

CHAPTER 18

Extension of the Maximum Entropy Principle Method for Directional Wave Spectrum Estimation

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Abstract

This paper presents an extension of the maximum entropy principle method (MEP), named the extended MEP (EMEP), as a general and practical method for estimating directional wave spectra. Since the EMEP is formulated to consider errors in the cross-power spectra, it proves to be an accurate, reliable, and robust method against such errors. In addition, we also examine the EMEP using numerical simulation and field wave data, with its validity being subsequently discussed.

1. Introduction

The directional spectrum of ocean waves expresses their fundamental properties by describing the energy distribution as a function of wave frequency and wave propagation direction. Many methods have been proposed for estimating the directional spectra of various types of ocean wave measurements, e.g., the direct Fourier transformation method (DFTM), parametric method, maximum likelihood method (MLM), extended maximum likelihood method (EMLM), maximum entropy principle method (MEP), and Bayesian directional spectrum estimation method (BDM).

The MEP (Hashimoto and Kobune, 1985) can estimate the directional spectra using three simultaneously measured quantities related to random wave motion, for instance, pitch, roll, and heave data, and when applied as such, estimates of directional spectra have better directional resolution than other existing methods. On the other

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hand, the MEP is not a general method because it is restricted to applications involving three-quantity measurements.

In comparison, the BDM (Hashimoto, 1987) can handle more than three arbitrary-mixed instrument array measurements and has the highest resolution among existing methods for estimating the directional spectrum under such conditions. This method is a robust method for estimating the directional spectrum using cross-power spectra contaminated by estimation errors, though it is also a general method and requires the use of time-consuming iterative calculations.

Consequently, a method needed to be developed that can be applied to arbitrary general measurements, while at the same time easily yielding an accurate and reliable estimate of the directional spectrum.

The present paper discusses inherent drawbacks of several existing methods for estimating the directional spectrum, and then describes a new method, the Extended MEP (EMEP), which correspondingly functions as a general yet practical method. Since the EMEP is formulated in the same manner as the BDM, i.e., considering errors in the cross-power spectra, it demonstrates both robustness and reliability. Here we also examine the EMEP using numerical simulation and field wave data, with its validity being subsequently discussed.

2. Fundamental Equation Related to Directional Spectrum

The general relationship between the cross-power spectrum for a pair of wave properties and the wave-number frequency spectrum was introduced by Isobe et al. (1984) as follows:

$$\Phi_{mn}(\omega) = \int_k H_m(k, \omega) H_n^*(k, \omega) \exp\{-ik(x_n - x_m)\} S(k, \omega) dk, \quad (1)$$

where ω is the angular frequency, \mathbf{k} the wave number vector, $\Phi_{mn}(\omega)$ the cross-power spectrum between the m -th and n -th wave properties, $H_m(\mathbf{k}, \omega)$ the transfer function from the water surface elevation to the m -th wave property, i the imaginary unit, \mathbf{x}_m the location vector of the wave probe for the m -th wave property, $S(\mathbf{k}, \omega)$ the wave-number frequency spectrum, and " * " denotes the conjugate complex. The wave number k is related to the frequency f by the following dispersion relationship:

$$\omega^2 = (2\pi f)^2 = gk \tanh kd, \quad (2)$$

hence, the wave-number frequency spectrum can be expressed as a function of the frequency f and wave propagation direction θ . Equation (1) can therefore be rewritten as

$$\Phi_{mn}(f) = \int_0^{2\pi} H_m(f, \theta) H_n^*(f, \theta) [\cos\{k(x_{mn} \cos\theta + y_{mn} \sin\theta)\} - \sin\{k(x_{mn} \cos\theta + y_{mn} \sin\theta)\}] S(f, \theta) d\theta, \quad (3)$$

where $x_{mn} = x_n - x_m$, $y_{mn} = y_n - y_m$, and $S(f, \theta)$ is the directional spectrum.

The directional spectrum is commonly expressed as a product of the frequency spectrum $S(f)$ and the directional spreading function $G(\theta|f)$, i.e.,

$$S(f, \theta) = S(f)G(\theta|f), \quad (4)$$

with $S(f, \theta)$ taking a non-negative value and satisfying the following relationship:

$$\int_0^{2\pi} S(f, \theta) d\theta = S(f). \quad (5)$$

Substitution of Eq. (4) into (5) yields

$$\int_0^{2\pi} G(\theta|f) d\theta = 1. \quad (6)$$

The transfer function $H_m(f, \theta)$ in Eq. (3) is generally expressed as

$$H_m(f, \theta) = h_m(f) \cos^{\alpha_m} \theta \sin^{\beta_m} \theta, \quad (7)$$

where the function $h_m(f)$ and parameters α_m and β_m obtained from small amplitude wave theory are specified for each measured quantity in Isobe et al. (1984).

Equations (1) and (3) are the fundamental equations for estimating the directional spectrum on the basis of simultaneous measurements of various wave properties. If the function $S(k, \omega)$ or $S(f, \theta)$ is determined which respectively satisfies Eq. (1) or (3), and it has a non-negative value, then this function is termed as the directional spectrum.

3. Existing Methods for Estimating the Directional Spectrum

If an infinite number of wave properties are measured, i.e., the cross-power spectra are known for infinite pairs of m and n in Eq. (1) or (3), the directional spectrum can be uniquely determined. However, only a limited number of wave properties can actually be measured at a limited number of locations; thus, the directional spectrum cannot be uniquely determined since the number of component waves is infinite. Existing methods for directional spectral estimation therefore attempt to determine the unique function by introducing some type of idea and/or trick, and as a result, their advantages and disadvantages depend on the employed concept. The most prominent methods and their advantages/disadvantages are briefly discussed as follows.

(1) Direct Fourier Transformation Method (DFTM)

This method was first proposed by Barber (1961) and uses the following estimation equation:

$$\hat{S}(f, \theta) = \alpha \sum_m \sum_n \Phi_{mn}(f) \exp\{ik(x_n - x_m)\}, \tag{8}$$

where α is a proportionality constant such that the estimate of the directional spectrum satisfies Eq. (5). Although the method's computation is easy, the directional resolution is low and a negative energy distribution sometimes occurs.

(2) Parametric Method

Parametric methods employed by Longuet-Higgins et al. (1963), Panicker and Borgman (1974), and Hasselman et al. (1980) assume a specific formulation for the directional spectrum, such the following truncated Fourier series or a cosine-powered function:

$$S(f, \theta) = a_0(f) + \sum_{n=1}^N \{a_n(f) \cos \theta + b_n(f) \sin \theta\} \tag{9}$$

$$S(f, \theta) = S(f) \sum_n \alpha_n(f) \cos^{2s_n(f)} \left\{ \frac{\theta - \theta_n(f)}{2} \right\}. \tag{10}$$

It should be realized that these methods are only able to approximate the real directional spectrum when it suitably fits the model.

(3) Extended Maximum Likelihood Method (EMLM)

This method was proposed by Isobe et al. (1984) who extended the maximum likelihood method (MLM) developed by Capon (1969) so as to handle an arbitrary combination of wave properties. The EMLM formula is derived using a window theory, namely,

$$S(\mathbf{k}, \omega) = \alpha / \left[\sum_m \sum_n \Phi_{mn}^{-1}(\omega) H_m^*(\mathbf{k}, \omega) H_n(\mathbf{k}, \omega) \exp\{ik(x_n - x_m)\} \right], \tag{11}$$

where $\Phi_{mn}^{-1}(\omega)$ is the mn element of inverse matrix $\Phi^{-1}(\omega)$, while α is a proportionality constant such that the directional spectrum $\hat{S}(\mathbf{k}, \omega)$ satisfies Eq. (5).

The EMLM has high-directional resolution and versatility, and consequently, has been widely employed in directional wave analysis. On the other hand, when the layout of the probe array is not proper or the cross-power spectra are contaminated by errors, this method sometimes estimates erroneous peaks or negative values, while also in some cases failing to yield a smooth and continuous estimation of the directional spectrum.

(4) Maximum Entropy Principle Method (MEP)

We previously developed the MEP (Hashimoto and Kobune, 1985), which provides a powerful means for estimating the directional spectrum when three-quantity point measurements are available. Such data can be obtained, for example,

using a disc buoy or a two-axis current meter and a wave gauge. The estimation equation of the MEP's directional spreading function is expressed by maximizing the entropy with an assumption that the directional spreading function is a type of probability density function, i.e.,

$$G(\theta|f) = \exp[a_0(f) + \sum_{n=1}^2 \{a_n(f) \cos n\theta + b_n(f) \sin n\theta\}], \quad (12)$$

where the coefficients a_n and b_n are the Lagrange multipliers. The advantage of this expression is that even though the Fourier series in the power of the exponential function is truncated by $n = 2$, it has non-negative values and yields a wide range of shapes for $G(\theta|f)$.

It should be noted that although the MEP was originally developed to be used with three-quantity point measurements, Nwogu (1989) expanded it for application to array measurements. Since this expansion was carried out using the same procedure as the original one, the directional spreading function results in a complex form including Bessel functions; thereby making the computation more difficult than the original MEP.

(5) Bayesian directional spectrum estimation method (BDM)

The BDM (Hashimoto, 1987) provides an accurate and reliable estimate of the directional spectrum for array measurements consisting of more than three arbitrary quantities. The assumed estimation equation of the BDM's directional spreading function is not a formal mathematical function, instead being a piece-wise constant function expressed as

$$\hat{G}(\theta|f) = \sum_{k=1}^K \exp\{x_k(f)\} I_k(\theta), \quad (13)$$

where

$$x_k(f) = \ln\{G(\theta_k|f)\} \quad (14)$$

$$I_k(\theta) = \begin{cases} 1 & : (k-1)\Delta\theta \leq \theta < k\Delta\theta \\ 0 & : \text{otherwise} \end{cases} \quad (15)$$

$(k = 1, \dots, K)$.

Equation (13) can be determined by minimizing Akaike's Bayesian information criterion (ABIC) (Akaike, 1980) and applying an additional condition that the directional spreading function be smooth and continuous, namely,

$$\{x_k - 2x_{k-1} + x_{k-2}\}^2 \rightarrow \text{small} \quad (16)$$

$(k = 1, \dots, K \text{ and } x_0 = x_K, x_{-1} = x_{K-1})$.

The advantage of Eq. (13) is that even though the additional condition of Eq. (16) is

imposed, it generates non-negative values and yields a wide range of arbitrary shapes for $G(\theta|f)$. However, since it consists of many unknown parameters $x_k(f)$; ($k = 1, \dots, K$), the BDM involves the use of time-consuming iterative calculations.

After reviewing the various methods, it is clear that each formulates a tailored model for approximating the directional spectrum, being characterized by some unknown parameters. Though the principle and method of deriving each model are different, inherent advantages and disadvantages arise due to the model's different characteristics. When considering the MEP and BDM, since they are characterized by an exponential function incorporating a power function, an advantage exists in that these models result in non-negative values and flexibly yield a wide range of arbitrary shapes of the directional spectrum.

4. Formulation of Extended Maximum Entropy Principle Method (EMEP)

To simplify the nomenclature in the equations, Eq. (3) is rewritten using the upper triangular components of the hermitian matrix $\Phi(f)$, such that

$$\phi_i(f) = \int_0^{2\pi} H_i(f, \theta)G(\theta|f)d\theta \quad (i = 1, \dots, K), \tag{17}$$

where K is the number of equations, and

$$H_i(f, \theta) = H_m(f, \theta)H_n^*(f, \theta)[\cos\{k(x_{mn} \cos\theta + y_{mn} \sin\theta)\} - \sin\{k(x_{mn} \cos\theta + y_{mn} \sin\theta)\}] / W_{mn}(f) \tag{18}$$

$$\phi_i(f) = \Phi_{mn}(f) / \{S(f)W_{mn}(f)\} \tag{19}$$

$$G(\theta|f) = S(f, \theta) / S(f), \tag{20}$$

with $W_{mn}(f)$ being a weighting function introduced for normalizing and non-dimensionalizing the errors of the cross-power spectra. This function is represented by the following equation (standard deviations of error of co-spectrum and quadrature-spectrum, Bendat and Pirsol, 1986) for the real and imaginary part of Eq. (17), respectively:

$$\sigma[\hat{C}_{mn}(f)] \approx [\{ \Phi_{mn}(f)\Phi_{mn}(f) + C_{mn}(f)^2 - Q_{mn}(f)^2 \} / 2N_a]^{1/2} \tag{21}$$

$$\sigma[\hat{Q}_{mn}(f)] \approx [\{ \Phi_{mn}(f)\Phi_{mn}(f) - C_{mn}(f)^2 + Q_{mn}(f)^2 \} / 2N_a]^{1/2}, \tag{22}$$

where $\Phi_{mn}(f) = C_{mn}(f) - iQ_{mn}(f)$ and N_a is the number of the ensembled average.

The directional spreading function normally takes values greater than or equal to zero. However, in the EMEP, the function is treated as a function which always takes positive values. Then, as a general expression for $G(\theta|f)$, we can extend Eq. (12) (or simplify Eq. (13)) to obtain

$$G(\theta|f) = \frac{\exp\left[\sum_{n=1}^N \{a_n(f) \cos n\theta + b_n(f) \sin n\theta\}\right]}{\int_0^{2\pi} \exp\left[\sum_{n=1}^N \{a_n(f) \cos n\theta + b_n(f) \sin n\theta\}\right] d\theta} \quad (23)$$

where $a_n(f)$, $b_n(f)$; ($n = 1, \dots, N$) are unknown parameters.

For the sake of convenience, the complex values $\phi_i(f)$ and $H_i(f, \theta)$ are rewritten as

$$\left. \begin{aligned} \phi_i &= \text{Re}\{\phi_i(f)\} & \phi_{K+i} &= \text{Im}\{\phi_i(f)\} \\ H_i(\theta) &= \text{Re}\{H_i(f, \theta)\} & H_{K+i}(\theta) &= \text{Im}\{H_i(f, \theta)\} \end{aligned} \right\} \quad (24)$$

so that $\phi_i(f)$ and $H_i(f, \theta)$ are real. For simplicity, the frequency f is omitted on the LHS of Eq. (24), and is also omitted hereafter.

When Eq. (17) is applied to the observed data, the errors contained in the cross-power spectra must be taken into account. Thus, after making the substitution of Eq. (23) into (17), it can be modified by considering the existence of errors ϵ_i , i.e.,

$$\epsilon_i = \frac{\int_0^{2\pi} \{\phi_i - H_i(\theta)\} \exp\left\{\sum_{n=1}^N (a_n \cos n\theta + b_n \sin n\theta)\right\} d\theta}{\int_0^{2\pi} \exp\left\{\sum_{n=1}^N (a_n \cos n\theta + b_n \sin n\theta)\right\} d\theta} \quad (25)$$

$(i = 1, \dots, M)$

where M is the number of independent equations left after eliminating the meaningless equations such as the zero co-spectrum and zero quadrature-spectrum.

Now, if ϵ_i ; ($i = 1, \dots, M$) are assumed to be independent of each other, and the probability of their occurrence is expressed by the normal distribution having a zero mean and variance σ^2 , the optimal $G(\theta|f)$ is one which minimizes $\sum \epsilon_i^2$. However, Eq. (25) is nonlinear with respect to a_n , b_n ; ($n = 1, \dots, N$), and is therefore difficult to solve. Consequently, let us apply Newton's technique of local linearization and iteration to solve the problem.

If approximate solutions \tilde{a}_n , \tilde{b}_n ; ($n = 1, \dots, N$) are known, the solution a_n , b_n ; ($n = 1, \dots, N$) can be written as

$$\left. \begin{aligned} a_n &= \tilde{a}_n + a_n' \\ b_n &= \tilde{b}_n + b_n' \end{aligned} \right\} \quad (26)$$

where a_n' , b_n' are the residuals between the solution a_n , b_n and the approximate solution \tilde{a}_n , \tilde{b}_n .

Substitution of Eq. (26) into (25) and rearranging yields the following linearized equation with respect to a_n' , b_n' :

$$\epsilon_i = Z_{N,i} - \sum_{n=1}^N (a_n' X_{n,i} + b_n' Y_{n,i}) \quad (i = 1, \dots, M), \quad (27)$$

where

$$Z_{N,i} = \frac{\int_0^{2\pi} \{\phi_i - H_i(\theta)\} F_N(\theta) d\theta}{\int_0^{2\pi} F_N(\theta) d\theta} \quad (28)$$

$$X_{n,i} = Z_{N,i} \left\{ \frac{\int_0^{2\pi} F_N(\theta) \cos n\theta d\theta}{\int_0^{2\pi} F_N(\theta) d\theta} - \frac{\int_0^{2\pi} \{\phi_i - H_i(\theta)\} F_N(\theta) \cos n\theta d\theta}{\int_0^{2\pi} \{\phi_i - H_i(\theta)\} F_N(\theta) d\theta} \right\} \quad (29)$$

$$Y_{n,i} = Z_{N,i} \left\{ \frac{\int_0^{2\pi} F_N(\theta) \sin n\theta d\theta}{\int_0^{2\pi} F_N(\theta) d\theta} - \frac{\int_0^{2\pi} \{\phi_i - H_i(\theta)\} F_N(\theta) \sin n\theta d\theta}{\int_0^{2\pi} \{\phi_i - H_i(\theta)\} F_N(\theta) d\theta} \right\} \quad (30)$$

$$F_N(\theta) = \exp\left\{ \sum_{n=1}^N (\tilde{a}_n \cos n\theta + \tilde{b}_n \sin n\theta) \right\}. \quad (31)$$

Equation (27) can be solved iteratively by assuming a proper approximate solution \tilde{a}_n , \tilde{b}_n ; ($n = 1, \dots, N$).

Minimizing $\sum \epsilon_i^2$ for a particular data set also introduces an additional problem of choosing the optimal finite order N for the model (Eq. (23)); hence to overcome this, the minimum Akaike's Information Criterion (AIC) procedure (Akaike, 1973) is incorporated into the above iterative calculations to yield a reasonable and smooth estimate of $G(\theta|f)$. The AIC is given by

$$\text{AIC} = M(\ln 2\pi + 1) + M \ln \hat{\sigma}^2 + 2(2N + 1), \quad (32)$$

where M is the number of independent equations (Eq. (27)) and $\hat{\sigma}^2$ is the estimate of the variance of ϵ_i ; ($i = 1, \dots, M$).

5. Numerical computation of the EMEP

To estimate the directional spectrum using the EMEP, the computation must be performed from lower ($N = 1$) to higher orders. During the iterative computation, however, the computation occasionally becomes unstable in special cases. If so, a control parameter δ is introduced into Eq. (26) to under-relax the iterative computation:

$$\left. \begin{aligned} a_n &= \tilde{a}_n + \delta a_n' \\ b_n &= \tilde{b}_n + \delta b_n' \end{aligned} \right\} \quad (33)$$

that is, when the iterative computation is unstable, the control parameter δ is changed to a smaller value, followed by $\delta = (0.5)^k : (k = 0, \dots, 4)$.

The numerical computation procedure including the minimization of the AIC is summarized as follows:

- 1) Select the lowest model order $N = 1$ and compute $X_{n,i}$, $Y_{n,i}$, and $Z_{N,i}$ of Eq. (28) \sim (30) assuming the initial approximate solutions of \tilde{a}_1 and \tilde{b}_1 being equal to zero. Then, to obtain solutions a_1 and b_1 , carry out the iterative computation of Eq. (27) and perform the least square method for $\sum \epsilon_i^2$ until the absolute values of residuals $|a'_1|$ and $|b'_1|$ become small enough ($|a'_1|, |b'_1| < 0.01$).
- 2) Compute the AIC by Eq. (32).
- 3) Substitute a_1 and b_1 obtained in step 1) into Eq. (31) vice \tilde{a}_1 and \tilde{b}_1 , leaving \tilde{a}_2 and \tilde{b}_2 equal to zero, and carry out the same procedure as step 1) to obtain the solutions $a_i, b_i : (i = 1, 2)$ of the 2nd order model ($N = 2$). If during the iterative computation one of the absolute values of residuals $|a'_i|, |b'_i| : (i = 1, 2)$ has a value greater than 30, terminate the computation and adopt $N = 1$ as the optimal model order, with the values a_1 and b_1 obtained in step 1) being chosen as the solutions. If the iterative computation does not converge after 100 iterations, terminate the computation and similarly choose the solutions obtained in step 1). If $|a'_i|, |b'_i| : (i = 1, 2)$ become less than 0.01, compute the AIC using Eq. (32) and proceed to the next step.
- 4) If the AIC obtained in step 3) is greater than that of step 2), or if the absolute value of the difference between the two AICs is much less than 1, adopt $N = 1$ as the optimal model order and choose the values a_1 and b_1 of step 1) as the solutions. If the above cases do not hold, proceed to the next step.
- 5) Change the model order into a higher one (3rd, 4th,) and repeat in the same manner as steps 3) and 4).
- 6) During the computation of step 5), if the AIC of the model order $N+1$ is greater than that of the previous order N , or when the absolute value of the difference between the AIC of the model order $N+1$ and that of the previous order N is much less than 1, or when the iterative computation does not converge at the model order $N+1$, then terminate the computation and choose the model order

N and $\alpha_i, b_i : (i = 1, \dots, N)$ as the optimal solutions.

7) Substitute the optimal solutions $\alpha_i, b_i : (i = 1, \dots, N)$ into Eq. (23) to get the optimal directional spreading function $\hat{G}(\theta|f)$.

In addition, since the number of unknown parameters should be less than or equal to the number of independent equations, i.e., $2N \leq M$, steps 1) ~ 6) should be performed at most within this order.

6. Examination of EMEP by Numerical Simulation

Here we will employ numerical simulation to examine the validity of using the EMEP to estimate the directional spectrum. The procedure is the same as that used for examining the EMLM (Isobe et al., 1984), where the employed directional spreading function is a cosine-powered type function expressed as

$$G(\theta) = \sum_i \alpha_i \cos^{2S_i} \left(\frac{\theta - \theta_i}{2} \right). \tag{34}$$

When $i = 1$ only, Eq. (34) yields a unimodal directional spreading function, while a bimodal function is formulated by the superposition of two unimodal directional spreading functions, i.e., $i = 1$ and 2, having a different coefficient α_i , mean direction θ_i , and spreading parameter S_i . The cross-power spectra utilized for the numerical simulation are obtained by numerically integrating Eq. (3).

Figure 1 compares the given directional spreading function (TRUE) and the estimated ones by the EMEP, MEP, BDM, and EMLM, where the three measured quantities (sea surface elevation and two orthogonal slopes on the surface at the same location) are assumed to be the simulated observation condition. The ordinate is normalized by dividing the value of the directional spreading function by the maximum value of the TRUE directional spreading function. As indicated, the EMEP yields the same estimate as the MEP and can detect the small peak in proper direction in **Fig. 1(e)** and **(f)**, though it overestimates the main peak and underestimates the

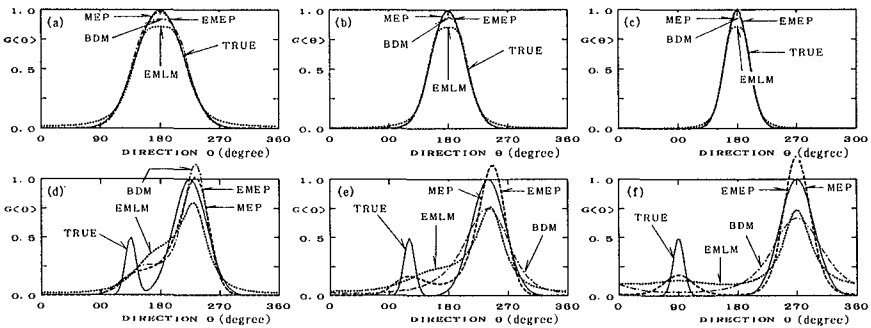


Fig. 1 Comparison of EMEP, MEP, BDM and EMLM (Three-quantity measurement)

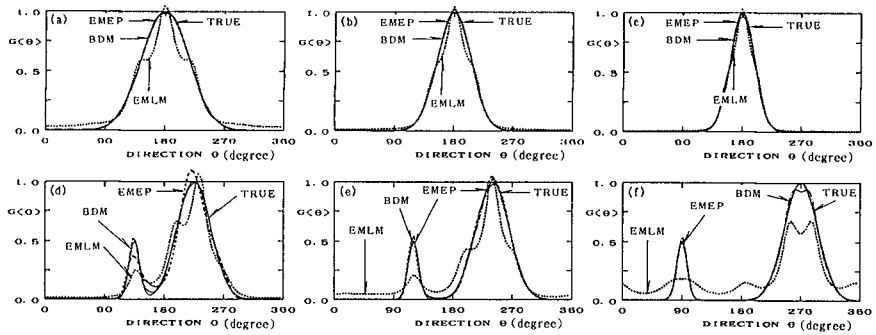


Fig. 2 Comparison of EMEP, MEP, BDM and EMLM (Star array measurement)

small peak. Also note that the EMEP (MEP) shows the minimum leakage of the wave energy into neighboring directional bands. On the other hand, the EMLM appears to recognize the existence of the small peak in **Fig. 1(e)**, but its estimated direction is not proper, while the BDM does not even recognize the small peak.

Figure 2 shows results when a star array consisting of four wave gauges is assumed as the simulated observation condition. The minimum distance D between the wave gauges is assumed as $D/L=0.2$, where L is the wave length. In comparison with **Fig. 1**, the directional resolution of the EMEP, BDM, and EMLM is improved in **Fig. 2(d)**, **(e)**, and **(f)**. In particular, the EMEP is very close to the BDM and shows good agreement with TRUE. In contrast, the EMLM underestimates the small peak and shows some energy leakage around the peaks, especially in **Fig. 2(e)** and **(f)**.

7. Field Data Analysis

Here we apply the EMEP to analyze wave records acquired at an offshore oil rig (Iwaki-oki Station) located 42 km off the Iwaki coast, the northeastern coast of the main island of Japan (**Fig. 3**), where the water depth is 155 m. **Figure 4** shows the oil rig, which has installed on its legs, four step-resistance wave gauges, a two-axis ultrasonic current meter, and a pressure sensor (**Fig. 5**). The simultaneous measurement of seven elements is performed over 20 min every 2 h. The Onahama Port Construction Office, Second Port Construction Bureau, Ministry of Transport, has been conducting this multi-element measurement of directional waves since 1986.

Figure 6 shows typical directional spectra of a swell having a significant wave height $H_{1/3} = 3.81$ m, period $T_{1/3} = 12.3$ s, and directional spreading parameter $S_{\max} = 75$, which are estimated by the EMEP, BDM, and EMLM. Note that the shape of the EMEP estimate is very similar to that of the BDM, but different from the EMLM's. The major difference between them is that the EMLM is more diffuse and does not show a concentration around the peak. In addition, the peak of the EMLM is obviously much lower than that of the EMEP and BDM.

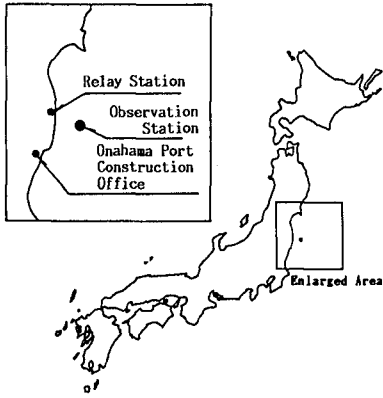


Fig. 3 Location of the wave observation station.

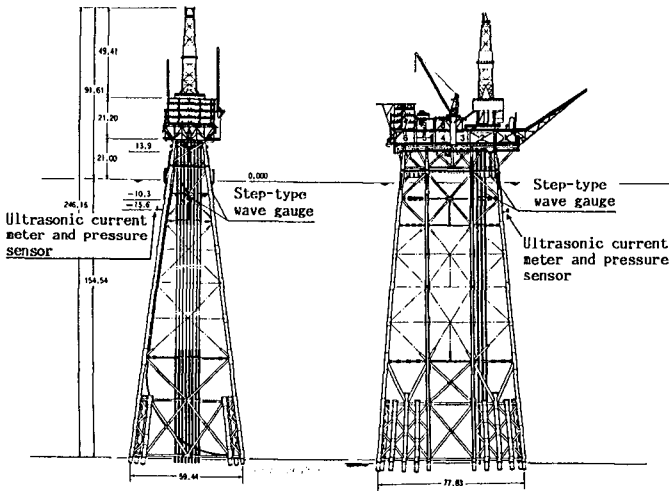


Fig. 4 Offshore oil rig and the location of wave instruments.

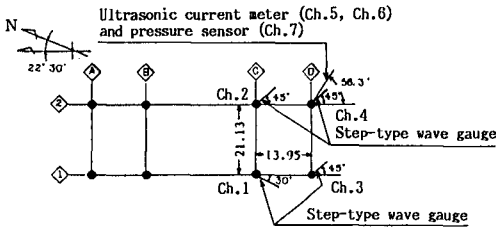


Fig. 5 Wave instrument array.

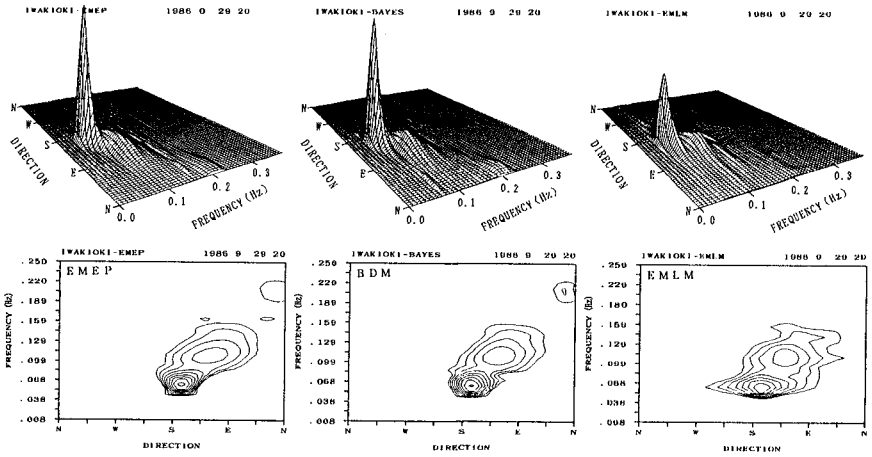


Fig. 6 Typical directional spectra of a swell estimated by EMEP, BDM and EMLM

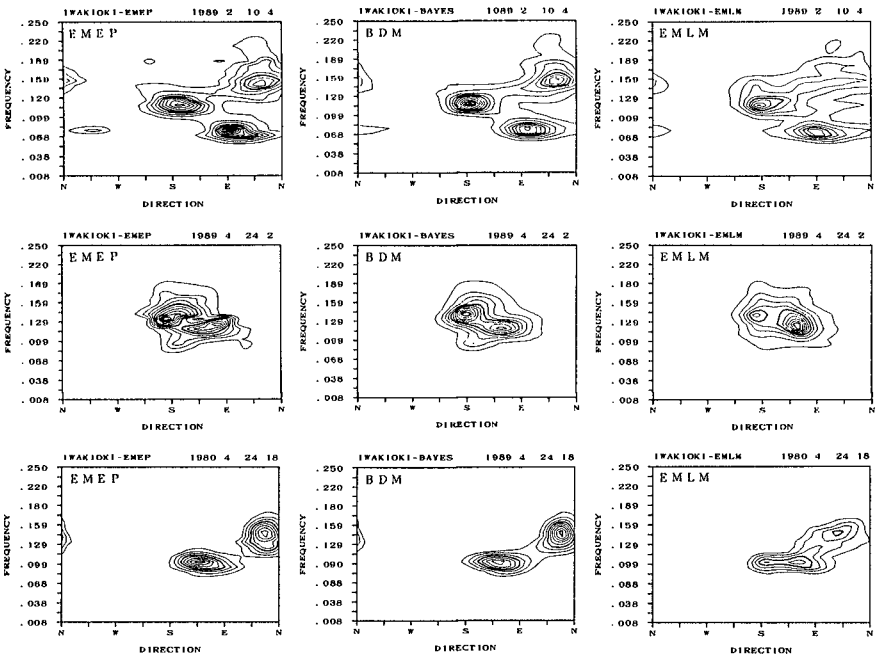


Fig. 7 Typical directional spectra of multiple wave systems estimated by EMEP, BDM, and EMLM.

Sometimes several wave systems exist from different sources, with Fig. 7 showing such an example measured at the Iwaki-oki Station for the EMEP, BDM, and EMLM. The contour lines of the relative spectral density are plotted by direction versus frequency domain. The upper figures show directional spectra having $H_{1/3} = 2.18$ m and $T_{1/3} = 9.0$ s, while the middle ones are $H_{1/3} = 2.02$ m and $T_{1/3} = 8.2$ s and the lower ones $H_{1/3} = 3.94$ m and $T_{1/3} = 8.8$ s. In the EMEP and BDM, several wave groups can be clearly seen coming from different directions with different peak frequencies, whereas some of the EMLM energy peaks are diffused and not as clear.

8. Conclusions

Precise determination of the directional spectral characteristics of ocean waves is an essential step in planning and designing of coastal and offshore structures. Since field measurements of directional wave spectra require deployment of multiple sensors, all of which must be maintained in good condition for successful data recording, the obtained data must be analyzed using an accurate, reliable, and robust method to estimate the directional wave spectrum. We believe the proposed EMEP meets these criteria and can be successfully employed for carrying out research on directional seas.

Our major conclusions are as follows:

- 1) The EMEP can be applied to handle arbitrary-mixed instrument array measurements.
- 2) When the EMEP is applied to three-quantity measurements, it yields the same estimate as the MEP and has higher resolution than the EMLM.
- 3) When the EMEP is applied to more than three arbitrary-mixed instrument array measurements, it yields almost the same estimate as the BDM and has the highest resolution among other existing methods.
- 4) A personal computer can be used to perform the EMEP; thereby obtaining real-time estimation of directional spectra.

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