

CHAPTER 212

Formation and Reduction Processes of River Deltas ; Theory and Experiments

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Abstract

Beach erosion has recently been recognized as accelerating due to change in sediment sources from rivers. This can be considered as reduction processes of river deltas. In this paper, the formation and reduction processes of river deltas are first investigated theoretically by introducing the non-uniformity of longshore sediment transport into the equation of continuity of shoreline changes. Theoretical solutions to shoreline changes in the formation and reduction processes are well explained. Secondly, experiments both on the formation and reduction processes were carried out. The theoretical results of the processes compared favorably with the experimental ones.

1. Introduction

Delta resulted from the relative importance of deposition of river sediments which build the delta seaward, versus the action of waves and currents which transport the sediments and erode the delta. If the rate of sediment deposition from the river is greater than the rate of sediment removal by the longshore currents, the formation process of river delta will take place. On the contrary, if the rate of sediment deposition from the river is less than the rate of sediment removal by the longshore currents, beach erosion occurs resulting in the reduction of river delta. Historically, the term *delta* was first applied by the Greek historian Herodotus, circa 450 B.C., to the triangular alluvial deposit at mouth of the Nile River.

The processes of formation and reduction of river deltas have been investigated through the study of analytical solutions of the so-called one-line theory. The one-line theory was first introduced by Pelnard-Considé (1956), and it has been demonstrated to be adequate in practical applications. Several authors have presented analytical solutions for the one-line theory (e.g. Bakker and Edelman, 1964; Iwagaki, 1966; Tsuchiya, 1973, 1978; Tsuchiya and Yasuda, 1979; Kraus, Hanson and Harikai, 1985; and recently Hanson and Larson, 1987). The fundamental assumption of this theory is that the bottom profile moves in parallel to itself without changing shape. The second major assumption of the theory is that the longshore sediment transport takes place averaging over the beach profile down to a certain offshore limiting depth of the sediment drift zone.

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2. Theory of Formation and Reduction Processes of River Deltas

In the formation and reduction processes of river deltas, the non-uniformity of longshore sediment transport along the shoreline must be introduced. Tsuchiya and Refaat (1992) recently developed theoretical approaches to the longshore sediment transport in the non-uniform condition. Introducing this theoretical formulation of the total rate of non-uniform longshore sediment transport, a set of the governing equations of shoreline change is proposed.

2.1 The governing equations and the field equations of delta formation process

The equations governing the formation and reduction processes of river deltas are composed of the equations of continuity of shoreline change and total rate of longshore sediment transport. Taking the coordinate system as shown in Fig. 1, the equation of continuity of shoreline change is expressed as (Tsuchiya ; 1978) :

$$\frac{\partial y_0}{\partial t} + \frac{1}{(1-\lambda)h_k} \frac{\partial Q_x}{\partial x} = \frac{1}{B h_k} Q_R(t) \delta(x-x_0) \quad (1)$$

in which y_0 is the shoreline position from the datum line, x is the longshore distance from the origin, t is the time, λ is the porosity of beach sediment, h_k is the limited water depth of littoral drift, B is the river width, Q_x is the total rate of longshore sediment transport, and $Q_R(t) \delta(x-x_0)$ is the sediment input effective to the shoreline change from a river located at $x=x_0$ where $\delta(x-x_0)$ is Dirac's delta function.

The phenomenon of longshore sediment transport around the river deltas are non-uniform owing to the non-uniform characteristics of waves and the effect of the boundary conditions of both longshore currents and beach changes on the longshore sediment transport. The equation of longshore sediment transport in the unsteady, nonuniform condition is given by Tsuchiya and Refaat (1992) as:

$$\frac{\partial Q_x}{\partial t} + \frac{\alpha_2}{\alpha_1 k} \frac{\partial}{\partial x} \left[\frac{Q_x}{h_b} \right] = k f(x) - \frac{\alpha_3 \sqrt{g} \gamma C_f}{\alpha_1 \sqrt{h_b}} Q_x \quad (2)$$

where

$$f(x) = \frac{\gamma^2}{16} g h_b^2 \{ \sin 2\alpha_b - 2 \cos^2 \alpha_b \frac{\partial y_b}{\partial x} - (5\beta_1 - 6\beta_2 + 6\beta_3 \sin^2 \alpha_b) \frac{\partial h_b}{\partial x} - 2\beta_3 h_b \sin 2\alpha_b \frac{\partial \alpha_b}{\partial x} \} \quad (3)$$

where α_b is the breaker angle which is defined as the angle between the wave crest and the shoreline, t is the time, g is the acceleration of gravity, y_b is the offshore distance of the breaking point from the shoreline, k , α_i and β_i ($i = 1, 2, 3$), are the coefficients being nearly constants (Tsuchiya and Refaat, 1992).

In order to express the total rate of longshore sediment transport by a function of the breaker angle and breaker depth, and their derivatives, it can be expressed by Taylor series up to the first order as:

$$Q_x = Q_0 + \frac{\partial Q_x}{\partial \alpha_{b0}} (\alpha_b - \alpha_{b0}) + \frac{\partial Q_x}{\partial h_{b0}} (h_b - h_{b0}) + \dots \quad (4)$$

in which Q_0 denotes the total rate of longshore sediment transport at the initial wave conditions presented by the subscript 0. Reference Fig. 1 the small changes in the breaker angle and depth can be expressed geometrically as:

$$\Delta \alpha_b = \alpha_b - \alpha_{b0} \approx -\frac{\partial y_0}{\partial x} \quad \text{and} \quad \Delta h_b = m \Delta y_0 \quad (5)$$

Substituting Eqs. (4) and (5) to Eq. (1) yields

$$\frac{\partial y_0}{\partial t} + \varepsilon_1 \frac{\partial y_0}{\partial x} - \varepsilon_2 \frac{\partial^2 y_0}{\partial x^2} = q_R(t) \delta(x - x_0) \quad (6)$$

where

$$\varepsilon_1 = \frac{m}{(1-\lambda)h_k} \left(\frac{\partial Q_x}{\partial h_b} \right)_{h_{b0}}, \quad \varepsilon_2 = \frac{1}{(1-\lambda)h_k} \left(\frac{\partial Q_x}{\partial \alpha_b} \right)_{\alpha_{b0}},$$

and $q_R(t) = \frac{I}{B h_k} Q_R(t)$ (7)

The coefficient ε_2 which has the dimensions of length squared over time, is interpreted as a diffusion coefficient. While the coefficient ε_1 having the dimension of length over time, is interpreted as the traveling speed of shoreline towards the longshore direction. In Eq. (7) it is clearly seen that the coefficient ε_1 is proportional to the beach slope. Thus, the coefficient ε_1 can not be ignored when the beach slope changes along the delta. By specifying initial and boundary conditions in the areas which represent conditions prevailing in a specific shoreline evolution, the analytical solutions are directly applicable.

2.2 Solution to shoreline changes in the formation process of river delta by normally incident waves

Since the river delta has an infinite length and waves are approached the shoreline normally, the coefficient ε_1 becomes very small and can be neglected. The general solution of river delta formation can be expressed from Eq. (6) (Carslaw and Jaeger, 1959) as:

$$y_0(x, t) = h_k q_{R0} \sqrt{\frac{t}{\varepsilon_2}} \operatorname{jerfc} \left(\frac{x - x_0}{2 \sqrt{\varepsilon_2 t}} \right) \quad \text{for } t > 0 \text{ and } -\infty < x < \infty \quad (8)$$

where jerfc denotes the integral of the complementary error function erfc .

In Fig. 2 the solution to Eq. (8) is illustrated. The dimensionless sediment input used to normalize the shoreline position is expressed as:

$$\frac{h_k q_{R0}}{\varepsilon_2} = \frac{q_{R0}}{(\partial Q_x / \partial \alpha_b)_{\alpha_{b0}}} \quad (9)$$

which can be interpreted as a ratio of the sediment input from the river to the derivative of longshore sediment transport rate with respect to the breaker angle.

2.3 Solution to shoreline change in the formation process of river delta by obliquely incident waves

When the waves approach the delta coast obliquely, the expected shape of the delta will be asymmetric. Consider the case where a river mouth has a finite width, $2a$, the continuity equation of sediment transport in the full form of Eq. (6) is applied as:

$$\frac{\partial y_{01}}{\partial t} = -\varepsilon_1 \frac{\partial y_{01}}{\partial x} + \varepsilon_2 \frac{\partial^2 y_{01}}{\partial x^2} + q_{R0}; -a \leq x \leq a \quad (10)$$

and

$$\frac{\partial y_{02}}{\partial t} = -\varepsilon_1 \frac{\partial y_{02}}{\partial x} + \varepsilon_2 \frac{\partial^2 y_{02}}{\partial x^2}; |x| > a \quad (11)$$

The problem consists of two coupled partial differential equations. By specifying the boundary conditions, which are the beach must be continuous at all times over the two zones and the shoreline is unaffected by the sediment input from the river as x approaches infinity. The solution to Eqs. (10) and (11) are then obtained as:

$$y_0(x, t) = \begin{cases} \exp\left[\frac{3\varepsilon_1}{2\varepsilon_2}x\right] F_1(x, t); & -\infty < x \leq -a \\ \exp\left[\frac{3\varepsilon_1}{2\varepsilon_2}x\right] F_2(x, t); & -a \leq x \leq 0 \\ \exp\left[-\frac{\varepsilon_1}{2\varepsilon_2}x\right] F_2(x, t); & 0 \leq x \leq a \\ \exp\left[-\frac{\varepsilon_1}{2\varepsilon_2}x\right] F_1(x, t); & a \leq x < \infty \end{cases} \quad (12)$$

where

$$F_1(x, t) = 2q_{R0}t \left\{ j^2 \operatorname{erfc}\left(\frac{x-a}{2\sqrt{\varepsilon_2 t}}\right) - j^2 \operatorname{erfc}\left(\frac{x+a}{2\sqrt{\varepsilon_2 t}}\right) + \frac{\varepsilon_1 a}{\varepsilon_2} j^3 \operatorname{erfc}\left(\frac{x-a}{2\sqrt{\varepsilon_2 t}}\right) - \frac{\varepsilon_1 a}{\varepsilon_2} j^3 \operatorname{erfc}\left(\frac{x+a}{2\sqrt{\varepsilon_2 t}}\right) \exp\left(-\frac{\varepsilon_1}{\varepsilon_2}a\right) \right\} \quad (13)$$

and

$$F_2(x, t) = q_{R0}t \left\{ 1 - 2j^2 \operatorname{erfc}\left(\frac{a-x}{2\sqrt{\varepsilon_2 t}}\right) - 2j^2 \operatorname{erfc}\left(\frac{a+x}{2\sqrt{\varepsilon_2 t}}\right) + \frac{2\varepsilon_1 a}{\varepsilon_2} j^3 \operatorname{erfc}\left(\frac{x-a}{2\sqrt{\varepsilon_2 t}}\right) - \frac{2\varepsilon_1 a}{\varepsilon_2} j^3 \operatorname{erfc}\left(\frac{x+a}{2\sqrt{\varepsilon_2 t}}\right) \exp\left(-\frac{\varepsilon_1}{\varepsilon_2}a\right) \right\} \quad (14)$$

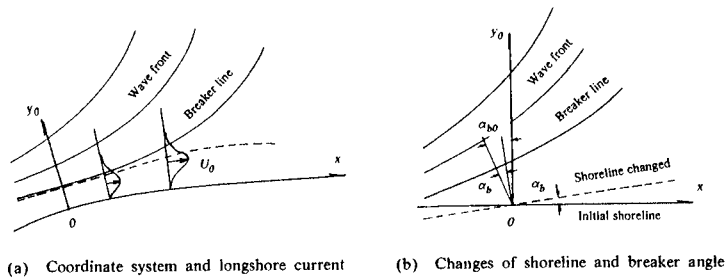


Fig. 1. The coordinate system used in the formation.

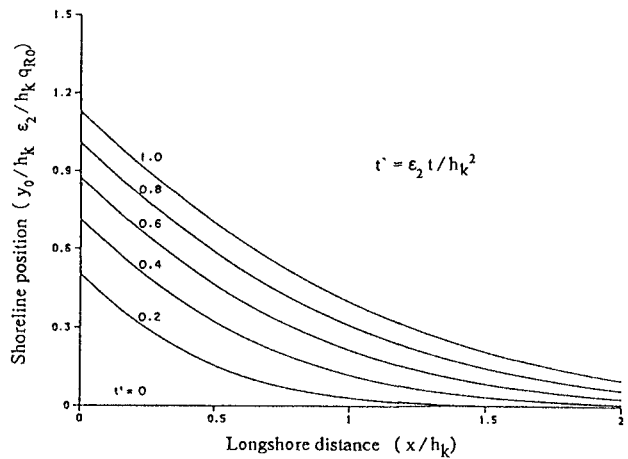


Fig. 2. Shoreline evolution in the formation of river delta of infinite length.

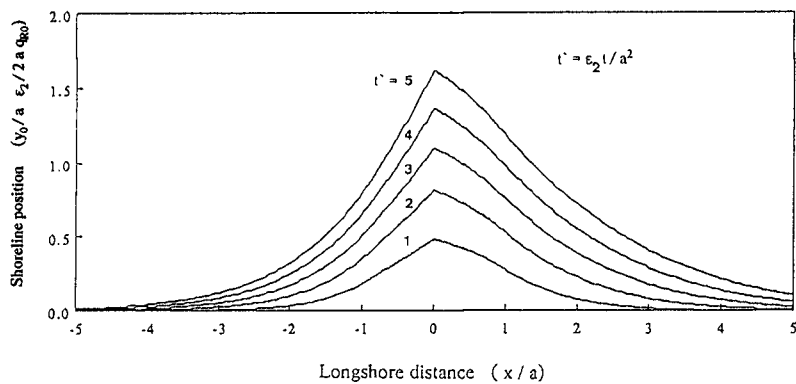


Fig. 3. Shoreline evolution in the formation of river delta by obliquely incident waves

The function ierfc is the integral of the complementary error function and n exponent represents n time integrations of the complementary error function. The time evolution of river delta by Eq. (12) is illustrated in Fig. 3. It is well understood in this figure that the asymmetric plane shape of the delta is clearly formed by obliquely incident waves.

2.4. Solution to shoreline changes in the reduction process of river delta

The reduction process of river delta occurs due to decrease or lack of sediment deposit from the river. Generally, the basic equations describe the shoreline changes in reduction process of river delta are the same equations used to describe the shoreline changes in formation process of river delta except that the river sediment discharge is no longer considered. Therefore, the linear partial differential equation described the shoreline change is given as:

$$\frac{\partial y_0}{\partial t} + \varepsilon_1 \frac{\partial y_0}{\partial x} - \varepsilon_2 \frac{\partial^2 y_0}{\partial x^2} = 0 \quad (15)$$

By specifying the initial and boundary conditions in the areas which represent conditions prevailing in a specific shoreline evolution, the corresponding analytical solutions are directly applicable. In the following cases of reduction of river delta, the initial shoreline position will be determined with the aid of the formation process of river delta.

(1) Reduction of river delta of infinite length by normal incident waves

Since the river delta has an infinite length and waves are assumed to approach the shoreline normally, the coefficient ε_1 becomes very small and can be neglected. Also, the reduction process will take place symmetrically with respect to the center of the river delta. Thus, the solution to shoreline changes is obtained as:

$$y_0(x, t) = \frac{h_k q_R}{2 \varepsilon_2 \sqrt{\pi}} \sqrt{\frac{A}{t}} \int_{-\infty}^{\infty} \text{ierfc} \left(\frac{x'}{2 \sqrt{\varepsilon_2 A}} \right) \exp \left\{ -\frac{(x - x')^2}{4 \varepsilon_2 t} \right\} dx' \quad (16)$$

In Fig. 4 the solution to Eq. (16) is illustrated. In this figure the reduction

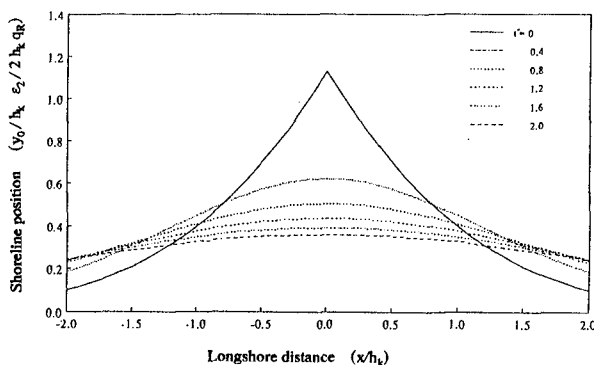


Fig. 4. Time variation of shoreline change in the reduction process of river delta of infinite river length.

process of river delta is taking place symmetrically with respect to the center of the river. The rate of reduction of river delta at the center of the river is diminished with time, while the shoreline at the longshore ends of the river delta is replenished with time. The final stage for the configuration of the delta will become a straight beach.

(2) Asymmetric change of shoreline position of river delta

When the waves approach the coast of river delta obliquely, the shoreline change is expected to occur asymmetrically. Consider the case where the river mouth has a finite width, $2a$, the general solution to shoreline change is obtained as:

$$y_0(x, t) = z_0(x, t) \exp\left(\frac{\varepsilon_1}{2\varepsilon_2}x - \frac{\varepsilon_1^2}{4\varepsilon_2}t\right) \quad \text{for } t > 0 \text{ and } -\infty < x < \infty \quad (17)$$

where

$$\begin{aligned} z_0(x, t) = & \frac{1}{2\sqrt{\pi\varepsilon_2 t}} \left[\int_{-\infty}^a F_1(x') \exp\left\{-\frac{(x-x')^2}{4\varepsilon_2 t} + \frac{\varepsilon_1 x'}{\varepsilon_2}\right\} dx' \right. \\ & + \int_a^0 F_2(x') \exp\left\{-\frac{(x-x')^2}{4\varepsilon_2 t} + \frac{\varepsilon_1 x'}{\varepsilon_2}\right\} dx' + \int_0^a F_2(x') \exp\left\{-\frac{(x-x')^2}{4\varepsilon_2 t} - \frac{\varepsilon_1 x'}{\varepsilon_2}\right\} dx' \\ & \left. + \int_a^{\infty} F_1(x') \exp\left\{-\frac{(x-x')^2}{4\varepsilon_2 t} - \frac{\varepsilon_1 x'}{\varepsilon_2}\right\} dx' \right] \quad (18) \end{aligned}$$

In Fig. 5 the solution to Eq. (18) is illustrated. The asymmetric shape of the delta is clearly seen in this figure. Since the waves approach the shoreline obliquely, the longshore sediment is predominant. Consequently, most of the sediment moves in the predominant direction of littoral drift. As a result, the

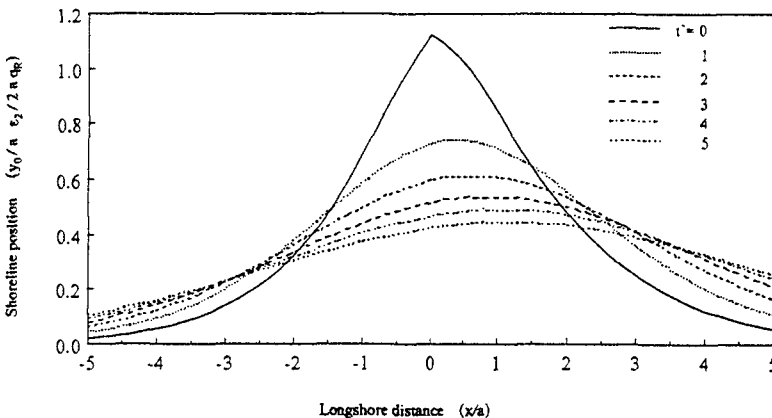


Fig. 5. Time variation in shoreline change in the reduction process of river delta by obliquely incident waves in the case where $\varepsilon_1 a / \varepsilon_2 = 0.2$.

reduction rate of the shoreline on the upcoast side is greater than that on the downcoast. The final stage for the configuration of the delta may become an inclined straight beach which the waves tend to approach normally. Also, the river mouth moves predominantly in the direction of littoral drift, as the reduction process of river delta takes place.

3. Experiments on Formation and Reduction Processes of River Deltas

The main objectives of this experimental study are, to demonstrate the process of formation and reduction of river deltas, to determine the changes of beach profile along the shoreline of river delta, and finally to study the effect of oblique wave incident on the symmetrical configuration of the river delta, and on the formation and reduction processes of river deltas.

3.1 Experiments on Formation Process of River Deltas

(1) Experimental setup

The experiments were performed in the fan-shaped wave basin (semicircular part : $r=17.5$ m ; rectangular part : 35×10 m) of Ujigawa Hydraulics Laboratory, Disaster Prevention Research Institute, Kyoto University. A smooth concrete beach was constructed with a slope of 1:10. The beach was roughed by bonding light weight aggregate, the same material as the one whose used for modeling river sediment discharge, on to the smooth concrete. The wave guide walls, which are composed of smooth steel plates, were installed in the normal to the wave generator. On the land side, a sediment feeder machine with a controlling speed motor discharging sediment at a constant rate, was set about 1.0 m from the initial shoreline. Two smooth asbestos pipes were connected to the sediment feeder machine at the output opening in order to carry the sediment to the desire position of the point source, (see Fig. 6).

(2) Experimental procedure

Measurements of sediment discharge rate were performed: 1) at the beginning of the experiments, and 2) at every 10 min interval time during the experiments, then the rate of sediment discharge was calculated as the average value of the measured sediment ratios. Measurements of shoreline positions were made at 10 min intervals along the delta at 13 stations (50 cm interval distance), see Figure 12. At the end of each experiment, beach profiles as well as shoreline changes were measured every 10 cm interval distance along the delta. The formation process of river delta was observed by taking photographs every 10 minutes interval time.

Measurements of wave heights in the constant depth part were made using capacitance type wave gauges. While on the sloping part, the measurements were made using a wave gauge mounted on a carriage controlled by a personal computer. The angles of incoming wave incidence were measured in the constant depth part by measuring the angles of inclination of the wave generator to the beach. Snell's law and linear wave theory were used to estimate the angles of wave incidence at the breaker line. The longshore currents were visually observed by using colored paper tracers.

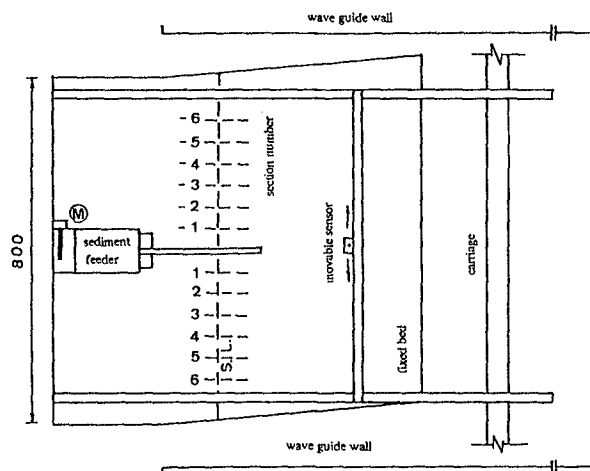
(3) Experimental results

Six experiments were performed, (see Table 1), for a still water depth in the constant depth part of 30 cm, a wave height of 2.0 cm and a wave period of 0.8 sec. The incoming waves were normally incident during the experiments series A and series B, while they were oblique incident during experiments C-1 and D-1, with 7.5° and 15° , respectively. The experimental results are drawn as follows :

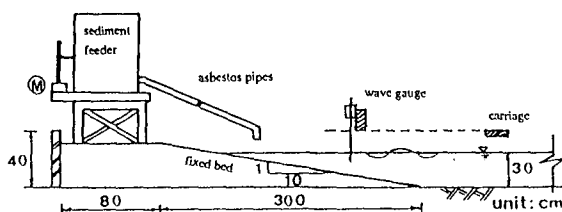
a) General description of the formation process: Once the sediment feeder machine supplies sediment to the wave basin, the sediment particles spread along the coast by the effect of the longshore current. Figure 7 shows the behaviors of the growth of river delta in experiments series A. The configuration of the shoreline of river delta in this experiment is nearly symmetric with respect to the center of the river.

Table 1. Experimental conditions of formation process of river delta.

Run	No.	Q_s cm ³ /sec	Run time (min)	α_0 (deg.)
Series A	A-1	7.06	50	0
	A-2	7.06	30	0
Series B	B-1	15.08	30	0
	B-2	8.12	30	0
Series C	C-1	11.0	90	7.5
Series D	D-1	11.0	180	15.0



(a) Plane view.



(b) Side view.

Fig. 6. Schematic diagram of experimental arrangement of formation and reduction processes of river deltas.

b) River delta configuration in case of oblique wave incidence; In the case of oblique wave incident, an asymmetrical shape of river delta is expected. Figure 8 illustrates the measured data of shoreline positions in Run C-1 where the angle of wave incident is 7.5° . The black symbols represent the measured data at the upcoast side, whereas the measured data at the downcoast side are represented by the white symbols. It is clear that asymmetrical river deltas are formed in the case of oblique wave incidence. In general, oblique waves generate strong longshore currents, consequently, most of the river sediment discharge is moved in the predominant direction of littoral drift and deposited on the downcoast side. Therefore, the growth rate of the shoreline on the downcoast side is faster than that on the upcoast side.

c) Variation of beach profile and beach slope along a delta coast; The beach profiles measured along the river delta for experimental Run A-2, $t = 80$ min, for the selected sections are illustrated in Fig. 9. It is seen from this figure that the beach profiles are convex in shape.

Figure 10 demonstrates the variation of beach slope along the river delta for Run B-1, $t = 30$ min, represented by a white symbol, and Run B-2, $t = 60$ min, represented by a black symbol, respectively. It is obvious from this figure that; 1) in Run B-1, the beach slope around the river mouth is steeper than the beach slope at the longshore ends of the river delta, where the configuration of river delta was also sharply curved, and 2) when the rate of river sediment input was reduced by half in Run B-2, the configuration of the shoreline of the river delta changed, becoming more gently curved, and therefore the variation of beach profiles along the river delta lessened.

(4) Applicability of analytical solutions to experiments

The measurements of shoreline evolution of a river delta for experimental runs A-1 and A-2 are compared with the analytical solution given by Eq. (8), the results are shown in Fig. 11. It is seen from the comparison that the evolution of the shoreline near the river mouth is well predicted. However, near the longshore ends of the river delta less agreement between the measured and computed shoreline position is observed. This may be due to the effect of the roughness of fixed bed in the wave tank, which induced rapid transport of the sediment downcoast, causing the shoreline to rapidly decrease.

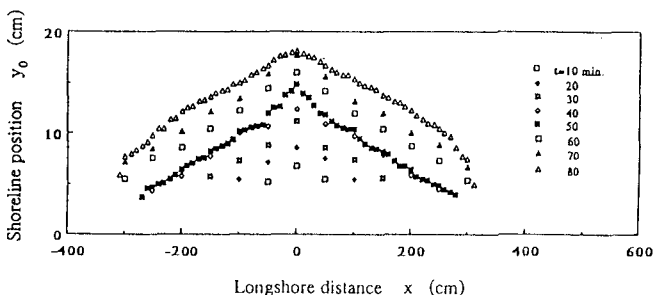


Fig. 7. Accretion process of shoreline in the formation of river delta in the case of Series A.

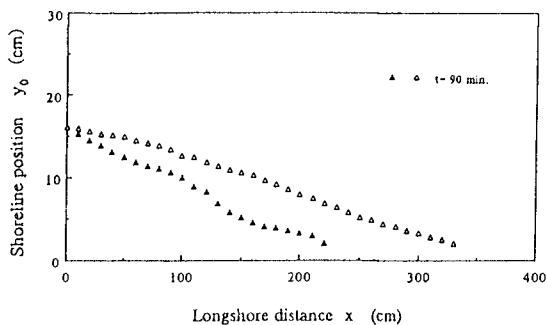


Fig. 8. Asymmetrically of river delta configuration in the case of oblique wave incidence, Run C-1.

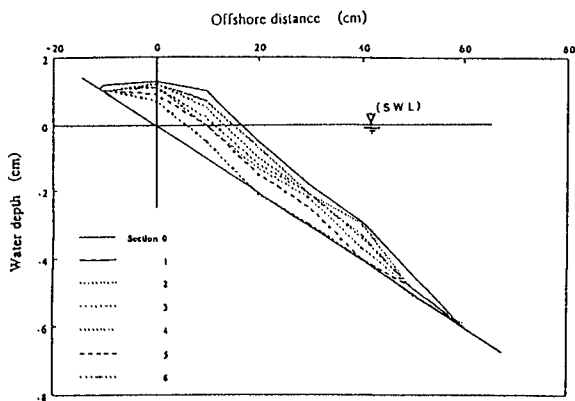


Fig. 9. Variation of beach profiles formed along the shoreline of river delta in the formation process, Run A-2.

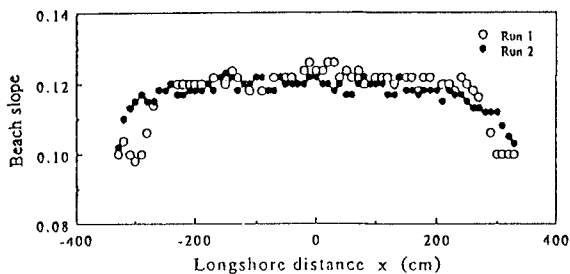


Fig. 10. Longshore changes in beach slope at the shoreline along the river deltas in the case of Series B.

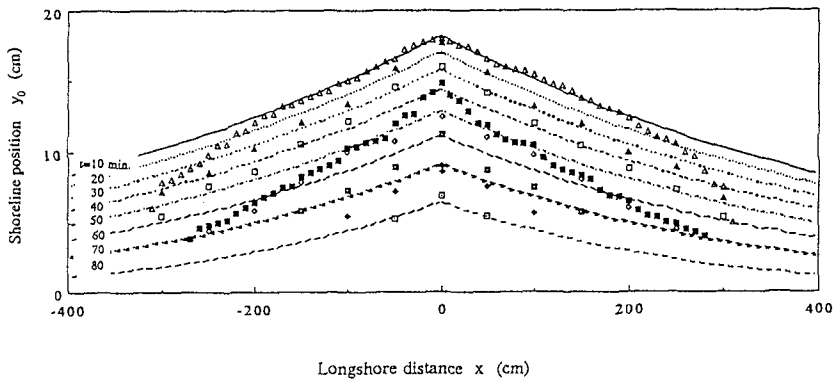


Fig. 11. Comparison between theoretical and experimental shorelines in the formation process of a river delta.

3.2 Experiments on Reduction Process of River Deltas

(1) Experimental procedure

The initial shoreline and beach profile in this experiment simulated the final ones in the experiment of the formation process of river delta presented in the previous section, (Series A and B). An experiment was halted once no remarkable longshore current existed, with only cross-shore movement. To verify this condition, color tracers were used and the trajectories were observed. Measurements of shoreline changes were made in the same way as in the formation process of river delta.

(2) Experimental results

Two experiments were performed, A-3 and B-3, with a still water depth of 30 cm, a wave height of 2.0 cm and a wave period of 0.8 sec. The incoming waves were normally incident during the experiments.

a) General description of river delta reduction process; The reduction process of the river delta is significantly different from the formation process of the river delta. Figure 12 shows the process of river delta reduction for experiment A-3. Due to the lack of sediment input, the shoreline of the delta rapidly becomes to a nearly straight, resulting in erosion near the center of river delta and deposition near the longshore ends of the river delta.

b) Variation of beach profiles; The variation of the beach profiles at selected experiment A-3, is shown in Fig. 13. It is noted that the beach profile changes from a convex type which was in the formation process to a concave one. This phenomenon is completely different from the formation process, and may be due to the rapid erosion in the surf zone to maintain the longshore sediment transport by waves.

4. Conclusions

The formation and reduction processes of river deltas were investigated theoretically and experimentally in this paper. The formation and reduction processes are governed by the relation between the rates of sediment input from rivers and longshore sediment transport by waves and currents. From the results of the theoretical solutions, the following conclusions can be drawn:

(1) The solutions of formation process of river deltas of infinite and finite river mouth widths were obtained from linearized equations of shoreline change. It was theoretically found that the configuration of river delta becomes asymmetric when the dimensionless quantity ϵ_1/ϵ_2 is greater than zero.

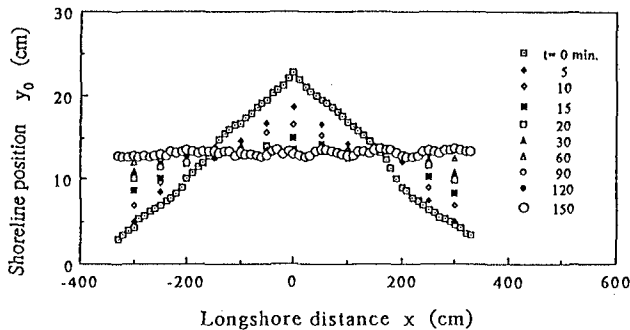


Fig. 12. Time variation of shoreline change in the reduction process of river delta in the case of Series B.

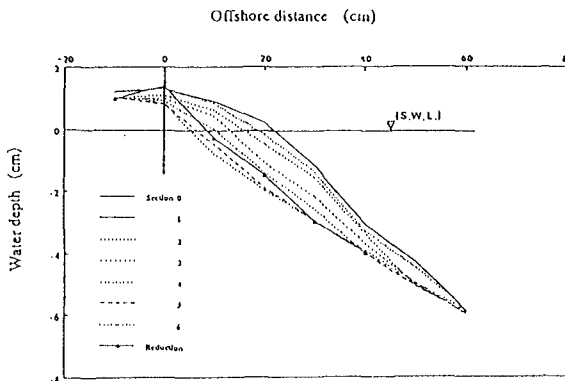


Fig. 13. Variation of beach profiles formed along the shoreline of river delta in the reduction process, Run B-3.

(2) By normal wave incidence, the reduction of the river delta takes place symmetrically with respect to the center of the river. The shoreline at the vicinity of the river mouth is rapidly eroded, while the shorelines at the longshore ends of the river delta retreat slowly. At the final stage, the configuration of the river delta may become a straight beach. By oblique wave incidence, the reduction of the river delta takes place asymmetrically, at a faster rate of reduction on the upcoast side of the delta than on the downcoast side. Also, the river mouth moves predominantly in the direction of littoral drift, as the reduction process of river delta takes place.

In order to understand the physical processes of formation and reduction of river deltas, and to compare the results of the formation process with the theoretical one, experiments on the formation and reduction processes of river deltas were carried out. The following conclusions can be drawn from the experiments:

(1) In the formation process, the shoreline of river delta propagates symmetrically with respect to the center of the river, when the waves is normally incident. On the contrary, by obliquely waves incidence, the shoreline of river delta propagates asymmetrically with respect to the center of the river at faster rate of formation on the downcoast side than that on the upcoast side.

(2) On the contrary of the formation process of river deltas, in the reduction process, the shoreline of river delta is rapidly eroded. It was noted in the reduction process that the beach profile changes from convex type, which was shown in the formation process, to a concave-upward one. This fact is completely different from the formation process, and may be due to the rapid erosion in the surf zone to maintain the longshore sediment transport by waves.

(3) Since the waves approach the shoreline obliquely, the longshore sediment transport is predominant. Consequently, the reduction rate of the shoreline on the upstream coast side is greater than that on the downstream coast side. The final stage for the configuration of the delta may be an inclined straight beach where the waves tend to approach it normally.

(4) In comparison, all experimental parameters in the theoretical solutions must be determined experimentally. It was concluded from the comparison that the evolution of the shoreline near the river mouth is well-predicted.

Finally, further extension of the present study should be directed towards both theoretical investigations of formation and reduction processes of river deltas as one of diffusion processes, which is the task of the near future.

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