CHAPTER 137

NUMERICAL MODELLING OF THE STABILITY OF RUBBLE BASES

By

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ABSTRACT

A numerical model is developed to predict the stability of rubble bases exposed to wave action. A boundary element method is applied to solve the problem of wave interaction with a caisson and its rubble base, and to calculate the velocity field. The predicted velocities are used to determine the destabilizing forces on individual stones. Theoretical results reveal two local minima in stability: in very shallow water and at intermediate depths. The rubble base stability increases with decreasing incident wave height, the rubble base height and the bench width. Preliminary results also show that the stability depends on the damping properties and that the stability increases with increasing permeability.

INTRODUCTION

A rubble base exposed to a wave action has to satisfy several requirements. The design of a rubble base requires an estimation of the weight of stones used to build the rubble base which is one of the most important design parameters. These stones have to provide stability of the rubble base during large waves.

There are several empirical formulae for the determination of the stability of a rubble base. The empirical formulae are mainly a result of some modifications of the classical Iribarren or Hudson equations. However, it is a well known fact that the empirical formulae have a number of limitations. Additionally, an analysis of various parameters involved in the stability based on laboratory experiments is very time-consuming and expensive. Moreover, some conclusions regarding the stability

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based on laboratory experiments may be severely contaminated by scale effects. An alternative method for the analysis of the effect of various parameters on the stability is still a widely recognized need.

A numerical modelling of the stability of rubble mounds seems to be the alternative method. This method has been given more and more attention recently (McDougal and Sulisz 1990; Sulisz and McDougal 1991). The numerical modelling is employed in the present work by modelling theoretically the interaction of water waves with a caisson and its rubble base and then employing predicted flow fields in a stability model to determine the required stone size. A similar approach was applied to determine the stability of rubble mounds beneath caissons by Sulisz and McDougal (1991). However, they used a constant value of the damping coefficient, which may underestimate or overestimate damping properties of the rubble base.

NUMERICAL MODEL

The model for waves interacting with a caisson and its rubble foundation is based on the unsteady Forchheimer equation of motion in the pores of a coarse, granular medium (Sulisz 1983; Sulisz 1985). The wave flow in the porous domain $\boldsymbol{R}$ and the adjacent wave field is governed by the following equations

$$\nabla^2\Phi = 0 \ , \tag{1a}$$

$$S\frac{\partial\Phi}{\partial t} + \frac{1}{\rho}P + gz + f\omega\Phi = 0 \ , \tag{1b}$$

where the damping coefficient f, is

$$f = \frac{1}{\omega}\frac{\int\limits_{\boldsymbol{R}} \mathrm{d}\boldsymbol{R} \int\limits_{t}^{t+T} \varepsilon^2 \left(\frac{\nu V^2}{K} + \frac{C_f \varepsilon}{\sqrt{K}} |V| V^2\right) \mathrm{d}t}{\int\limits_{\boldsymbol{R}} \mathrm{d}\boldsymbol{R} \int\limits_{t}^{t+T} \varepsilon V^2 \mathrm{d}t} \ , \tag{1c}$$

and S is the inertial coefficient, ρ is the density of fluid, g is the acceleration due to gravity, ν is the kinematic viscosity, ε is the porosity, K is the intrinsic permeability, C_f is the turbulent damping coefficient, T is the wave period, P is the pressure, Φ is the velocity potential, and the velocity vector V=V $[\Phi_x, \Phi_z]$.

The above model, Eqs. (1), has successfully been applied to describe the interaction of waves with a rubble-mound breakwater (Sulisz 1985), a composite breakwater (Sulisz & McDougal 1988; Sulisz 1992b) and a rubble toe protection (Sulisz *et al.* 1989). The boundary element method has been used to determine Φ. Various results from the theoretical model have been confirmed by experimental data.

Velocity fields determined by a solution of the problem of wave interaction with a composite breakwater (Sulisz 1992b) are used as the input data for a stability model. Forces acting on an individual stone are computed tangential and normal to the slope from the following Morison-type equations

$$F_s = \frac{1}{2} C_{Ds} \rho A_s \boldsymbol{V}\cdot\boldsymbol{s} \, |\boldsymbol{V}\cdot\boldsymbol{s}| + C_{Ms} \rho \forall \frac{\partial \boldsymbol{V}}{\partial t}\cdot\boldsymbol{s} \pm \frac{1}{2} C_{Ln} \rho A_n (\boldsymbol{V}\cdot\boldsymbol{n})^2 , \qquad (2a)$$

$$F_n = \frac{1}{2} C_{Dn} \rho A_n \boldsymbol{V}\cdot\boldsymbol{n} \, |\boldsymbol{V}\cdot\boldsymbol{n}| + C_{Mn} \rho \forall \frac{\partial \boldsymbol{V}}{\partial t}\cdot\boldsymbol{n} \pm \frac{1}{2} C_{Ls} \rho A_s (\boldsymbol{V}\cdot\boldsymbol{s})^2 , \qquad (2b)$$

where $\boldsymbol{s}$ and $\boldsymbol{n}$ are the tangential and normal unit vectors with respect to the slope, respectively; (C_{Ds}, C_{Dn}), (C_{Ms}, C_{Mn}), (C_{Ls}, C_{Ln}) are the drag, inertia and lift force coefficients in the direction of $\boldsymbol{s}$ and $\boldsymbol{n}$, respectively; A_s and A_n are the cross-sectional areas of the stone in the direction of $\boldsymbol{s}$ and $\boldsymbol{n}$, respectively; $\forall$ is the volume of the stone.

The velocity vector in Eqs. (2) is calculated at various positions along the slope from a velocity potential corresponding to a fluid domain rather than rubble base domain. Due to uncertainties in determining the direction of the lift forces it is assumed that the lift forces maximize the instability.

Stability is based on the static equilibrium of the stone by examining lifting, sliding or rolling. An analysis shows that for a nearly spherical stone, the stability condition for rolling is the most critical. The detailed analysis of the static equilibrium of a sphere located on a layer of spheres in contact, shows that the critical condition for rolling is

$$W_B (\tan\beta - \tan\alpha) \cos\alpha < F_n \tan\beta - F_s \qquad (3)$$

where W_B is the stone weight in water, β is the angle associated with location of spheres in relation to each other, and α is the slope angle.

Several parameters like density, size, shape and placement scheme of the armor unit are involved in the stability model. Of course, the stability is also a function of the wave period, wave height, caisson shape, rubble base shape, porosity, permeability and turbulent damping properties of the rubble base. The model makes possible a detailed examination of the contribution of various parameters on the overall stability. Because calculations can be conducted on personal computers, the model may provide engineers with a useful tool for examining a variety of cases at a very low cost.

RESULTS

The described model is first employed to calculate the flow field for a problem of interaction of water waves with a caisson founded on a rubble base (Fig. 1). The boundary element method is used to solve a boundary-value problem and to determine the velocity potential in the rubble base and in the vicinity of the caisson and its rubble base. The caisson of the width $2b/h = 0.5$ is analyzed. The rubble base of the caisson is considered to be of trapezoidal cross-section and of various widths and heights, but of fixed slopes 1:2. Additionally, it is assumed that the inertial coefficient $S = 1$, porosity $\varepsilon = 0.4$, and that the damping coefficient is known for each analyzed wave frequency ($0.04 \le kh \le 4$). The calculations are conducted for a constant value of a dimensionless coefficient $\hat{f}$ which is related to the damping coefficient f via the following relation

$$f = \hat{f} / (kh \tanh kh)^{0.5} \qquad (4)$$

where k is the wave number and h is the water depth.

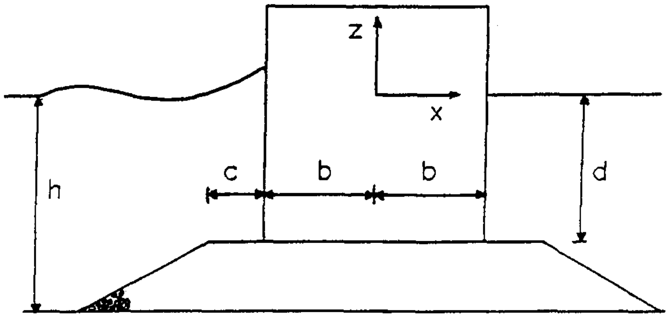

Fig. 1. Definition sketch

A presentation of the results based on Eq. (1b) may be confusing and their analysis difficult to follow if the damping coefficient f is kept constant over a wide range of wave frequencies, as is stressed by Sulisz (1992a). This is because the damping coefficient f is a function of the wave frequency even in a case where a pure Darcy motion law is applicable ($C_f = 0$) . Further analysis shows that the calculations based on a constant value of the damping coefficient usually underestimate rubble mound damping properties in shallow water or overestimate them for waves of intermediate lengths and in deep water. Of course the dimensionless coefficient $\hat{f}$ is still a function of wave frequency which is evident from Eq. (1c) and Eq. (4). However, in a case of a pure Darcy flow the coefficient $\hat{f}$ is directly related to a Darcy coefficient and the wave frequency is not involved in the relation. This implies that a presentation of the results versus a function of wave frequency, based on a constant value of the dimensionless coefficient $\hat{f}$, properly reflects features of a case with the pure Darcy flow in a rubble mound. The presentations of the results versus a function of wave frequency for different values of $\hat{f}$ also provide insight into a significance of the nonlinear damping term included in Eq. (1c) for an analyzed quantity.

A predicted flow field is used to calculate the required stone size of the rubble base. The calculations are conducted for the drag force coefficient $C_{Ds} = C_{Dn} = 0.7$, the inertia force coefficient $C_{Ms} = C_{Mn} = 0.5$, the lift force coefficient $C_{Ls} = C_{Ln} = 0.5$. These values of the force coefficients are chosen based on the results of some previous works on the stability problem. Of course the force coefficients are parameters of the stability model and they require further research based on experimental verifications.

The stability number, N_s, is used to present the output of the calculations conducted to estimate the required stone size of the rubble base. The stability number is defined as follows

$$N_s = \frac{(6/\pi)^{1/3}}{(S_r-1)} \frac{H_d}{D} , \tag{5}$$

where S_r is the relative stone density, H_d is the incipient damage wave height, and D is the equivalent spherical stone diameter.

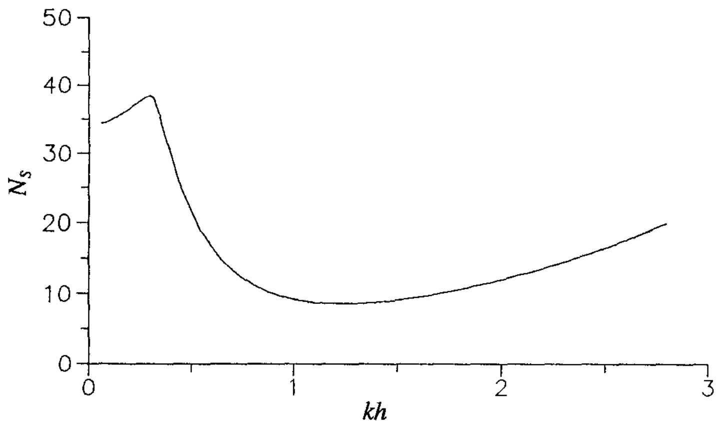

Fig. 2. Stability number versus kh**;** d/h **= 0.6,** c/h **= 0.2,** H/h **= 0.1,** $\hat{f}$ **= 1.**

A typical dependency of the stability number on dimensionless wave number, kh, is presented in Fig. (2). In general, for relatively low rubble bases analyzed here, two local minima in stability may occur: a local minimum in very shallow water and a local minimum at intermediate depths. The local minimum in very shallow water is of interest only for rubble bases of small damping properties

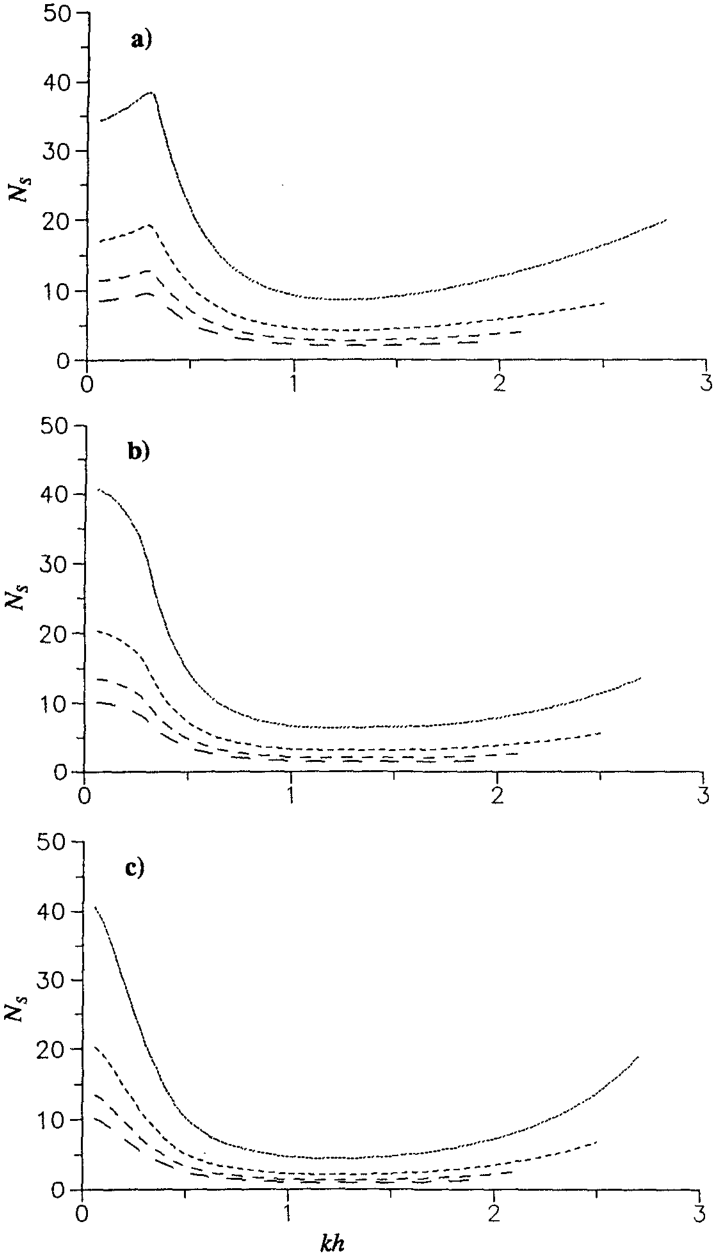

Fig. 3. Stability number versus kh; $d/h = 0.6, \hat{f} = 1$, ···· $H/h = 0.1$, - - $H/h = 0.2$, – – $H/h = 0.3$, —— $H/h = 0.4$, a) $c/h = 0.2$, b) $c/h = 0.4$, c) $c/h = 0.6$.

where it may become a global minimum of the stability number. The local minimum in stability at intermediate depths is, in a majority of cases, our main interest, because this minimum is usually a global minimum of the stability number and requires further investigation. Of course in some cases, the plots of the stability number versus the dimensionless wave number may become more complex and additional minima may occur.

The results presented in Fig. (2) show that the incident wave length is essential to predict the stability number. An additional parameter involved in the stability that belongs to a group of parameters associated with wave excitation properties, is the incident wave height (H). Figs. (3)-(4) show the stability number plotted versus dimensionless wave number for four ratios of the incident wave height to the water depth (H/h = 0.1, 0.2, 0.3, 0.4). Some plots for the steepest waves are omitted because the corresponding parameters of the incident waves exceed the breaking limits for progressive waves. Breaking may still occur for some waves included in Figs. (3)-(4) due to large reflection from a composite breakwater.

The results in Figs. (3)-(4) are intuitive and the general conclusion follows that reported by Sulisz (1992a). The stability increases with decreasing incident wave heights. However, it is necessary to point out that the calculations are conducted for a constant value of the dimensionless coefficient $\hat{f}$. Thus the presented results correspond to a case of the pure Darcy flow in the rubble base. Although, a contribution from the nonlinear damping term in the motion equation that is included in Eq. (1c) is expected to be rather small due to large reflection, some changes in the presented results may occur if the incident wave height is additionally included in the calculations of velocities via Eq. (4) and Eq. (1c). The main changes are expected in shallow water where a contribution from the nonlinear damping term may be significant and where the stability model is sensitive to the rubble base damping properties.

The results presented in Figs. (3)-(4) also enable us to examine some parameters from a second group of parameters affecting the rubble base stability. This group is associated with the shape of the composite breakwater. The stability number is plotted versus dimensionless wave number for six rubble bases. An effect of the rubble base height and the bench width on the rubble base stability is investigated. The results show that deeper rubble bases are more stable. This conclusion is fairly well supported by experimental data (Brebner and Donnelly 1962). A somewhat surprising result refers to the effect of the bench width on the rubble base stability. The plots indicate an increase in stability with decreasing the bench width. This is observed for both analyzed rubble base heights.

The local minimum of the stability number in very shallow water was reported by Sulisz and McDougal (1991), who presented the stability number versus the dimensionless wave number based on a constant value of the damping coefficient. Since the damping coefficient is a function of wave frequency, the calculations based on a constant value of the damping coefficient usually, as is pointed out above, underestimate media damping properties in shallow water, or overestimate them at intermediate depths and in deep water. Further calculations conducted by applying the present approach with $\hat{f}$ = constant, confirm an occurrence of the local

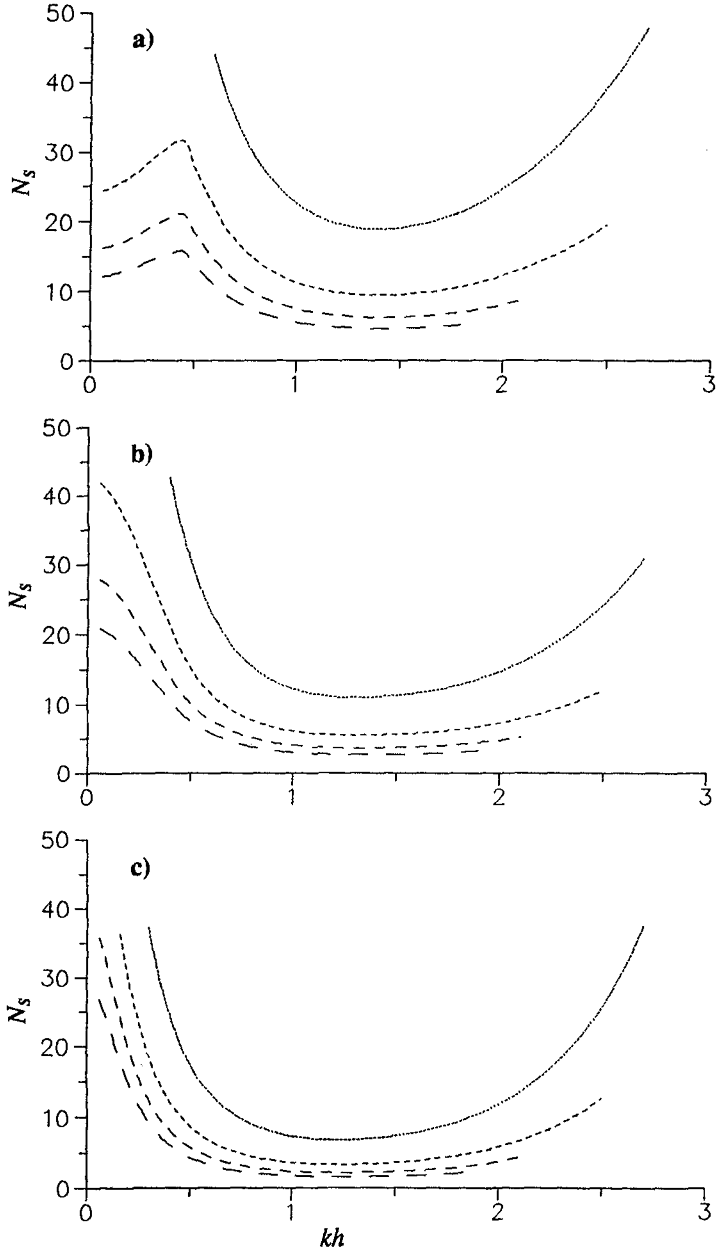

Fig. 4. Stability number versus kh; d/h **= 0.8,** $\hat{f}$ **= 1, ···· H/h = 0.1, - - H/h = 0.2, – – H/h = 0.3, —— H/h = 0.4, a) c/h = 0.2, b) c/h = 0.4, c) c/h = 0.6.**

minimum in very shallow water for rubble bases of small damping properties, however, this local minimum does not occur for rubble bases of significant damping properties as is shown in Fig. (5).

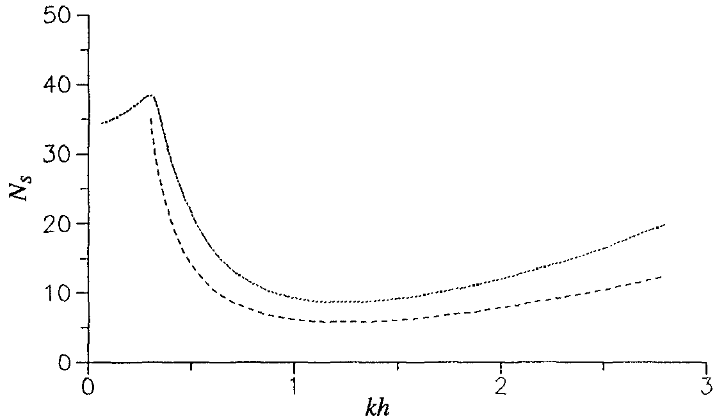

Fig. 5. Stability number versus kh; d/h = 0.6, c/h = 0.2, H/h = 0.1,···· $\hat{f}$ = 1, - - $\hat{f}$ = 10

The results presented in Fig. (5) also show the importance of the rubble base damping properties for the stability of the rubble bases. Several conclusions may be drawn from the results presented in Fig. (5). A preliminary analysis indicates that if the damping property of the analyzed rubble base decreases, the minimum stability number increases. It has already been pointed out that in a case of a pure Darcy flow, the coefficient $\hat{f}$ is directly related to the Darcy coefficient. The results in Fig. (5) indicate that for a pure Darcy flow in the analyzed rubble base, the stability of the rubble base increases with increasing its permeability. This conclusion is drawn on a base of the minimum stability number at intermediate depths which is our main interest. An opposite conclusion may be drawn in very shallow water. Additionally, the results indicate the importance of the nonlinear damping term included in the present approach in Eq. (1c) for the stability analysis. The analysis shows that this term is of relatively minor importance for the minimum stability number associated with intermediate depths, but may be of major importance in shallow water.

CONCLUSIONS

A theoretical analysis of the stability of rubble bases is conducted, applying an approach based on numerical modelling. The flow in the rubble base of a caisson-type breakwater is described by a linearized Forchheimer equation of motion. The boundary element method is applied to solve the problem of wave interaction with the caisson and its rubble base, and to calculate the velocity field. The predicted velocities are used in a Morison-type equation to determine the destabilizing forces on individual stones.

The stability is shown to depend on several parameters. There are three or even four groups of parameters involved in the stability analysis. The detailed analysis is conducted to investigate the parameters associated with incident wave properties and rubble base shapes. Preliminarily examined are some parameters associated with hydraulic properties of the rubble base.

The incident wave properties are shown to be essential to predict the stability number. Theoretical results reveal two local minima in stability: a local minimum in very shallow water and a local minimum at intermediate depths, but in some cases additional minima may occur. The local minimum at intermediate depths is usually becoming a global minimum. The stability, as intuitively expected, is also a function of the incident wave height and the stability increases with decreasing wave height. The rubble base shape belongs to the second group of parameters involved in the stability. The results show that the stability increases with decreasing rubble base height and the bench width. The preliminary results indicate the necessity of including damping properties in the stability analysis and show that for the analyzed rubble base the minimum stability number increases with increasing permeability.

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REFERENCES

Brebner, A. and Donnelly, P. 1962. Laboratory study of rubble foundations for vertical breakwaters. Proc. of the 8th Coastal Engineering Conference, ASCE, 408-429.

Mettam, J. D. 1980. Can we develop new breakwater armor formulae. Proc. of the 17th Coastal Engineering Conference, ASCE, 2304-2323.

Sulisz, W. 1983. Transmission of waves through permeable breakwaters of arbitrary cross-sections. Ph. D. Thesis, Polish Academy of Sciences, Institute of Hydroengineering, Gdańsk.

Sulisz, W. 1985. Wave reflection and transmission at permeable breakwater of arbitrary cross-section. Coastal Engineering, 9, 371-386.

Sulisz, W. & McDougal, W. G. 1988. Wave interaction with a composite breakwater. Proc. of an International Conference on Computer Modelling in Ocean Engineering, Venice, 715-721.

Sulisz, W., McDougal, W. G. and Sollitt, C. K. 1989. Water wave interaction with rubble toe protection. Ocean Engineering, 16, 463-473.

Sulisz, W. & McDougal, W. G. 1991. Wave interaction with a composite breakwater. Proc. of the Second International Conference on Computer Modelling in Ocean Engineering, Barcelona, A. A. Balkema, 715-721.

Sulisz, W. 1992(a). Stability analysis for multilayered rubble bases. Manuscript in preparation.

Sulisz, W. 1992(b). Wave loads on a caisson founded on a multilayered rubble base. Manuscript in preparation.