}\right)+\frac{9}{100i}\right]^{1/2} \tag{17}$$

If the maximum value of η at $x = 0$ is derived by the differential calculation of Eq.(14) and the wave set-up at the shoreline is considered by using Eq.(5), the following equation can be obtained as an approximation for hm.

$$\frac{hm}{H_b} = \frac{1}{2}\frac{H_a}{H_b} \cdot J_0\left(\frac{4\pi\sqrt{H_b}}{\sqrt{g}\,T\,i}\sqrt{\frac{hm}{H_b}}\right) + 0.8\,i^{0.6} \qquad (18)$$

The results of the calculations by using Eq.(18) are shown as the dotted lines in Figure 7. The plotted data are the measured values of the experiment for non-overtopping. The length, width and height of the two-dimensional flume used were 18m, 40cm and 75cm respectively, while the slope of the seabed was 1/10. The front slopes of the seawalls were 1:4, 1:2, upright. The wave height, period and water depth were variously changed. H_b was obtained by using Eq.(6). The values calculated by using Eq.(18) are larger than the measured values. It takes long time to calculate the Bessel function J_0 in Eq.(18). Therefore the use of the approximate expression of the Bessel function and the substitution of realistic values for hm induce the following equation:

$$\frac{hm}{H_b} \fallingdotseq 0.7\left[\frac{0.375}{\pi^{3/4}}\left(\frac{i}{\sqrt{0.8H_b/L_0}}\right)^{1/2} + 0.8\,i^{0.6}\right] \qquad (19)$$

The results of the calculations by using Eq.(19) are shown as the solid lines in Figure 7.

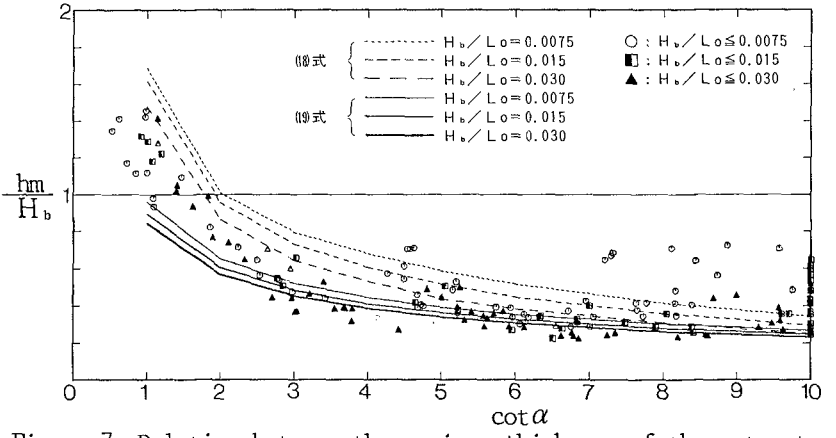


Figure 7. Relation between the maximum thickness of the water tongue and the bottom slope.

Finaly the overtopping coefficient C was investigated. The overtopping coefficient represents the percentage of the imaginary area of the wave run-up profile that passes over the sea wall crown and that actually reaches the inland side. It can be assumed that the overtopp-

ing coefficient increases with the increase of the lateral component of the water block movement of the wave run-up. Therefore this coefficient increases as the front slope angle of the seawall or the wave steepness becomes smaller. Because of this, the two parameters $(\cos\theta + \cos\alpha)/2$ and $(L_o/H_b)^{1/3}$ were considered, and the relationship between the non-dimensional wave overtopping rate and these parameters was investigated. In the above expression, θ is the front slope angle of the seawall. The experiment for overtopping was conducted by using the same flume and models that were used for the experiments of non-overtopping. The wave height, period and water depth were variously changed. The experimental results are shown in Figure 8 which indicates that the non-dimensional wave overtopping rate increases as the two parameters increase. By substituting experimental data into Eqs.(12), (13) and (19), Eq.(20) for the coefficient c can be obtained.

$$c = 0.1(L_o/H_b)^{1/3}(\cos\theta + \cos\alpha)/2 \qquad (20)$$

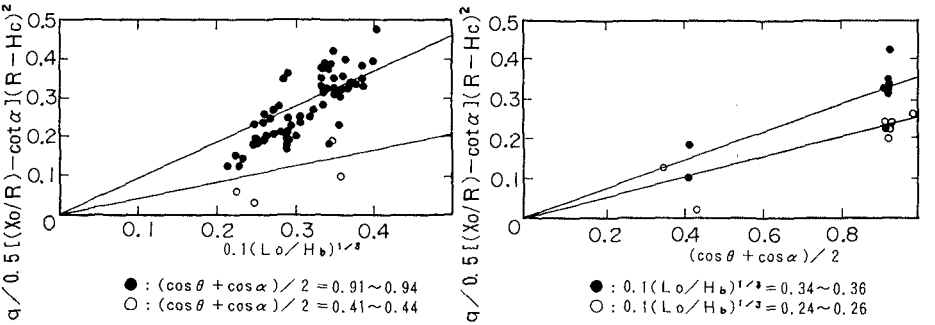


Figure 8. Relation between the wave overtopping rate and the wave overtopping coefficient.

Figure 9 depicts the comparison between the calculated values of q by Eqs.(12), (13), (19) and (20) and the experiment data. This figure suggests that there is a good fit between the calculated values of q and the measured ones. Figure 10 depicts the comparison of the values of q computed by using Eq.(12) with the ones computed by using the equation proposed by Kikkawa et al.(1967). Figure 11 is the same comparison between the values obtain by Eq.(12) and the experiment data for a compound bottom profile by Inoue et al.(1972). However the breaking water depths for non-overtopping were used for these calculations.

(2) For non-breaking waves

It can be assumed that the effect of the seabed profile on the wave overtopping rate is small for non-breaking waves. Therefore the following experimental equation by Takada(1977) was used.

$$q = 0.65\,(R - H_c)^2 \qquad (21)$$

where R is the wave run-up height determined by Eq.(9).

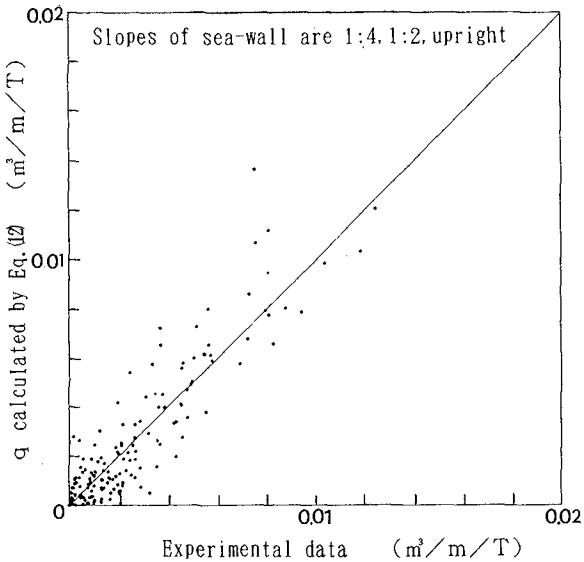


Figure 9. Comparison of q calculated by Eq.(12) with experimental data (regular waves).

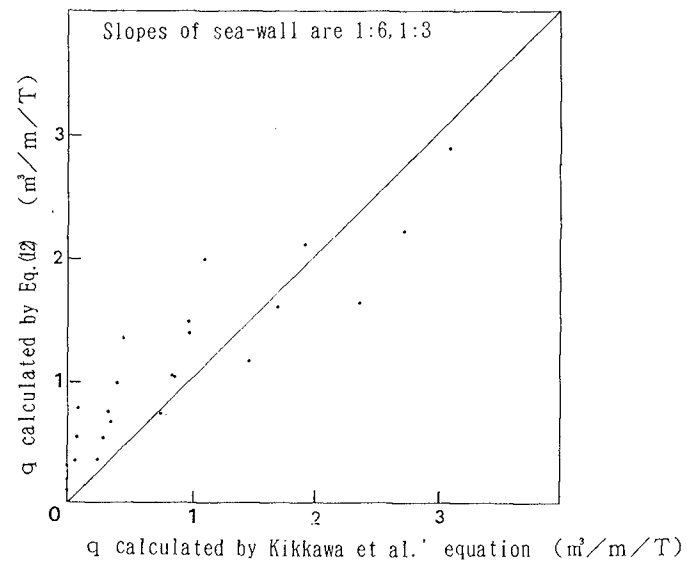


Figure 10. Comparison of q calculated by Eq.(12) with q calculated by the Kikkawa et al.' equation (regular waves).

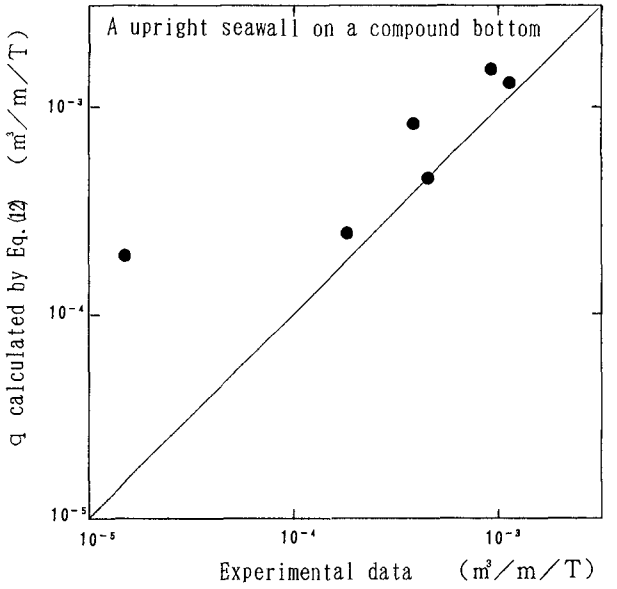


Figure 11. Comparison of q calculated by Eq.(12) with experimental data (regular waves).

(3) For irregular waves

The wave overtopping rate for irregular waves can be calculated by the following equation:

$$Q = \int_0^\infty \int_0^\infty q \cdot p \, dH \, dT \tag{22}$$

where, Q is the overtopping rate of irregular waves, p is the joint distribution function of wave heights and wave periods, q is the overtopping rate of the component waves and H and T are the wave height and the wave period of the component waves respectively. The term p proposed by Watanabe et al.(1984) can be expressed as follows:

$$\left.\begin{aligned} &p = p(\tau)\, p(\chi_f \mid \tau) / \chi_m(\tau), \\ &p(\tau) = \frac{\sqrt{1+\nu^2}}{1+\sqrt{1+\nu^2}} \cdot \frac{\nu^2}{[\nu^2 + (\tau-1)^2]^{3/2}}, \\ &\nu = \sqrt{(m_0 m_2 / m_1^2) - 1}, \\ &p(\chi_f \mid \tau) = (32/\pi^2)\chi_f^2 \exp[-4\chi_f^2/\pi], \\ &\chi_f = \chi / \chi_m(\tau), \\ &\chi_m(\tau) = \sqrt{S(f)f} \Big/ \int_0^\infty \sqrt{S(f)f}\, p(\tau)\, d\tau \end{aligned}\right\} \tag{23}$$

where $\chi = H/\overline{H}$, $\tau = T/\overline{T}$ (the overbar indicates an average value), f is the frequency, m_k is the kth order moment of the spectrum and S(f) is the Bretschneider-Mitsuyasu Spectrum.

The overtopping rate for breaking waves is calculated by using Eqs.(1), (7), (8), (12), (13), (19) and (20). That for non-breaking waves is

calculated by using Eqs.(9) and (21).

Figure 12 is comparison of the calculated values by this method with the prototype data. From this figure, it is remarkable that this method is applicable for the prototype conditions.

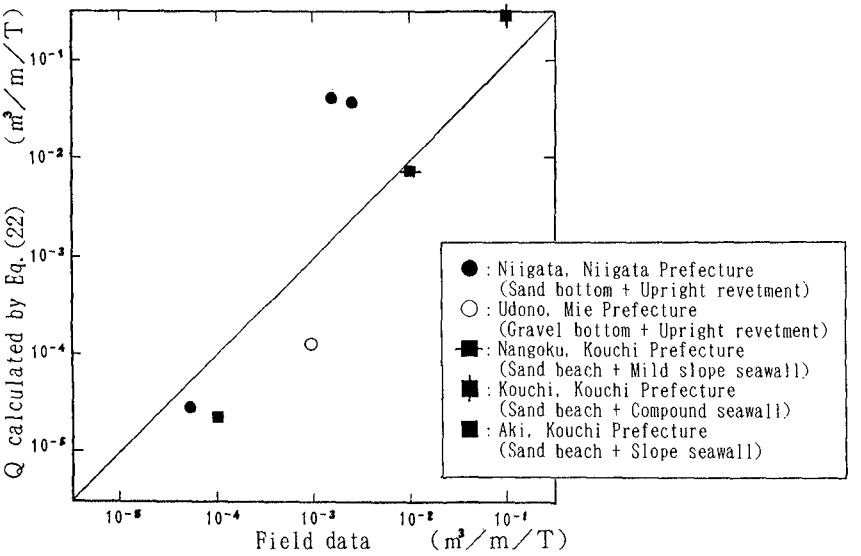


Figure 12. Comparison of Q calculated by Eq.(22) with field data. (irregular waves)

Application of The Present Method to Predict The Overtopping Rate in Case of The Mean Sea Level Rise Due to Greenhouse Effect

If the lapse-rate of air temperature is assumed to increase by greenhouse effect in future, there is a possibility that the intensity of typhoon becomes stronger. Manabe et al.(1990) predicted that the lapse-rate of air temperature is increased by this effect in the zone where many typhoons are generated. Therefore it is important to investigate the effect of the increment of the typhoon intensity on the wave overtopping rate in addition to the mean sea level rise.

Emanuel(1987) stated that 3° C increment in sea surface temperature leads to 30~40% increase in the maximum pressure drop of tropical cyclone and 15~20% increase in the maximum wind speed.

Now if these conditions are applied to a typhoon model (radius of the isopressure line 1000mb is 600km and central pressure 930mb), a remarkable increase in the wave overtopping rate were predicted by the present method and the results are shown in Figure 13. In this figure, type A is the case when there is a seawall of 10m height on the uniform sea bottom of 1/10 slope and type B is the case when there is the same seawall on the compound sea bottom of 1/10 and 1/100 slopes.

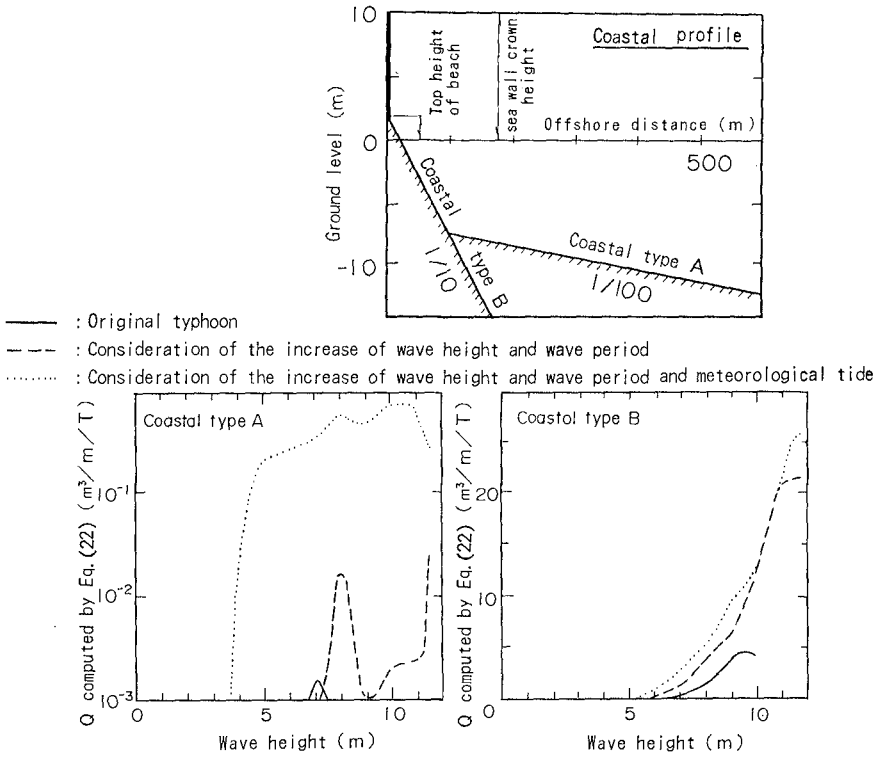


Figure 13. Increase in wave overtopping rate due to a typhoon.

Conclusions

(1) The proposed methods have been checked with the laboratory data as well as the field data. The agreement between the calculated values and the available data is favourablly good.

(2) If the intensity of typhoon becomes stronger by the greenhouse effect, the rate of wave overtopping may increase remarkably by the small increment of the typhoon intensity.

In case the bottom slope of sea is mild, the heights of infragravity waves become large in the surf zone, and the wave overtopping rates by these waves cannot be ignored. This problem should be a subject of the future study.

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