CHAPTER 155

SHEET FLOW UNDER NONLINEAR WAVES AND CURRENTS

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Abstract

Experiments were conducted on initiation and transport rate of sheet flow under asymmetric oscillations. Values of net sand transport rates under asymmetric oscillations and superimposed currents were also measured. In this paper, based on physical interpretations, a new parameter representing the observed transport mechanism is derived. Using this parameter a general transport formula is then presented and applied to estimating the net transport rate and its direction; fairly good agreement with the experimental results is observed. It is shown that the newly introduced parameter together with the sediment Reynolds number can fairly well express the inception of sheet flow.

1. Introduction

This paper is concerned with intense sand transport under high oscillatory shear stress conditions when ripples have been washed out and the bed is flat; the so-called sheet flow transport. As the orbital velocity of fluid over a rippled bed is increased, the oscillatory ripples lose their height and finally will all disappear. Sediment motion then occures as a sheet of sand within a few centimeters of the bottom, moving with the intense orbital motions. A nearly flat bed with a high sediment transport rate is a condition that can be expected over much of the surf zone during storms (see for example Dingler and Inman, 1976). Therefore,

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detail knowledge of sheet flow transport is of decisive importance in a successful modeling of beach topography change.

There have previously been several experimental studies of sheet flow transport from different points of view. Among them Manohar (1955), Chan *et al.* (1972), Komar and Miller (1975), Kaneko (1981), and Horikawa *et al.* (1982) have considered the inception of sheet flow, while Horikawa *et al.* (1982), Sawamoto and Yamashita (1986), and Ahilan and sleath (1987) have been concerned with the rate of sediment transport. These studies, however, have all considered the sheet flow under purely sinusoidal oscillatory motions. Hence, only transport rates over half a cycle have been obtained, because in a sinusoidal oscillation the net transport over a complete cycle is zero. In reality, however, waves are nonlinear and asymmetry of the near-bottom velocity during the two half cycles of the orbital motion or steady currents superimposed on the waves can produce significant net transport.

The objectives of the present study were to investigate the initiation of sheet flow under asymmetric oscillations, to measure the net sheet sand transport rate under asymmetric oscillations and superimposed steady currents, and, to establish a transport rate formula to be used in modeling of beach topography change.

2. Experiments

The present experiments were carried out in a loop-shape oscillatory/steady flow water tunnel at the University of Tokyo. The tunnel is driven by a piston to generate oscillations of any given arbitrary shape and by a pump to superimpose a steady current on the oscillation. It has a horizontal test section of rectangular cross section with glass side walls and removable acrylic ceilings. The test section is of 2.0 m length, 0.22 m height, and 0.24 m width. For the present experiments the width of the test section was reduced to 0.12 m in order to provide large values of velocity. Sand traps made of honeycombs were installed at both ends of the test section in order to trap the sand which would otherwise move out of the test section. Toyo-ura sand with median grain size of 0.2 mm and mean fall velocity, W, of 2.3 cm/s was used. The depth of the bed of sediment was approximately 7 cm with a length of 1.6 m.

Five oscillation periods ranging from 1 s to 4 s and four nonlinearity indices $u_{max}/\hat{u} = 0.5, 0.6, 0.7, 0.8$ (see Fig. 1) were selected. Longer oscillation periods could not be used because of the limited length of the test section. Time histories of the water particle displacement were simulated on the base of the first-order cnoidal wave theory which is valid in relatively shallow water. A laser-Doppler velocimeter was used to measure the water particle velocities through the side wall of the tunnel. Velocity profiles corresponding to each case were measured over a fixed bed and were then repeated over the sandy bed.

At first, conditions for disappearance of ripples and inception of sheet flow were investigated and net transport rates due to pure oscillatory motions were

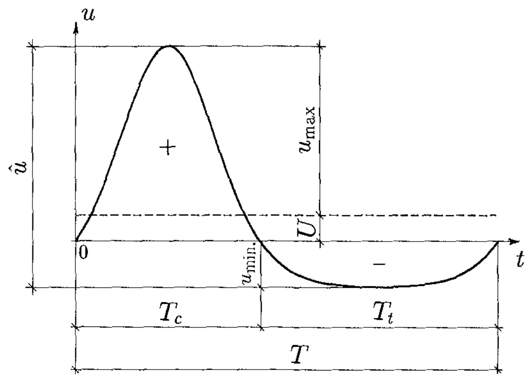

Figure 1: **A typical velocity profile.**

measured. Then four steady flow velocities ($U \simeq -20, -10, 10, 20$ cm/s) were superimposed on each of the oscillations and resulting net transport rates were measured.

To investigate the initiation of sheet flow, the normal test procedure was for the tunnel to be set in motion with the required frequency and nonlinearity index, while the bed was flat. At the begining the amplitude of the oscillation was small in order to let the ripples appear and form. Then the amplitude was gradually increased until the ripples disappear and the bed became flat again. For the measurement of net transport rate the procedure was as follows. First a thin plate was carefully placed at the middle of the test section, to separate its two sides completely, and the sand was weighted and placed in each side in such a way to make a uniform flat bed. Then the plate was removed and the tunnel was set in the required motion for as long as 100 cycles of oscillations provided that the bed remained nearly flat. The test time duration was shorter if the bed started to distort. Finally the thin plate was again placed at the middle of test section and the sand from each side was removed and weighted to give the net transport rate.

3. Initiation of Sheet Flow

A total of 18 cases covering periods of 1 to 4 s and nonlinearity indices $u_{max}/\hat{u} = 0.5$ to 0.8 were tried. There was no steady current. In order to classify the data by using the conventional parameters, a knowledge of near-bottom orbital velocity amplitude and period is required. Unlike a sinusoidal velocity profile, appropriate definitions for the above mentioned quantities are less clear for an asymmetric velocity profile and they could be different according to the phenomenon under investigation.

Analysis of the current data showes that the maximum velocity for the incep-

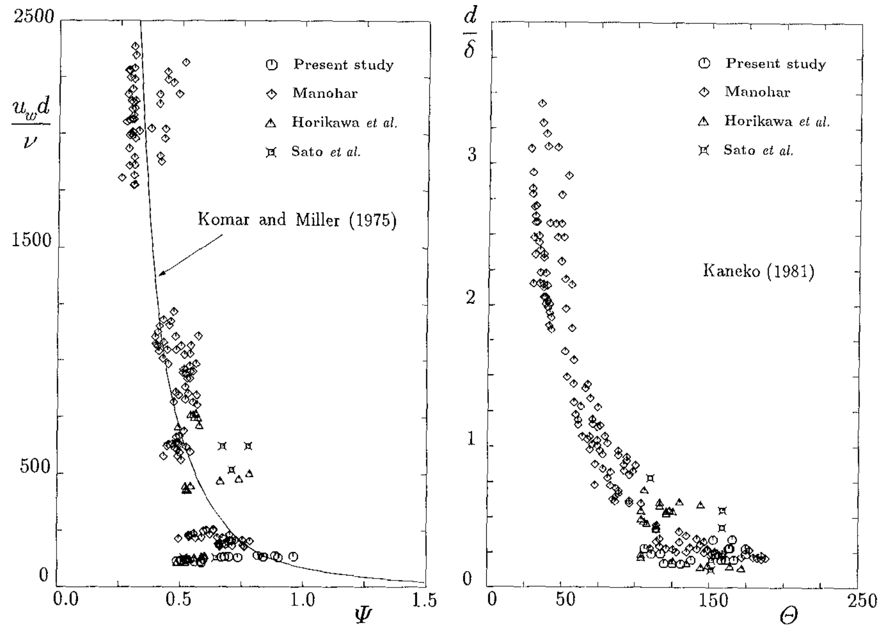

Figure 2: **Behavior of two of the available criteria for inception of sheet flow.**

tion of sheet flow changes with both the wave period and nonlinearity but the effect of nonlinearity is much more significant; namely, for a given wave period, a more asymmetric velocity profile requires a higher maximum velocity to wash out the ripples and initiate the sheet flow transport. It is concluded that as far as the disappearance of ripples and inception of sheet flow is concerned, it is the energy contained in the near-botton velocity profile which is the important factor in defining the near-bottom orbital velocity amplitude, u_w. That is why for a more asymmetric profile, with a slender crest and a wide but small trough, a higher maximum velocity is required. Hence, u_w is defined as

$$u_w^2 = 2u_{\rm rms}^2 = \frac{2}{T}\int_0^T u^2 dt \qquad (1)$$

in which $u_{\rm rms}$ is the root mean square value of u and the other definitions are given in Fig. 1. In case of a sinusoidal velocity profile u_w obtained by (1) is equal to the velocity amplitude. Hence, u_w defined by (1) is an equivalent sinusoidal velocity amplitude for the asymmetric velocity profile.

Manohar (1955) has reported data on disappearance of ripples for a wide range of sediments and under different flow conditions. His data are reanalyzed and used together with the data of Horikawa *et al.* (1982), Sato *et al.* (1987), and the present data. Several different parameters and their combinations are examined. It is found that the two parameters used by Kaneko (1981),

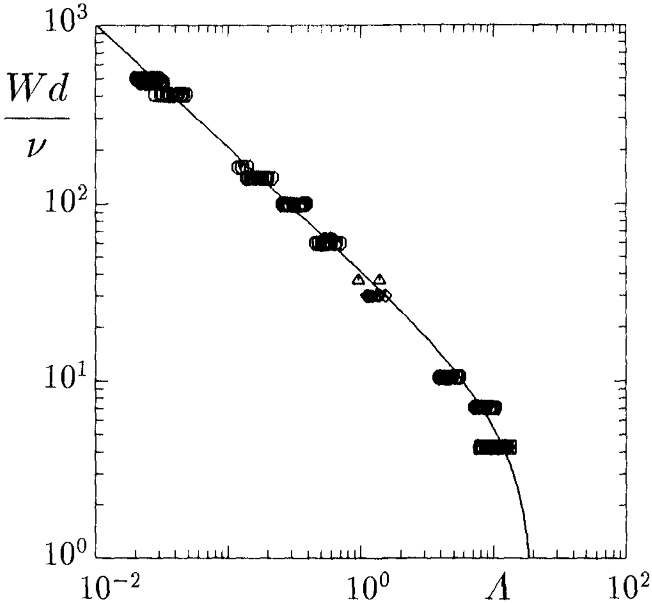

Figure 3: **Sheet flow inception criterion of this study.**

$$\Theta = \frac{u_w^2}{sgd} \quad \text{and} \quad \frac{d}{\delta} \ ,$$

give a more continuous and clear trend, shown in Fig. 2, than the other existing criteria including that of Komar and Miller (1975). Here d is the grain size, $\delta = \sqrt{2\nu T/\pi}$ the Stokes layer thickness, ν the fluid kinematic viscosity, g the acceleration of gravity, and $s = (\rho_s - \rho)/\rho$ with ρ_s and ρ being the densities of sand and fluid, respectively. One can say that under an intense fluid motion it is the strong turbulence structure near the bottom which destroy the ripples rather than a shear mechanism. Therefore, Θ which represents the flow energy or intensity is more appropriate than the relative bottom shear stress, Ψ, used by Komar and Miller.

In the next section a new parameter, ω, is introduced which may be further changed to

$$\Lambda = \frac{u_w^2}{sgWT} \cdot \left(\frac{\delta}{d}\right)^2 \ . \tag{2}$$

The following relation is then found to give a good inception criterion (Fig. 3).

$$\frac{Wd}{\nu} = 42.5\left(\frac{1}{\Lambda} - 0.05\right)^{0.69} \tag{3}$$

It should be pointed out again that the present data are for the initiation of sheet flow over a rippled bed. In case of an initially flat bed, however, sheet flow could occur in lower flow velocities. This is because that over a rippled bed an excess amount of energy, comparing with a flat bed, is required to level the bed.

4. Net Transport Rate

More than 100 experiments were carried out and the net transport rates were measured. According to Ahilan and Sleath (1987), the sheet sand transport rate over a half cycle of a sinusoidal oscillation can be well estimated as a function of the Shields stress, $\varPsi$, by the formula of Shibayama and Horikawa (1980),

$$\phi = 19.0\,\varPsi^3 \quad , \tag{4}$$

which is same as the formula of Madsen and Grant (1976) with a different value of the coefficient. In this relation $\phi = (1-\lambda)q/Wd$ is the nondimensional transport rate, q the volume of transported sand per unit time, and λ the porosity of sand. Madsen and Grant suggested that under an asymmetric oscillation the overall net sediment transport rate, $\varPhi = (1-\lambda)Q/Wd$, could be calculated by integrating their formula with respect to time, half cycle by half cycle and taking the difference between the rates during successive half cycles. Denoting this by $\varPhi_{\text{cal.}}$, one may write

$$\varPhi_{\text{cal.}} = \frac{19.0}{T}(T_c\varPsi_c^3 - T_t\varPsi_t^3) \tag{5}$$

in which

$$\varPsi_c = \frac{1}{2}f_c\frac{u_c^2}{sgd} \quad , \quad \varPsi_t = \frac{1}{2}f_t\frac{u_t^2}{sgd}$$

and

$$u_c^2 = \frac{2}{T_c}\int_0^{T_c} u^2 dt \quad , \quad u_t^2 = \frac{2}{T_t}\int_{T_c}^{T} u^2 dt \tag{6}$$

where u_c and u_t are equivalent sinusoidal velocity amplitudes for the positive and negative portions of the velocity profile, respectively, and f_c and f_t are their corresponding friction factors. Figure 4 shows the comparison between calculated and measured net transport rates, $\varPhi_{\text{cal.}}$ and $\varPhi_{\text{meas.}}$, for all the cases. It is seen that except for few cases the disagreement is generally high and this method fails to predict not only the magnitude of the net transport rate but also its direction. It is clear that such a procedure could be correct only if the sand transports during the two half cycles are completely independent of each other. Then, under such a condition and for a velocity profile similar to the one shown in Fig. 1 (*i.e.* $|\, u_{\max} \,| \gg |\, u_{\min} \,|$ and $T_c \leq T_t$), the net transport rate should usually be in the direction of positive velocity. However, during the present experiments it

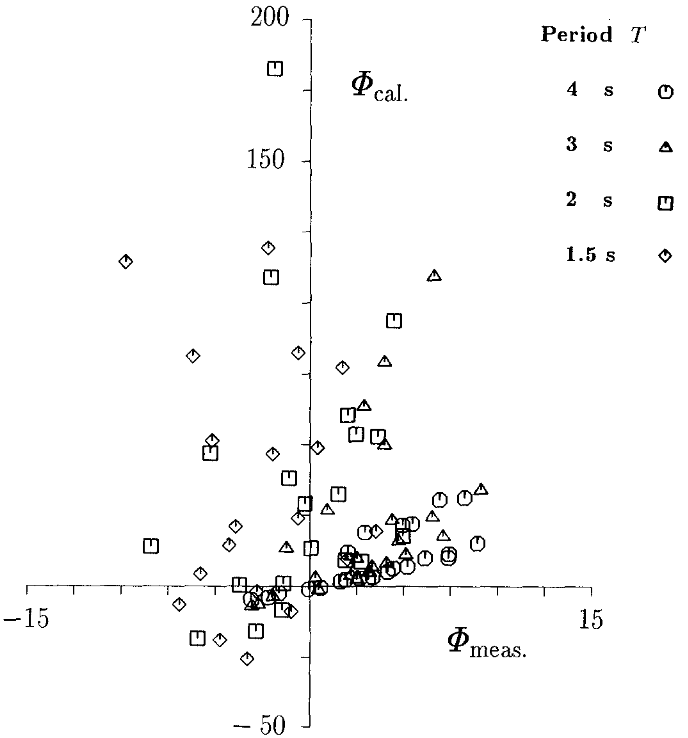

Figure 4: **Comparison between $\Phi_{meas.}$ and $\Phi_{cal.}$.**

was noticed that especially for highly asymmetric profiles (*i.e.* $u_{max}/\hat{u} = 0.8$) the sand which had been entrained during the positive half cycle, was brought back into the negative direction by the successive negative half cycle; and this mechanism in some cases was strong enough to make the net transport to be in the negative direction. In fact, it was observed that sheet flow is the motion of a condense suspended sand layer which is confined to the region near the bottom and its thickness depends on the magnitude of velocity. Once there was a positive velocity large enough to raise up sand particles to such a level that they could not reach the bottom before the negative velocity occured, then these particles tended to be carried to the negative direction.

In order to evaluate this mechanism we assume, as a first approximation, that the fluid and sediment should move together with the same velocity, and that the flow kinetic energy is transferred to the required potential energy to raise up sand particles through the strong but confined eddies which exist inside the sand sheet layer. Hence, the kinetic energy, E_k, of a sand particle with volume V moving with its ambient fluid, averaged over the time under the positive velocity, T_c, may be estimated as

$$E_k = \frac{1}{2}\rho V u_c^2 \quad . \tag{7}$$

The potential energy, E_p, required to raise that particle as high as one unit of length is

$$E_p = (\rho_s - \rho)Vg \quad . \tag{8}$$

Therefore, the distance that a sand particle travels upward, or in other words, the thickness of sand sheet layer, Δ_s, can be obtained as

$$\Delta_s = \frac{E_k}{E_p} = \frac{1}{2}\frac{u_c^2}{sg} \quad . \tag{9}$$

According to this expression the thickness of sheet layer is proportional to the square of velocity and independent of the grain size. Now lets consider a sand particle which is entrained. It will fall into the bottom by its free fall velocity, W. The time required for this particle to reach the bed, T_{fall}, will be

$$T_{\text{fall}} = \frac{\Delta_s}{W} = \frac{1}{2}\frac{u_c^2}{sgW} \quad . \tag{10}$$

Suppose a case in which T_{fall} is longer than T_c. Then what will happen is that before the sand particle reaches the bottom, the negative velocity will occur and will carry it into the negative direction. This means that the ratio of T_{fall}/T_c can be an appropriate indication for the phenomenon of our interest. Thus, we define ω_c as

$$\omega_c = \frac{1}{2}\frac{u_c^2}{sgWT_c} \tag{11}$$

to represent the intensity of the above suspension mechanism. In Fig. 5 the measured net transport rates, $\Phi_{\text{meas.}}$, under pure oscillatory motion are plotted against their corresponding values of ω_c. It is seen that $\Phi_{\text{meas.}}$ increases with increasing ω_c until it reaches a maximum at around $\omega_c = 1$ and, due to the already-mentioned suspension mechanism, starts to decrease thereafter. For values of ω_c greater than about 2.5, the mechanism is so strong that makes the net transport to be in the negative direction. From this figure we can see that ω_c is an appropriate tool not only for the prediction of transport direction but also for estimating the magnitude of net transport rate. In fact ω_c is a time ratio which in turn is an indication of the intensity of suspension, and/or an indication of the sand concentration.

Up to now we have only considered the positive portion of a velocity profile. However the same suspension mechanism could happen during the negative half cycle. Therefore we introduce ω_t as

$$\omega_t = \frac{1}{2}\frac{u_t^2}{sgWT_t} \quad . \tag{12}$$

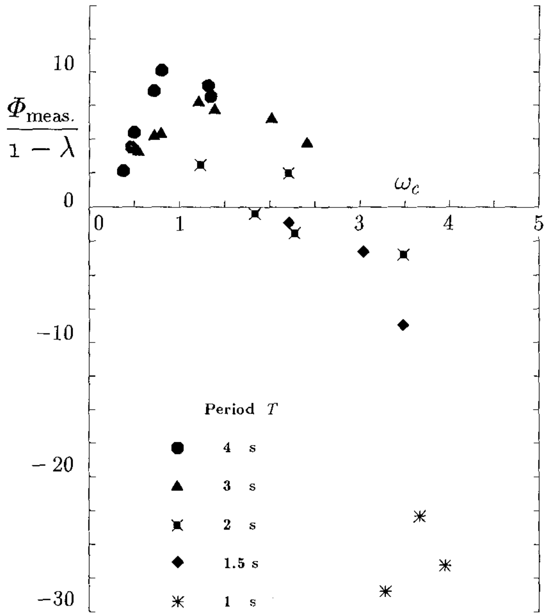

Figure 5: **Relation between the measured net transport rates and the parameter ω_c.**

Now, the net transport rate should be obtained as the difference between the rates during the two half cycles including the effect of suspension during one half cycle on its successive half cycle. It was already described that part of the sand entrained during the positive half cycle of velocity profile may remain in suspension and be carried by the velocity of the successive negative half cycle into the negative direction, and vice versa. Hence, for example, the negative velocity has two groups of sand to carry; one is the sand which is entrained by the negative velocity itself, and the other one is the sand remaining in suspension from the previous positive half cycle. Having this in mind, we define a parameter, Γ, to represent the net transport rate, as

$$\Gamma = \frac{u_c T_c(\Omega_c^3 + \Omega_t'^3) - u_t T_t(\Omega_t^3 + \Omega_c'^3)}{(u_c + u_t)T} \tag{13}$$

In this relation Ω_c' represents the amount of suspended sand remaining from the positive half cycle, to be carried by the negative velocity. Similarly Ω_t' stands for the amount of sand still in suspension from the negative half cycle which will be transported to the positive direction. On the other hand, Ω_c represents that amount of sand which is entrained and carried only by the positive velocity;

and Ω_t indicates the amount of sand which is entrained, transported, and settled during the negative half cycle.

It was already discussed that if the values of ω (ω_c and ω_t) are less than unity there will be no exchange of suspended sand between the two half cycles; both Ω'_c and Ω'_t will then be equal to zero. It was also mentioned that ω_c (or ω_t) is a parameter with two meanings; namely, it represents the time ratio T_{fall}/T_c (or T_{fall}/T_t) and also it indicates the intensity of sand concentration. Now suppose a case in which ω (ω_c or ω_t) is greater than one. Consider ω as representing the sand concentration. It should be divided into two parts; one part corresponding to the sand transported and settled during the current half cycle, ω_m, and the second part, ω', corresponding to the sand which will be delivered to the successive half cycle. This time consider ω as the time ratio. If this ratio is greater than one the suspension mechanism will be effective. The larger a value of ω the more suspended sand to be delivered to the next half cycle. Hence, we may assume that ω', as a first approximation, is equal to the amount that ω surpasses unity; namely,

$$\begin{cases} \text{if} \quad \omega \leq 1 & \begin{cases} \omega_m = \omega \\ \omega' = 0 \end{cases} \\ \text{if} \quad \omega > 1 & \begin{cases} \omega_m = 1 \\ \omega' = \omega - 1 \quad . \end{cases} \end{cases} \tag{14}$$

Based on the above interpretation, the following relations are used to estimate values of Ω_c, Ω'_c, Ω_t, and Ω'_t:

$$\begin{cases} \text{if} \quad \omega_c \leq 1 & \begin{cases} \Omega_c = \omega_c \cdot \dfrac{WT_c}{d} \\ \Omega'_c = 0 \end{cases} \\ \text{if} \quad \omega_c > 1 & \begin{cases} \Omega_c = \dfrac{WT_c}{d} \\ \Omega'_c = (\omega_c - 1) \cdot \dfrac{WT_c}{d} \end{cases} \\ \text{if} \quad \omega_t \leq 1 & \begin{cases} \Omega_t = \omega_t \cdot \dfrac{WT_t}{d} \\ \Omega'_t = 0 \end{cases} \\ \text{if} \quad \omega_t > 1 & \begin{cases} \Omega_t = \dfrac{WT_t}{d} \\ \Omega'_t = (\omega_t - 1) \cdot \dfrac{WT_t}{d} \quad . \end{cases} \end{cases} \tag{15}$$

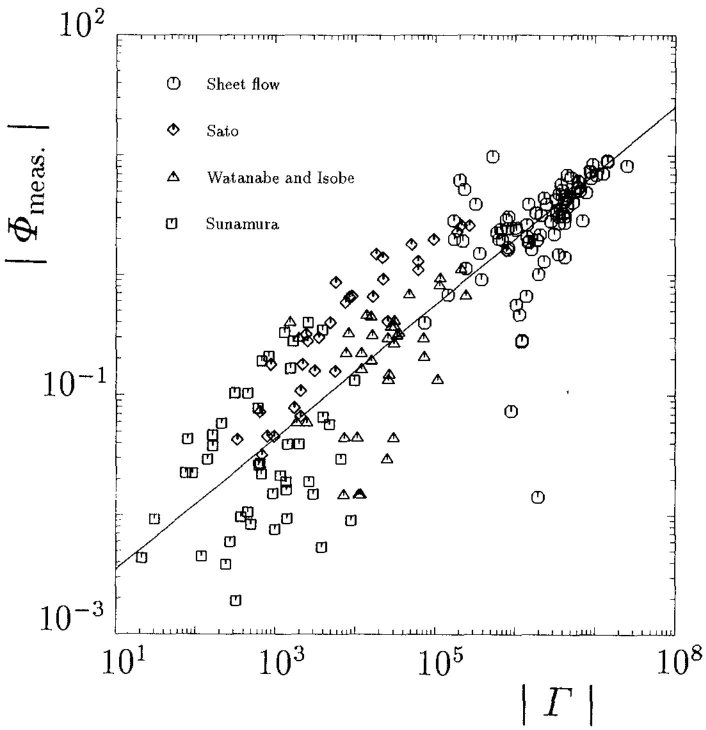

Figure 6: **Relation between $|\Phi_{meas.}|$ and $|\Gamma|$.**

Figure 6 shows values of $|\Gamma|$ plotted versus $|\Phi_{meas.}|$ for the present sheet flow data and for the previous data on suspended load above ripples. Fairly good relation is observed for the present data. The solid line which is fitted for the present data reads

$$|\Phi_s| = 0.001\,|\Gamma|^{0.55} \quad . \tag{16}$$

Values of Φ_s based on this relation are compared with the measured values in Fig. 7; Φ_s taking the same sign as Γ. It is seen that both the magnitude and direction of net transport rate under sheet flow condition are very well predicted.

The generality of Γ defined by (13) also permits its application to estimating the net transport rate over ripples. It is well known that suspended load is the predominant transport mode over sand ripples. Although the suspension mechanism above a rippled bed is different from that of sheet flow, but one may expect that the present concepts generally hold, though in a different scale, for suspended transport over ripples. It should be noticed that in the presence

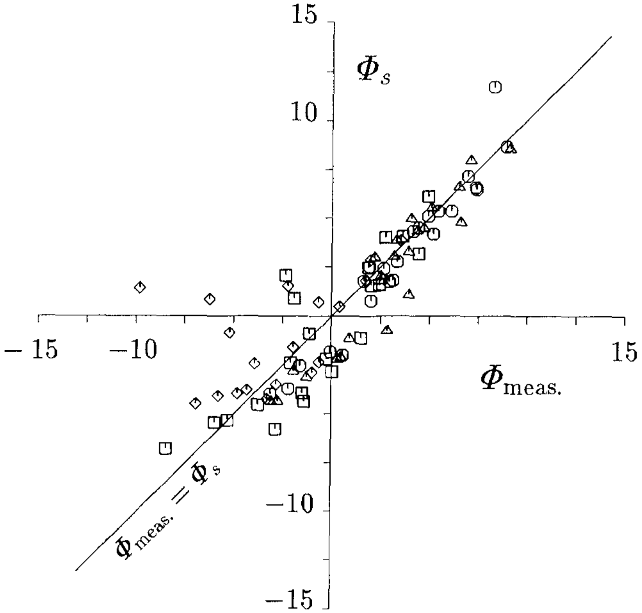

Figure 7: **Comparison between $\Phi_{\text{meas.}}$ and Φ_s.**

of ripples usually most of the sand suspended during one half cycle is carried by the velocity of the coming successive half cycle. This means that over the ripples values of Ω' should be much larger than their corresponding values of Ω. Hence, in relations (14), we may change the critical value of ω from unity to a value of the order of 0.001 corresponding to bedload condition. This is tried for the experimental data of Sato (1987), Watanabe and Isobe (1990), and Sunamura (1982) and the results are shown in Fig. 7. It should be mentioned that measured velocity profiles for the data of Sato and those of Sunamura were not available; they are calculated by using the Stream Function Method from the reported flow conditions. Also for the data of Watanabe and Isobe, only the maximum and minimum values of velocity and values of T_c and T_t were available. Sunamura's data have been obtained in a wave flume, while those of Sato, and Watanabe and Isobe are from the same flow tank as that of the present experiments. Considering these points we can see the reasonable performance of Γ in estimating the transport rate above ripples.

It is instructive to note that the third power of Ω which corresponds to the sixth power of velocity is used in relation (13), and then nearly the square root of Γ is taken in (16). This shows the correlation of transport rate with nearly the third power of velocity. However, using a power of 1.5 instead of 3 for Ω in (13) gives a slightly weaker correlation than what is seen in Fig. 6.

In a real field, there is a transition region between the rippled bottom of suspended load and the flat bed of sheet flow. Over this region, the ripple height decreases and the net transport rate changes its direction from the offshoreward suspended load to the shoreward sheet flow. The critical value of ω, should then be gradually changed from 0.001 to unity over this transition region. Further study on the critical value of ω for suspended load is also required.

5. Conclusions

Experiments were conducted on the initiation of sheet flow under asymmetric oscillations, and on the net sand transport rate under asymmetric oscillations and superimposed currents. Concerning the inception of sheet flow it is found that for a velocity profile with an arbitrary shape, it is the energy contained in that velocity profile which is the important factor in defining the near-bottom orbital velocity amplitude. Among the available criteria, it is found that the parameters used by Kaneko (1981) give a more continuous and clear trend than those by the others. A new parameter is also introduced wich together with the sediment Reynolds number can fairly well express the inception of sheet flow with a better performance than that of Kaneko.

The validity of the method of Madsen and Grant (1976) in predicting the net transport rate and its direction under sheet flow condition is examined. It is found that except for few cases this method fails to predict not only the magnitude of the net transport rate but also its direction. Their procedure could be correct only if the sand transports during successive half cycles of oscillations are completely independent of each other. In the present study it is found that once there is a positive velocity large enough to raise up sand particles to such a level that they can not reach the bottom before the negative velocity ocurres, then these particles tend to be carried to the negative direction. Based on physical interpretations a new parameter representing this phenomenon is derived. Using this parameter a general transport formula is presented in which the above-mentioned mechanism is taken into account. It is shown that both the magnitude and direction of net transport rate under sheet flow condition are very well predicted by the new formula, and that the formula is potentially applicable to estimating the net transport rate of suspended load above ripples.

It should be mentioned that in the present formulation, the steady current velocity is added to the oscillatory velocity profile and the resulting velocity profile is then used and analyzed. This is different from those methods who consider the transports due to waves and currents separately. It is seen that the present method is successful and more realistic, while its application to the horizontal plane problems is also straightforward. Another point is that the Shields parameter is not used in this study for estimating the transport rate. This is because of the uncertainty still involved in estimating values of friction factor under intense flow conditions. Once the shear stress at the bottom could be accurately determined, then it should be used in the formulations.

6. References

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