CHAPTER 139

On the Chaotic Structure of Tide Elevation in the Lagoon of Venice

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Abstract

Sea level oscillations related to astronomical and meteorological tides in the Lagoon of Venice are analysed in order to find evidence of a possible chaotic nature of the phenomenon. On the basis of the analysis of the signal spectrum and by considering the values of the correlation dimension and of the largest Lyapunov coefficient of the attractor of the system, it is possible to infer that tidal oscillations inside the Lagoon of Venice are the result of deterministic dynamics, i.e. of a chaotic system characterized by few degrees of freedom. In the second part of the paper, taking advantage of the chaotic nature of the system, Farmer & Sidorowich's (1987) algorithm is used in order to make predictions of the time development of the system.

Introduction

The time development of dynamic variables of natural systems often shows random oscillations which sometimes are superimposed on a regular and predictable signal. Such behaviour can be the result either of a stochastic or deterministic non-linear process, highly influenced by initial conditions known in literature as "deterministic chaos". In the latter case, the time development of the system is somehow similar to that of a stochastic process even though from a mathematical point of view its dynamics are entirely deterministic.

From a practical point of view, the main difference between a stochastic process and a process showing deterministic dynamics is the different number of degrees of freedom necessary to describe the state of the system. In fact, when a system shows a chaotic behaviour, its dynamics can be described using a limited number of degrees of freedom. In other words, the time development of the system can be obtained by integrating a small number of ordinary differential

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equations. This result is not applicable to the behaviour of stochastic systems which are characterized by an infinite number of degrees of freedom. The detailed description of such systems is extremely complex and stochastic or statistical methods are recommended for the study of the time development of the system.

Recent studies have shown the existence of deterministic chaos in the time development of climate variables such as wind velocity (Tsionis & Elsner (1989)), atmospheric pressure (Henderson & Wells (1988)), rainfall (Sharifi, Georgakakos & Rodriguez-Iturbe (1990)). Those studies have opened up new fields for the study of the predictability of natural events.

In this paper, the time oscillations of water elevation at Venice are considered, depurated from the contribution due to sea waves, to obtain information about "high waters" and in particular about their frequency and their predictability.

Tidal levels recorded at Punta della Salute and Diga Sud Lido between 1975 and 1984 and published by the "Consorzio Imprese Veneto-Emiliane" are considered. The study is based on recent measurements, however a preliminary investigation performed on data recorded at the beginning of the century has shown similar results.

Because extreme events are of more practical interest, the analysis is performed also for the signal envelope of maxima which is obtained starting from the knowledge of diurnal and semidiurnal tidal oscillations.

The procedure used in the rest of the paper is the following: first the time oscillations of the water level in the lagoon of Venice are analysed by studying both the hourly measurements and the 'envelope' of maxima recorded during one year for different years. The presence of a predictable periodic component and of an irregular component is shown. Indeed, the Fourier spectra of the signals relative to the hourly measurements show peaks related to the diurnal, semidiurnal and moon tide and a broad band part, which is characteristic of random and/or chaotic systems. Moreover, the intensity of the continuous part of the spectra turns out to be of the same order of magnitude as the peaks. Then the dynamic behaviour of the system is analyzed in the pseudo-phase space where the attractor is reconstructed by means of the time-delayed coordinate technique. The characteristics of the attractor are evaluated and in particular the correlation dimension, which gives indications of the degrees of freedom of the system, is computed along with the largest Lyapunov coefficient which is a measure of the sensitivity of the system on initial conditions. The results obtained give strong indications of the presence of chaotic dynamics. For this reason, an attempt to predict the time development of the system by using the deterministic algorithm by Farmer & Sidorowich (1987) is made. The success in predicting the future time development of the system on the basis of historical records by means of deterministic methods is a further indication of the presence of deterministic chaos.

Signal and spectrum of tidal elevation

It is widely recognized that the understanding of the dynamics of currents and sea level oscillations is extremely important in most coastal regions and particularly in estuaries and lagoons. Typically, sea level oscillations related to the astronomical tide are the most significant even if sometimes meteorological factors such as wind, a non uniform distribution of atmospheric pressure and storms can induce sea level oscillations of the same order of magnitude as tidal oscillations.

Indeed, in Venice, "high" waters are often caused by the simultaneous presence of high tide and meteorological factors. While tidal oscillations, being the result of the periodic motion of celestial bodies, are deterministic and thus predictable, the effect of meteorological events is aperiodic and thus not easily predictable.

In fact, the astronomical tide is caused by spatio-temporal variation of the gravitational field due to the relative motion of the earth with respect to other celestial bodies. An analysis of the phenomenon taking into account all the possible influences is extremely complex. However, considering the order of magnitude of gravitational forces induced on the earth's surface by different celestial bodies, it can be inferred that in order to study sea level rise due to the astronomical tide it is sufficient to consider the relative motion of the earth, the moon and the sun. Usually tidal oscillations are decomposed into a number of sinusoidal time components, each with its own periodicity. Five basic periods are usually taken into account: 1 day due to the earth's rotation, 29.53 days due to the rotation of the moon around the earth, 365.24 days due to the rotation of the earth around the sun, 8.847 years due to the motion of the moon perigee and 18.616 years due to the rotation of the orbital plan of the moon.

Since in the present work we study sea level oscillations when meteorological factors prevail, the time scale of interest to us is of one week and thus periodicities of 365.24 days, 8.847 and 18.616 years can be ignored. In other words, we will assume that the sea level oscillates with periodicities of 1 and 29.53 days around a mean level slowly varying with moderate escursions during one year.

The periodic nature of the astronomic component of the tide is evident in figure 1 where the tidal levels (η) recorded during the months of August and September 1981 are plotted. Similar results are obtained considering different years. During summer and spring, the influence of wind, storms and other meteorological events on sea level oscillations is usually negligible. On the other hand, aperiodic meteorological events become relevant in autumn and winter. In figure 2 the tidal curve relative to November and December 1981 is reported in order to show the aperiodic character of exceptional events. The aperiodic character of extreme events is still more evident by looking at figure 3 where the envelope of maxima is shown for the years 1980-1984.

The spectrum of the signal, shown in figure 4(a) for the year 1981, supports this argument showing two peaks related to the periodicities of 12 and 24 hours

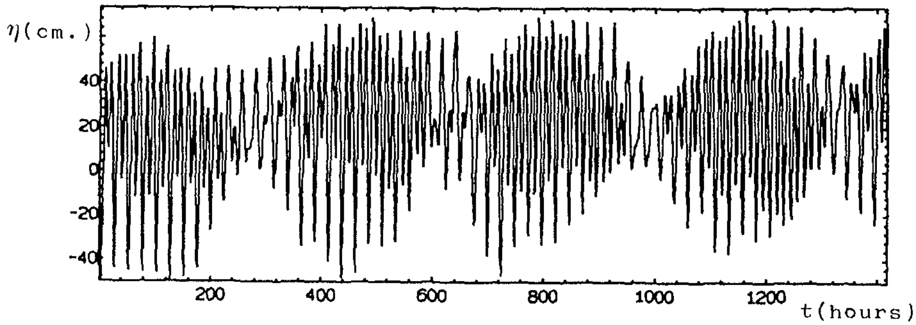

Figure 1: Tidal levels recorded at Punta della Salute during August and September 1981.

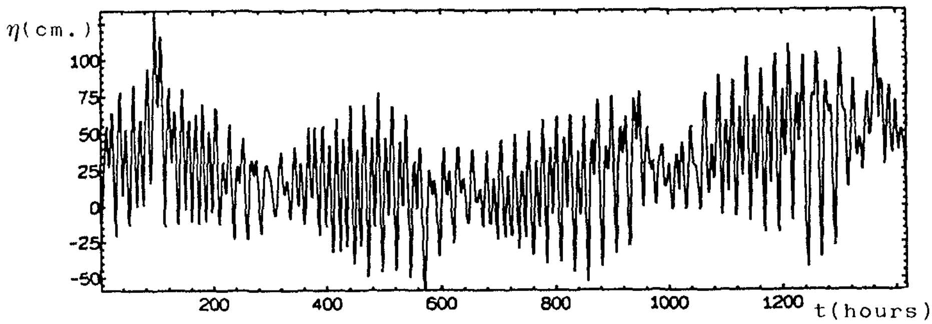

Figure 2: Tidal levels recorded at Punta della Salute during November and December 1981.

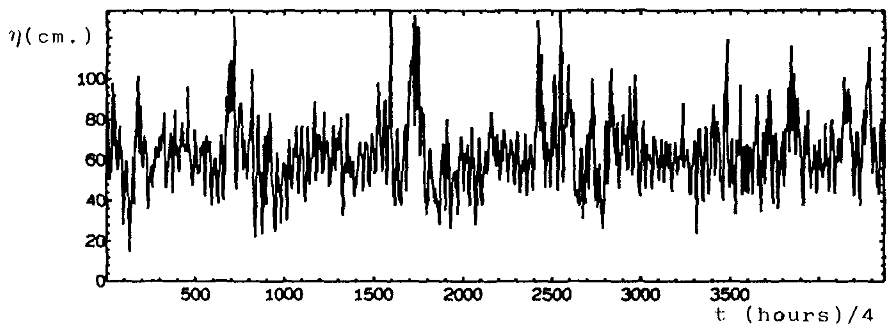

Figure 3: Envelope of maxima for the years 1980, 1981, 1982, 1983 and 1984.

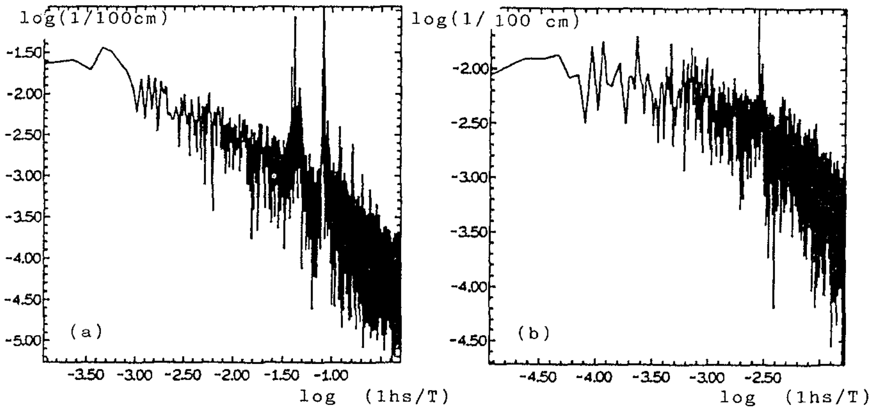

Figure 4: a) Spectra for the year 1981, b) Spectra of the 'envelope of maxima' for the years 1980, 1981, 1982, 1983 and 1984.

and a broad band part with the same order of magnitude.

The intensity of the broad band part varies when different years are considered: in fact, it is larger for years which have experienced more "high waters". However, considering different years, no relevant differences are found. The spectrum of the 'envelope' of maxima (see figure 4(b)) shows no peaks since the periodicities of 12 and 24 hours have been removed from the time sequence and the broad band part is larger and stronger with respect to that of the original signal.

So far, we have shown evidence of the non-periodic nature of sea level oscillations inside the lagoon of Venice; in the following we will investigate the nature of such oscillations. In fact, they could either be the result of a stochastic system, i.e. a dynamic system with a high number of degrees of freedom or the result of a chaotic system with few degrees of freedom.

The Chaotic Character of Tidal Elevation

In order to study the aperiodic character of tidal oscillations, it is necessary to perform a quantitative study of the attractor of the system. As suggested by Takens (1981) the trajectory of the system is reconstructed into a "pseudo-phase" space by using the time sequence of the values attained by one physical variable characteristic of the phenomenon. More specifically the $N-$dimensional vectors $\vec{s}$, describing the trajectory of the system in the pseudo-phase space, are obtained on the basis of the measurements $f(t)$ of the tidal level performed at the stations of Punta della Salute and Diga Sud Lido:

$$\vec{s}(t) = [s_1, s_2, ..., s_N] = [f(t), f(t-\tau), ..., f(t-(N-1)\tau)] \tag{1}$$

where f is the water level as a function of time and τ is an arbitrary time delay.

Takens (1981) showed that an attractor topologically equivalent to that of the original system, is obtained independently of the value of τ if N is sufficiently large. However, for computational reasons, it is necessary to accurately choose these parameters in order to have an accurate estimate of the characteristics of the attractor. In fact, if τ is too small, the attractor collapses along the line $s_1 = s_2 = s_3 = ...$ and the computation of the characteristics of the attractor becomes inaccurate.

In literature, it is suggested that the value of τ as the time delay relative to the first zero of the autocorrelation function be chosen. According to Battiston & Zambella this procedure gives a value of τ equal to 20 hours. However, in the following we have used values of τ equal to 50, 100 and 200 hours because these values have led to a better estimate of the Lyapunov exponent and of the correlation dimension. As far as the value of N is concerned, we have computed the characteristics of the attractor for increasing dimensions of the pseudo-phase space until non relevant differences were observed between N and $N+1$.

It is well-known that a chaotic attractor possesses a geometric structure called fractal attractor which has a finite and generally non integer dimension. To establish the nature of the aperiodic oscillations of the tidal wave, it is necessary first to evaluate the dimension of the attractor. In order to obtain a quantitative estimate of the possible fractal structure of the attractor, we computed the correlation dimension as defined by Grassberger & Procaccia (1983). As suggested by Takens (1981), the trajectory of the system into the pseudo-phase space is represented by a set of M points $\vec{s}_i$ defined by (1) and their relative distance is computed using the Euclidean distance. Then the correlation function is defined as the limit for M tending to infinity of the number of pairs with a relative distance d_{ij} less than r divided by M^2.

$$C(r) = \lim_{M\to\infty} \frac{1}{M^2} \; \{\text{Number of pairs } (\vec{s}_i, \vec{s}_j) \text{ such that } d_{ij} < r\} \tag{2}$$

Grassberger & Procaccia (1983) showed that for many attractors the correlation function for r tending to zero behaves like a power law, i.e.

$$\lim_{r\to 0} C(r) = ar^d \tag{3}$$

The correlation dimension is then defined as the exponent d of the power law, which can be expressed as:

$$d = \lim_{r\to 0} \frac{\log_{10} C(r)}{\log_{10} r} \tag{4}$$

In figure 5, the correlation function relative to the signal of tidal elevation is plotted for the years 1980 and 1982 and for a few dimensions N of the embedding (r_{max} denotes the maximum linear extent of the attractor). It is possible to observe that in both cases, by increasing N, the slope of the correlation function

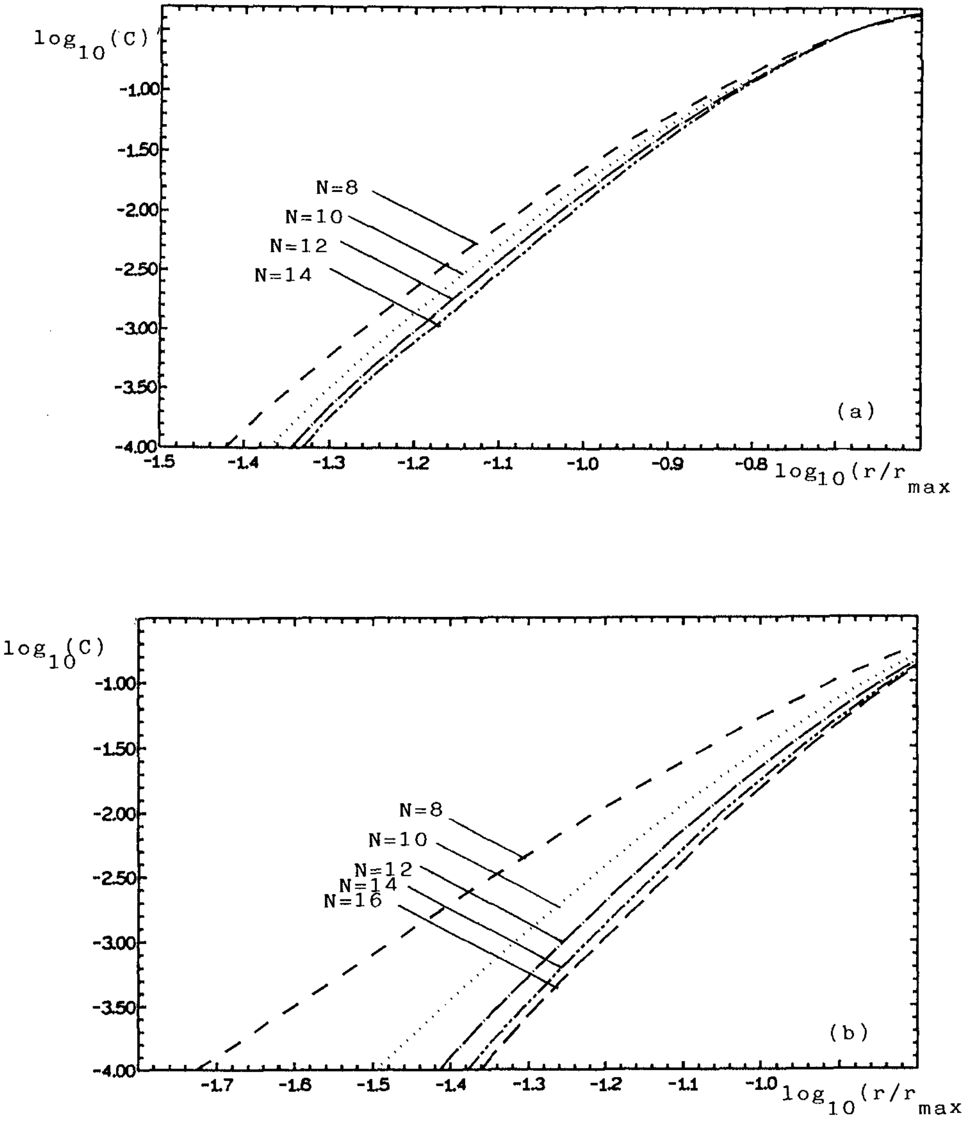

Figure 5: Correlation function for the years a) 1980 $\tau = 200$; b) 1982 $\tau = 200$.

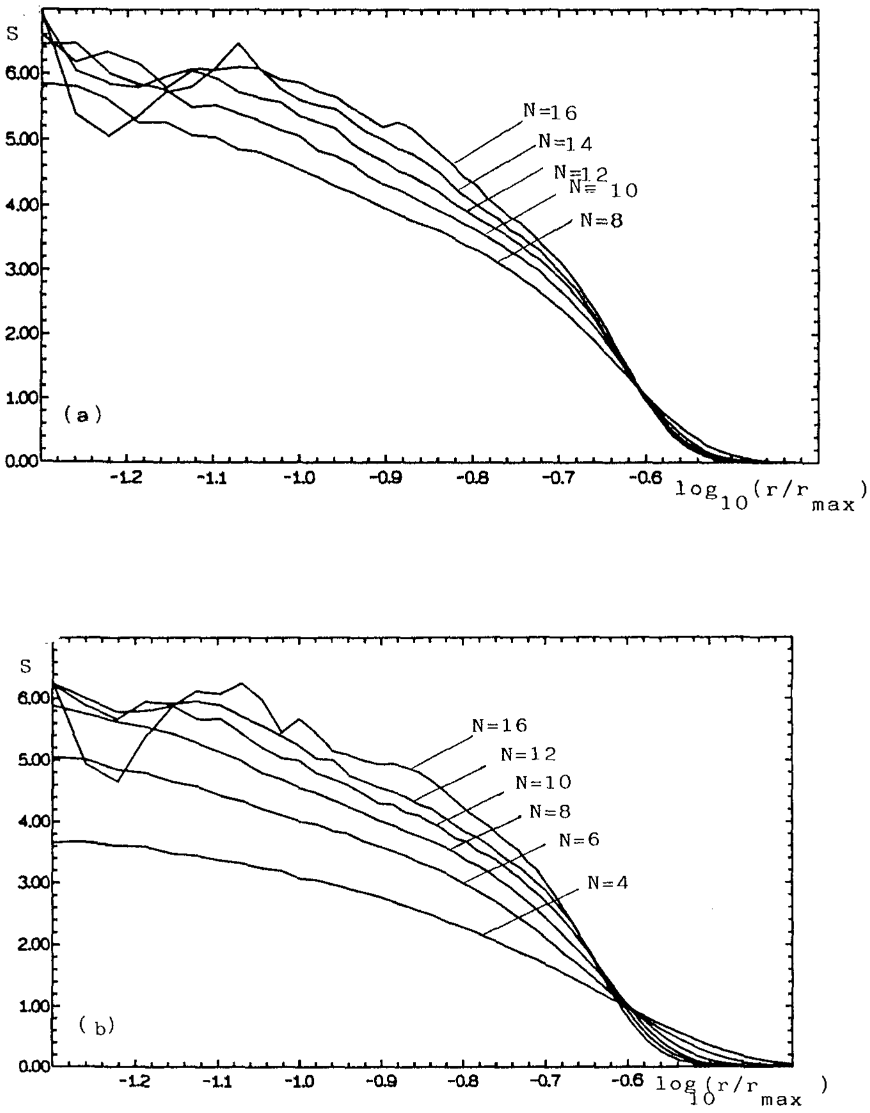

Figure 6: Slope of the correlation function for the years a) 1980 $\tau = 200$; b) 1982 $\tau = 200$.

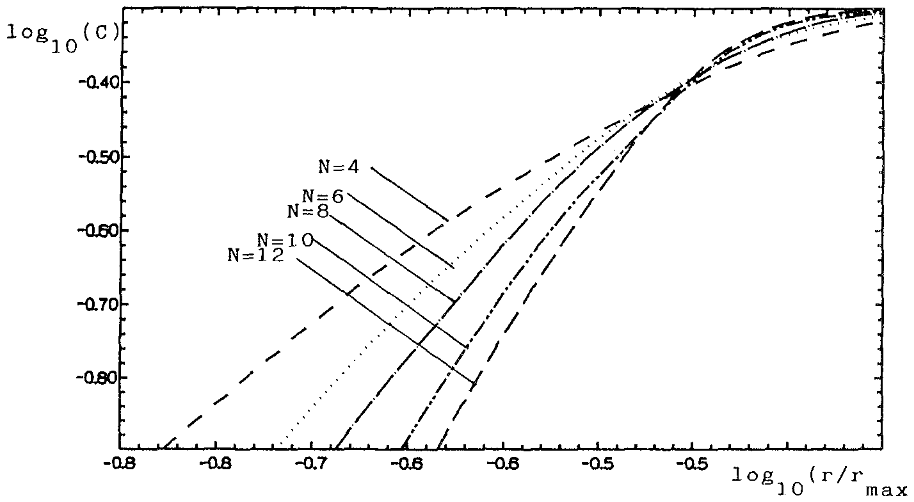

Figure 7: Correlation function for the 'envelope of maxima' relative to the years 1980, 1981, 1982, 1983 and 1984 $\tau = 40$.

reaches a limiting value for small but finite values of r where the limit defined by (4) can be evaluated and in the meantime boundary effects can be ignored. The value of the correlation dimension can best be estimated by looking at figure 6 where the function $S(r)$, corresponding to the slope of the correlation function, is plotted as a function of $\log_{10}(r/r_{max})$ for the same years as in figure 5.

It can be seen that for sufficiently large values of N and for small values of $\log_{10}(r/r_{max})$, S attains a constant non integer value close to 6. This value can be regarded as an approximation of the correlation dimension d.

A smaller value of the correlation dimension is obtained considering the attractor relative to the envelope of maxima. In figure 7, where the slope of the correlation function relative to the envelope of maxima is plotted for different values of N it can easily be seen that for small $r/r_{\max}$ the correlation dimension tends to a constant non integer value close to 2. The fact that the correlation dimension of the envelope of maxima is lower than that of the original signal is reasonable since by extracting the envelope of maxima a number of degrees of freedom related to the astronomic tide have been removed. As we shall see in the following, the lower dimension of the envelope of maxima turns out to be an advantage when making predictions of the temporal development of the system.

From the results shown so far, it is possible to conclude that sea level oscillations inside the lagoon of Venice are the result of a chaotic dynamic system since the number of degrees of freedom of the attractor is limited. Hence, in principle, it would be possible to predict the time development of tidal elevation by integrating a limited number of ordinary differential equations.

The following step is to determine the time interval within which a trajectory of the system can be predicted with a given uncertainty starting from an initial state known with an assigned error. It is well-known that chaotic attractors show an exponential divergence of the trajectories in the phase space. After some time, two states initially close to each other develop into two states far apart. Thus, the prediction of the time development of a given state, known with a given uncertainty, can be obtained only for a time interval which depends on the rate of divergence of the trajectories in the phase space. This rate of divergence is expressed by the largest Lyapunov coefficient.

Lyapunov coefficients can be defined by considering the time development of a hypersphere lying on the attractor reconstructed into the pseudo-phase space. Due to the chaotic nature of the system, the hypersphere will develop into an elipsoid. The i-th Lyapunov coefficient is defined in terms of the length of the i-th principal axis of the elipsoid (p_i) by means of the following relationship:

$$\lambda_i = \lim_{(t_1-t_o)\to\infty} \frac{1}{t_1 - t_0} \log_2 \frac{p_i(t_1)}{p_i(t_o)} \tag{5}$$

where the values of λ_i are ordered in ascending order and $t_1 - t_o$ is the time interval during which the computation is performed.

Thus, the Lyapunov coefficients are related to the average expansion or contraction of the hypersphere in the different directions of the phase space. Axes which expand on average originate positive values of λ_i while axes which on average contract give rise to negative values.

In the present paper the procedure suggested by Wolf et al (1985) to compute the largest Lyapunov coefficient is employed.

In table 1 the values obtained for the largest Lyapunov exponent (λ_1) are reported for the years ranging between 1980 and 1984.

TABLE 1

year	1980	1981	1982	1983	1984
λ_1(hours^{-1})	0.025	0.024	0.026	0.023	0.026

Similar values are obtained by taking into account the 'envelope of maxima'.

The values obtained for λ_1 would lead to the conclusion that sea level oscillations inside the Lagoon of Venice could be predicted for long periods, at least from a theoretical point of view. However, it should be considered that the use of a finite number of experimental data does not allow the desired infinitesimal length scales of the attractor to be tested. These scales are also inaccessible due to the presence of noise on finite length scales. Therefore, also taking into account that the chaos-producing structure of the attractor might be of small spatial extent, the estimate of the largest Lyapunov exponent of the system and thus of the timescale on which the system dynamics becomes unpredictable may be affected by a significant error. This fact can cause an underestimate of the largest Lyapunov coefficient.

The Predictions

Even though the characteristics of its attractor could not be quantified accurately due to the knowledge of a finite number of data, from the results previously described it is possible to infer that the system has a chaotic nature. Therefore, the prediction of the time development of the system can be attempted by using algorithms which take advantage of the deterministic nature of the system.

Even though the extreme sensitivity of the time development of the system on initial conditions poses some limits to the possibility of predicting its future time development, it is possible to make accurate predictions for small times and, at least in principle, to give an estimate of the error affecting the predictions.

In the present paper use has been made of the method proposed by Farmer and Sidorowich (1987). As a first step, a time sequence of data is used to represent a state on the attractor in the pseudo-phase space by means of (1). Secondly, a functional relationship between the current state $\vec{s}(t)$ and the future state $\vec{s}(t+\Delta t)$ is assumed to exist:

$$\vec{s}(t + \Delta t) = f_{\Delta t}(\vec{s}(t)) \qquad (6)$$

Due to the chaotic nature of the system, the function $f_{\Delta t}$ is certainly non-linear. To obtain an approximation to $f_{\Delta t}$, Farmer & Sidorowich (1987) suggest using the knowledge of the time development of a number of points on the attractor which are near to $\vec{s}(t_o)$, i.e. to the state whose time development we want to predict. Let us consider the $P + 1$ points of the attractor ($\vec{s}(t_j)$, $j = 1, 2, ..., P + 1$) which are the nearest to $\vec{s}(t_o)$. A local approximation of $f_{\Delta t}$ is obtained on the basis of the values $\vec{s}(t_j + \Delta t)$ attained by the P + 1 points. The easiest procedure would be a zero order approximation where $P = 0$. In this case $\vec{s}(t_1 + \Delta t)$ could be assumed to be the approximation of $f_{\Delta t}$. A better approximation is obviously obtained assuming $P + 1$ larger than N and determining, by means of the least square method, the coefficients of the linear relationship $a * \vec{s}(t_j) + \vec{b}$ which best approximate $\vec{s}(t_j + \Delta t)$.

After the values of the matrix a and $\vec{b}$ have been obtained, the prediction can easily be performed:

$$\vec{s}(t_o + \Delta t) = a * \vec{s}(t_o) + \vec{b}.$$

The results obtained are presented in figures 8 and 9 where predicted and measured levels are plotted as a function of time both for tidal levels and for the envelope of maxima. The measurements of tidal elevations during the years 1980, 1981, 1982 and 1983 have been used to reconstruct the attractor and the

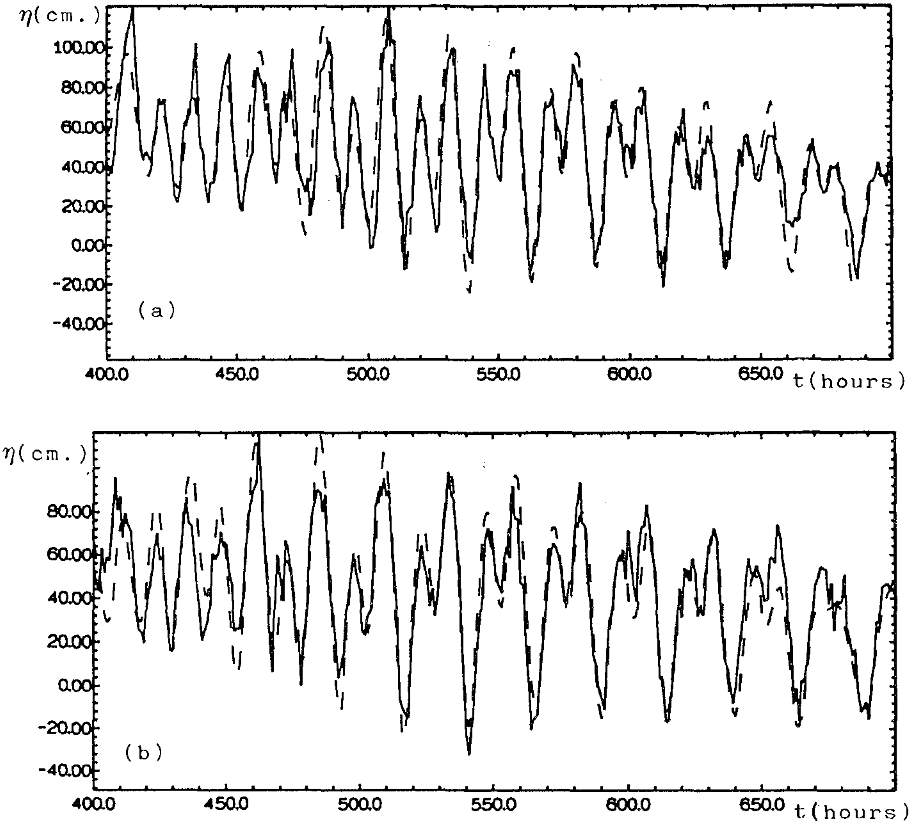

Figure 8: Predictions (—) and measurements (- - -) of tidal elevations at Punta della Salute in the last months of 1984. a) $\Delta t = 2$ hours; b) $\Delta t = 24$ hours.

prediction of tidal levels has been made for the last three months of 1984 when there were some exceptional events.

In figure 8 the observed (- - -) and the predicted (—) tidal levels are shown for November 1984 when three events of high water were observed. In figure 8(a) the prediction 2 hours into the future is shown while in figure 8(b) the prediction 24 hours into the future is presented. In figure 9(a) and (b) the predictions of the envelope of maxima are shown for 12 and 24 hours into the future respectively.

It is possible to see that the predictions relative to the envelope of maxima are more accurate than those of the original signal; this fact can be explained on the basis of the lower correlation dimension of the signal envelope of maxima with respect to that of the signal of tidal elevations.

As expected, by increasing Δt the predictions are less accurate both for the

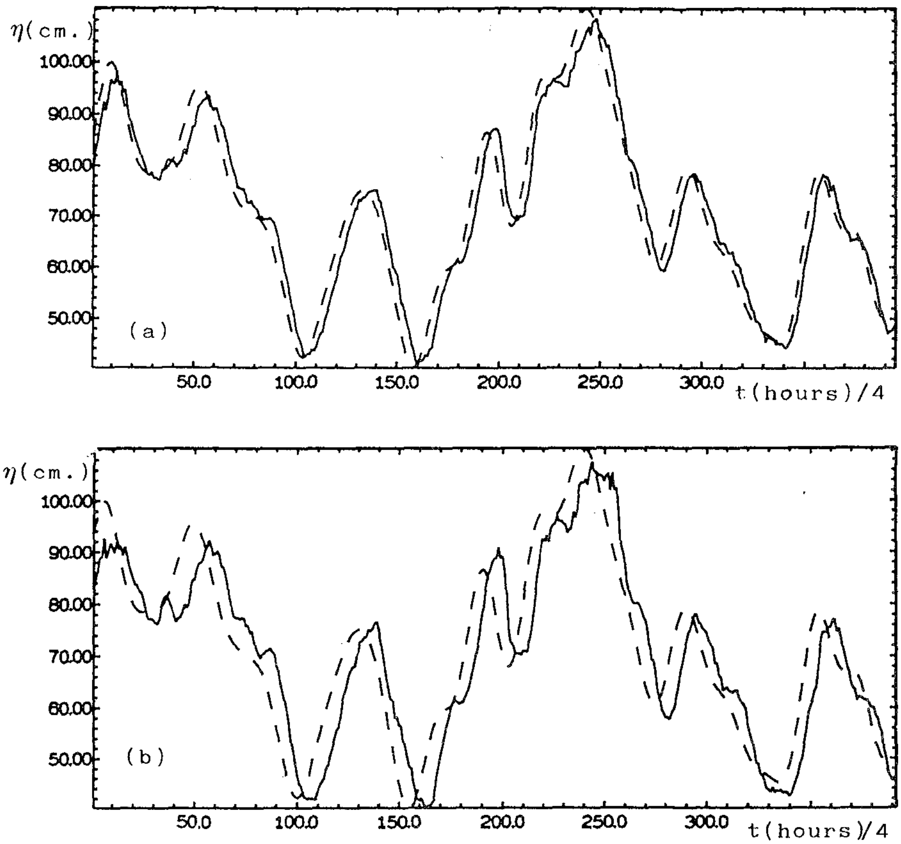

Figure 9: Predictions (—) and envelope of maxima (- - -) at Punta della Salute in the last months of 1984. a) $\Delta t = 12$ hours; b) $\Delta t = 24$ hours.

original signal and for the envelope of maxima.

However, the prediction could be improved by considering larger data set to reconstruct the attractor and by using a higher order approximation to obtain the local predictor $f_{\Delta t}$. In order to evaluate the performance of the proposed model it is necessary to take into account that, as reported by different authors (Battiston & Zambella (1981), Cecconi et al (1992)), the error of the statistical methods for predictions for a time in the future less than 15 hours ranges between 10 to 15 cm.

Acknowledgements

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