CHAPTER 196

Numerical Simulation of Pocket Beach Formation

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Abstract

The numerical model for three-dimensional beach change prediction is constructed to simulate a pocket beach formation. The model consists of three modules of waves, currents (include beach change) and sediment transport. Wave transformation module is based on the mild slope equation of hyperbolic type, current module is horizontally two-dimensional with direct interaction with sea bottom change which is evaluated by sediment transport model formulated by Bailard.

Numerical simulation of pocket beach formation was performed under two conditions of an artificial pocket beach formation which is the typical case of beach stabilization by headlands and an empirical pocket beach which is made by combining an equilibrium bay and beach profile.

1. Introduction

Beach erosion control by constructing a series of stable pocket beaches, the headland defense works by Silvester(1972), seems to be one of the effective methods to recover sandy beaches in the area which has been seriously eroded due to both reducing longshore sediment transport and increasing offshore going sediment transported by reflected waves from the sea wall. "Stabilization of Sedimentary Coastlines" is the title of Silvester's paper in Nature(1960) and "Use of Crenulate Shaped Bays to Stabilize Coasts" is the title of paper in the proceedings of 13th ICCE(1972). As can be recognized from these titles, this method is based on the idea of stabilizing beaches with pocket beach formation. Thirty years after Silvester, beach stabilization seems to be a word in vogue and the headland defense works has been recognized as a feasible way to protect sandy beaches in

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place of offshore detached breakwaters which thickly parasite on Japan's eroded beaches.

Following Silvester's frontier works, studies on mechanism of beach stabilization by forming pocket beaches (crenulate-shaped or zeta-shaped bays) have been carried out by several researchers. Hsu et. al.(1987) formulated bay shaped in terms of the 2nd order polynomial. Tsuchiya et. al.(1990) classified a pocket beach into two types of statically and dynamically stable beaches. Theoretical explanation of the existence of statically and dynamically stable pocket beaches has been done by Tsuchiya and Refaat(1990) in terms of the one-line theory with nonuniform longshore sediment transport model.

This study aims to simulate an artificial pocket beach formation by the numerical model for three-dimensional beach change prediction, in which knowledge of wave theory, nearshore dynamics, shoreline change model and sediment transport model are integrated. First numerical model is constructed and then simulation of pocket beach formation is performed under the condition of typical case of stable beach construction by headlands.

2. Model Outline

2.1 Wave Transformation

The steady state mild slope equation of the elliptic type including terms of uniform currents($\vec{U}$) interaction and wave energy dissipation due to breaking is employed to calculate wave field.

$$\frac{D^2\phi}{Dt^2} + (\nabla \cdot \vec{U})\frac{D\phi}{Dt} - \nabla \cdot (CC_g \nabla \phi) + (\sigma^2 - k^2 CC_g - i\sigma W)\phi = 0 \tag{1}$$

where ϕ is the velocity potential and the derivative operator is defined as $D/Dt = \partial/\partial t + U_j \partial/\partial x_j$ (Kirby 1984).

Wave energy dissipation rate W formulated by Izumiya & Horikawa(1984) is employed here. Using the complex surface elevation ζ, the mild slope equation can be written as:

$$\left\{1 + \frac{\sigma}{\omega}\left(\frac{C_g}{C} - 1 + \frac{i}{\sigma}W\right)\right\}\frac{\partial^2\zeta}{\partial t^2} + \frac{\partial}{\partial t}\left(\nabla \cdot (\vec{U}\zeta)\right) - \nabla \cdot \left\{\omega CC_g \nabla\left(\frac{\zeta}{\sigma}\right)\right\} = 0 \tag{2}$$

where ω and σ are the absolute and intrinsic angular frequencies, respectively with the following relation

$$\omega = \sigma + \vec{U} \cdot \vec{k}, \quad \sigma = \sqrt{gk \tanh kh} \tag{3}$$

and the wave celerity and group velocity are defined respectively by

$$C = \frac{\sigma}{k}, \quad C_g = \frac{\partial \sigma}{\partial k} \tag{4}$$

For effective numerical calculation, an equivalent hyperbolic system to the elliptic mild slope equation (2) can be derived as : (Warren et. al., 1985; Madsen and Larsen, 1987; Yamashita et. al., 1990).

$$\frac{\partial P}{\partial t} - i\omega P + \omega CC_g \frac{\partial}{\partial x}\left(\frac{S}{\sigma}\right) = 0 \tag{5}$$

$$\frac{\partial Q}{\partial t} - i\omega Q + \omega C C_g \frac{\partial}{\partial y}\left(\frac{S}{\sigma}\right) = 0 \tag{6}$$

$$\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{1}{\omega}\left\{1 + \frac{\sigma}{\omega}\left(\frac{C_g}{C} - 1 + \frac{i}{\sigma}W\right)\right\}\left(\frac{\partial S}{\partial t} - i\omega S\right) + \vec{U}\cdot\nabla S = SS \tag{7}$$

where S, P, and Q are the surface elevation and line discharges in the x and y direction and SS is the wave source which is given as

$$SS = \frac{C\Delta s}{\Delta x \Delta y} a_0 e^{i\chi} \tag{8}$$

where χ is the phase, a_o the wave amplitude, θ the wave angle and spacial increments are defined $\Delta x, \Delta y$, and $\Delta s = \Delta x/\cos\theta$ respectively.

The sponge layer boundary condition (Larsen and Dancy, 1983) is effective behind the wave source boundary and along the shoreline. An artificial dumping is introduced inside sponge layer of which dumping coefficient $\mu(x)$ is given by,

$$\mu(x) = \begin{cases} \exp\{(2^{-x/\Delta x} - 2^{-x_s/\Delta x})\ln\alpha\} & \text{for} \quad 0 \le x \le x_s \\ 1 & \text{for} \quad x_s \le x \end{cases} \tag{9}$$

where x_s is the thickness of sponge layer, α the parameter being dependent on x_s. This set of equations can be solved numerically by the ADI method.

2.2 Sediment Transport Model

The sediment transport rate q_i can be evaluated by the model of Bailard(1982), which was calibrated by Stive(1986), in which the sediment transport rate $\vec{q}$ consists of bed-load and suspended-load including the effects of wave asymmetry (suffix as), mean currents (suffix nc) and bottom slope (suffix sl). The total sediment transport vector is calculated by,

$$\vec{q} = \frac{1}{(\rho_s - \rho)g(1-\lambda)}(\vec{q}_{asb} + \vec{q}_{ass} + \vec{q}_{ncb} + \vec{q}_{ncs} - \vec{q}_{slb} - \vec{q}_{sls}) \tag{10}$$

where

$$\vec{q}_{asb} = \frac{9B_b u_{orb}^4 \cos\phi_2}{C\sinh^2(kD)}\vec{i}, \quad \vec{q}_{ass} = \frac{9B_s u_{orb}^5 \cos\phi_2}{5\pi C\sinh^2(kD)}\vec{i}$$

$$\vec{q}_{ncb} = \frac{3B_b u_{orb}^2 \vec{U}}{2}, \quad \vec{q}_{ncs} = 6.4 B_s u_{orb}^3 \vec{U} \tag{11}$$

$$\vec{q}_{slb} = \frac{1.6 B_b u_{orb}^3}{\tan\psi}\frac{\partial D}{\partial x}\vec{i}, \quad \vec{q}_{sls} = \frac{8\epsilon_s B_s u_{orb}^5}{w_f}\frac{\partial D}{\partial x}\vec{i}$$

where $\vec{i}$ is the unit vector, θ: the wave direction, u_{orb}: the near-bottom orbital velocity of wave motion(linear), $\vec{U}$: the near-bottom mean current vector, ϕ_2: the phase lag of second harmonics of the Stokes wave theory, $\epsilon_s = 0.02$, ρ_s, ρ: the densities of sea water and sediment, g: the gravitational acceleration, λ: the sediment porosity, w_f: the sediment falling velocity and k: the wave number. Moreover, the parameters for bed-load B_b and suspended-load B_s are $B_b = \rho C_f \epsilon_b/\tan\psi$ and $B_s = \rho C_f \epsilon_b / w_f$, respectively. The friction factor C_f is given by,

$$C_f = \frac{1}{2} f_w = \frac{1}{2} \exp\left\{5.2\left(2.5\frac{D_s}{a_w}\right)^2 - 6.0\right\} \tag{12}$$

where D_s is the characteristic grain diameter and a_w the orbital amplitude.

2.3 Nearshore Currents and Sea Bottom Change

Momentum equations including terms of interaction with sea bottom changes $(\partial D/\partial x_i)$ are derived as:

$$\frac{\partial Q_i}{\partial t} + \beta U_i \frac{\partial Q_j}{\partial x_j} = -gH\left(\frac{\partial H}{\partial x_i} - \frac{\partial D}{\partial x_i}\right) - \frac{1}{\rho}\left(\tau_i^B + \frac{\partial S_{ij}}{\partial x_j} + \frac{\partial T_{ij}}{\partial x_j}\right) \tag{13}$$

where , β:momentum correction coefficient, H: total depth, D:still water depth, Q_i and U_i are discharges and velocities of nearshore currents, τ_i^B:bottom stresses and S_{ij}:radiation stresses.

$$S_{ij} = \frac{\rho g}{4}\left[\mathrm{Re}\left(\frac{\partial S}{\partial x_i}\frac{\partial S^*}{\partial x_j}\right)\frac{1}{k^2}(1 + \frac{2kD}{\sinh 2kD}) + \delta_{ij}\left\{\mid S\mid^2 \frac{2kD}{\sinh 2kD}\right.\right.$$

$$\left.\left. + \frac{2kD\coth 2kD - 1}{2k^2}(\mid \frac{\partial S}{\partial x}\mid^2 + \mid \frac{\partial S}{\partial y}\mid^2 - k^2\mid S\mid^2)\right\}\right] \tag{14}$$

where δ_{ij} is Kroneker's delta and * indicates complex conjugate. Furthermore, horizontal mixing is formulated as

$$-T_{ji} = \rho H\left(\nu_e \cos(\theta + \frac{\pi\delta_{ij}}{2})\right)$$

$$\left(-\sin\theta\cos\theta\frac{\partial V}{\partial x_j} + \sin\theta\sin\theta\frac{\partial U}{\partial x_j} - \cos\theta\cos\theta\frac{\partial V}{\partial x_i} + \sin\theta\cos\theta\frac{\partial U}{\partial x_i}\right) \tag{15}$$

where ν_e is the eddy viscosity estimated by velocity scale $[u] \simeq \sqrt{gH)}$ and the length scale $[l]$ is the distance from the shore.

The continuity equations of water and sediment transport are given respectively by

$$\frac{\partial H}{\partial t} + \frac{\partial Q_i}{\partial x_i} = 0, \qquad \frac{\partial D}{\partial t} - \frac{\partial q_i}{\partial x_i} = 0. \tag{16}$$

The sediment transport rate q_i is evaluated by the model of Bailard(1982). The finite difference ADI method is also employed to solve the nearshore currents and sea bottom changes. The one-line model is employed to predict a long-term shoreline change under the dominant wave field calculated by the wave transformation model shown in 2.1. After determining the shoreline position of a stable beach, the sea bottom topography change is calculated up to getting the equilibrium beach topography. To save CPU time, wave field is renewed several times after calculations of nearshore currents and sea bottom change.

3. Pocket Beach Formation

3.1 Artificial Pocket Beach Formation

An initial beach topography is assumed the uniformly sloping bottom (s=1/23) constructed by the sand nourishment in the seriously eroded beach together with the construction of two headlands (100 and 50m long each) and groin system (see Figure 1). This beach is called an artificial pocket beach in this paper, of which dimension is typical in the headland defense works (beach stabilization works) under construction or planning in Japan's eroded coasts.

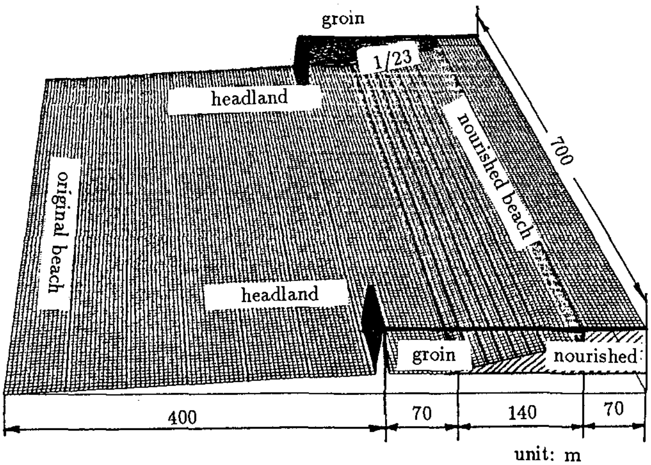
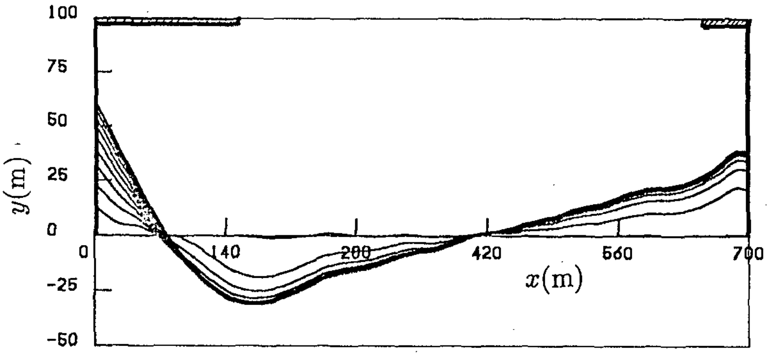


Figure 1. Initial beach topography

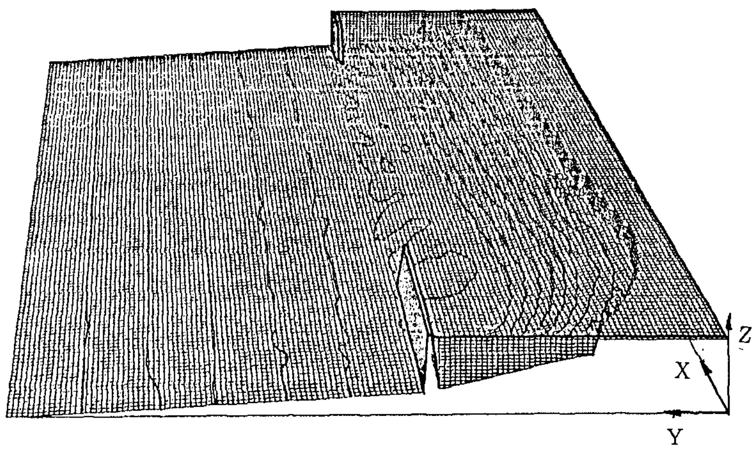
Operating the dominant incident waves (T=8s, H=2m, θ=8° are assumed here) and computing the stable shoreline position by the shoreline change model (one-line model including the effect of wave diffraction because the wave field is computed by the wave transformation model mentioned above), computations of nearshore current and sea bottom change start to obtain the statically stable topography of a pocket beach. Figures 2 (a)-(d) show the computed shoreline, beach topography, wave and current fields. It can be observed from these figures that nourished sand may not go out of headlands area to form a zeta-shaped bay in the case of dominant wave operation. Because perfect wave absorption is assumed at the boundaries such as headlands, beaches and side walls, smooth wave and current fields are computed in which wave diffraction and breaking play a main role in forming nearshore circulation.

3.2 Changes in Empirical Pocket Beach

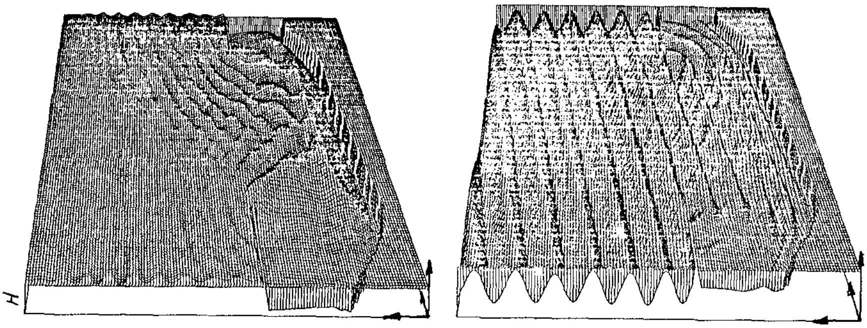
In addition to the case of an artificial pocket beach formation, an attempt of an empirical pocket beach formation is conducted. Combining the formulations of equilibrium bays (Hsu et. al., 1987) and equilibrium beach profiles, we can get the ideal stable beach topography which is called an empirical pocket beach in this paper. The stable shoreline of the pocket beach can be assumed by an equilibrium bay formulated by equation (18) (Hsu et. al.). Fixing the critical depth of beach changes, Dean's(1981) equilibrium beach profile evaluated by equation (17) is assumed along the line between shore and critical depth to determine the topography of stable beach, then we can get the stable beach topography shown in Figure 4.



(a) Computed shoreline of the pocket beach



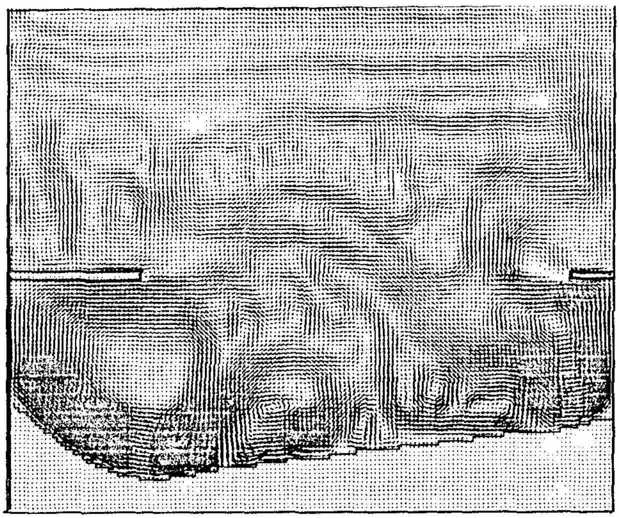
(b) Topography of artificial pocket beach



wave height

wave profile

(c) Wave field



(d) Current field

Figure 2. Artificial Pocket Beach Formation

$$h(x) = A(D_s)x^{2/3} \tag{17}$$

$$\frac{R}{R_o} = C_o + C_1(\frac{\beta}{\theta}) + C_2(\frac{\beta}{\theta})^2 \tag{18}$$

where the coefficients C_o, C_1 and C_2 are dependent on the incident wave angle β and the definition of variables are shown in Figure 3.

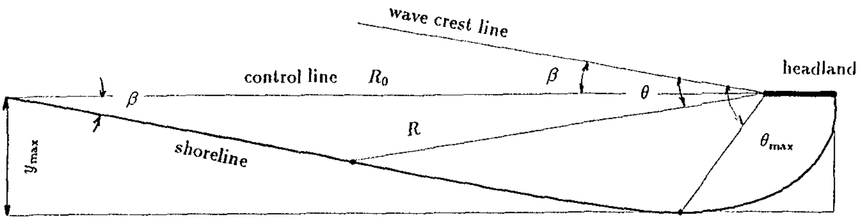


Figure 3. Definition of variables of equilibrium bay (Hsu et. al.)

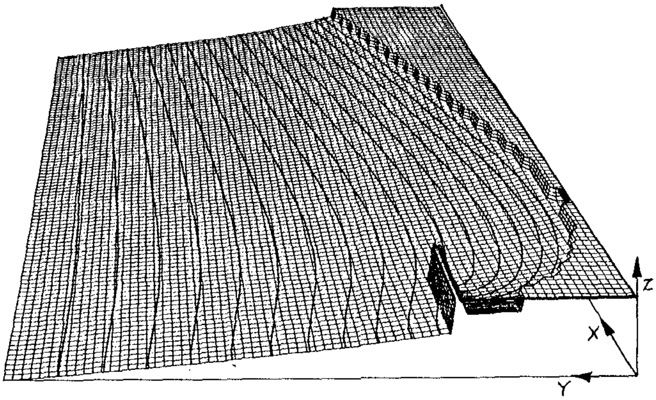


Figure 4. Topography of an empirical pocket beach

Calculations of nearshore currents and sea bottom changes are executed under the same conditions of wave and boundary as those of the case of an artificial pocket beach. Figure 5(a) shows the shoreline position calculated by equation (18)in which the length of headland is 57m. It can be recognized that both the embayment (about 30m at x=140m) and the salient length of 60m are almost same as artificial bay formation.

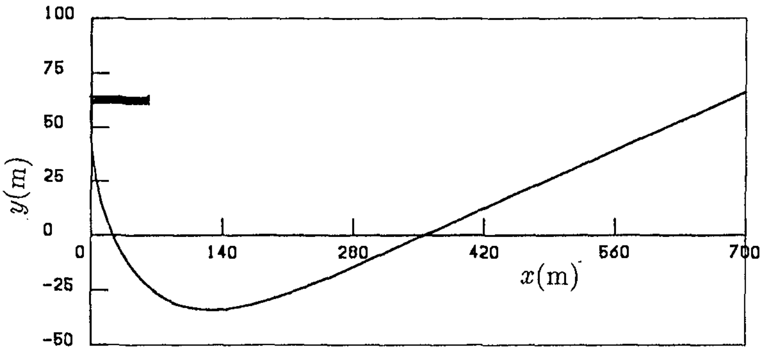
Figures 5(b) and (c) show currents and bottom topography in which no remarkable circulation cells except behind headland is observed and change in topography is also small. This means that pocket beach is stable against dominant waves.

4. Conclusions

A finite difference numerical model for sea bottom topography change was constructed by combining a wave transformation model(steady) and a three dimensional beach change model(unsteady) in which interaction between nearshore currents and bottom change was simultaneously considered. Numerical simulation of an artificial pocket beach formation was performed by the developed model together with the shoreline change model.

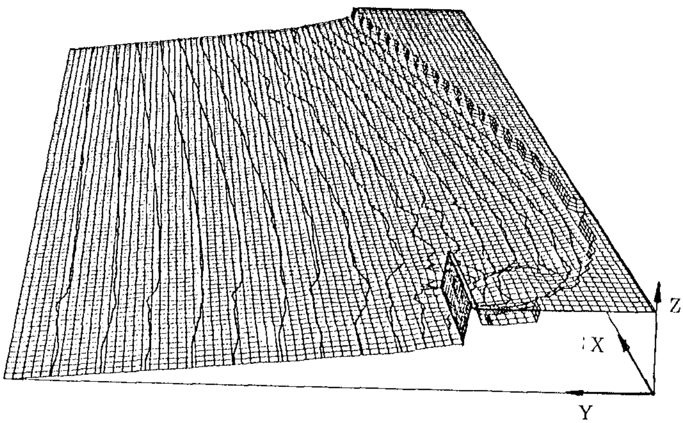
Applying the model to the case of artificial pocket beach of which dimension was similar to that of headland defense works (beach stabilization works) under construction in Japan's eroded coast, it was recognized that nourished sand might not go out of headlands area.

Moreover, combining the formulations of equilibrium bays and equilibrium beach profiles by Dean the initial beach topography was assumed(it was called an empirical pocket beach in this paper). It was shown that no remarkable circulation cells except behind headland are formed and topography change was also small to confirm beach stabilization.



(a) Computed shoreline of the pocket beach

(b) Nearshore currents



(c) Beach topography

Figure 5. Changes in Empirical Pocket Beach

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