CHAPTER 47

Bragg Scattering of Waves over Porous Rippled Bed

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ABSTRACT: A time-dependent and a time-independent wave equations are developed for waves propagating over porous rippled beds taking account of the effects of porous medium. The mean water depth and the thickness of porous layer are assumed to be slowly varying compared to the wavelength of surface gravity waves, and the spatial scale of ripples is assumed to be the same as the wavelength of surface waves. By using the time-independent equation, the Bragg scattering is examined in one-dimensional case. The results show that the reflected and transmitted waves become smaller than those in the case of impermeable rigid rippled bed due to energy dissipation in porous medium.

Introduction

Davies and Heathershaw (1984) studied the reflection from sinusoidal undulation over a horizontal bottom and derived a solution of reflection coefficient. Their experimental results showed a resonant Bragg reflection at the condition where the wavelength of the bottom undulation is one half the wavelength of the surface wave as predicted by their theory. Mei (1985) and Naciri and Mei (1988) developed theories of wave evolution at and close to the resonant condition by shore-parallel sinusoidal bars and two-dimensional doubly sinusoidal undulations over a slowly varying topography. For more realistic natural topography, Kirby (1986) derived a general wave equation which extends the mild slope equation of Berkhoff (1972). These existing theories don't take account of the effects of seabed permeability.

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The strong reflection of incident waves due to the Bragg reflection results in rough offshore sea. Since it is desirable to reduce the transmitted and reflected waves, artificial porous ripples or bars would be more convenient.

In this study, a time-dependent wave equation is developed, by extending the theory of Kirby (1986), for waves propagating over permeable rippled beds in order to take into account the effects of porous medium. Some numerical calculations are carried out to show the effects of seabed permeability on wave transformations or on the Bragg scattering by ripples in one-dimensional case.

Derivation of Wave Equation over Porous Rippled Bed

The coordinate system and main quantities are shown in Fig.1. The actual depth, $h'(\boldsymbol{x})$, is divided into the rapidly varying small amplitude undulation, $\delta(\boldsymbol{x})$, and the slowly varying mean water depth, $h(\boldsymbol{x})$:

$$h'(\boldsymbol{x}) = h(\boldsymbol{x}) - \delta(\boldsymbol{x}) \tag{1}$$

where $\boldsymbol{x} = (x, y)$. Thickness of the porous layer, $h_s'(\boldsymbol{x})$, is expressed as

$$h_s'(\boldsymbol{x}) = h_s(\boldsymbol{x}) + \delta(\boldsymbol{x}) \tag{2}$$

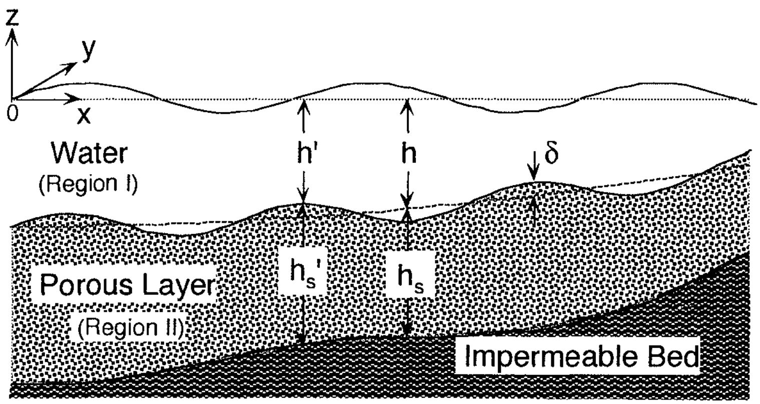


Fig.1 Definition of variables

where $h_s(\boldsymbol{x})$ is the slowly varying mean thickness. The bottom beneath the porous layer is assumed to be impermeable and rigid.

The horizontal scales of changes of $h(\boldsymbol{x})$, $h_s(\boldsymbol{x})$ and $\delta(\boldsymbol{x})$ are

$$O\left(\frac{\nabla_h h}{kh}\right) \approx O(k\delta) << 1 \tag{3}$$

$$O\left(\frac{\nabla_h (h + h_s)}{kh}\right) \approx O(k\delta) << 1 \tag{4}$$

and

$$O\left(\frac{\nabla_h \delta}{k\delta}\right) \approx O(1) \tag{5}$$

where ∇_h is the gradient operator as $(\partial / \partial x, \partial / \partial y)$, and k is the wave-number.

The analytical domain is divided into two regions: the region (I) is the fluid domain above the porous layer; the region (II) is the porous layer, as shown in Fig.1.

In the region (I), the irrotational motion of incompressive and inviscid fluid is described by a velocity potential, ϕ, as follows:

$$\nabla_h^2 \phi + \phi_{zz} = 0 \ ; \quad -h \le z \le 0 \tag{6}$$

$$\phi_{tt} + g\phi_z = 0 \ ; \quad z = 0 \tag{7}$$

$$\phi_z = -\nabla_h h \cdot \nabla_h \phi + \nabla_h \cdot (\delta \nabla_h \phi) + w^{(\mathrm{I})} \ ; \quad z = -h \tag{8}$$

Eq.(6) is the Laplace equation, Eq.(7) is the free surface boundary condition combined dynamic and kinematic boundary conditions, and Eq.(8) is the bottom boundary condition expanded about z=-h to the order of $O(k\delta)$, where t is the time, g is the acceleration of the gravity, and $w^{(\mathrm{I})}$ is the discharge velocity at the interface between the region (I) and (II). The pressure, $p^{(\mathrm{I})}$, is given by

$$p^{(\mathrm{I})} = -\rho(\phi_t + gz) \ ; \quad -h \le z \le 0 \tag{9}$$

where ρ is the density of the fluid.

In the region (II), after Sollit and Cross (1972) and Madsen (1974), the unsteady motion of the fluid in the porous medium is described by a continuity equation

$$\nabla \cdot \boldsymbol{u} = 0 \tag{10}$$

and by a momentum equation

$$\frac{\tau}{n}\frac{\partial \boldsymbol{u}}{\partial t} = -\frac{1}{\rho}\nabla\left(p^{(\mathrm{II})} + \rho g z\right) - f\frac{\omega}{n}\boldsymbol{u} \tag{11}$$

where $\boldsymbol{u}$ is the discharge velocity vector, ∇ is the gradient operator vector as $(\partial/\partial x,\ \partial/\partial y,\ \partial/\partial z)$, n is the porosity, τ is the inertia coefficient, f is the linearized friction factor, $p^{(\mathrm{II})}$ is the pressure, ω is the angular frequency. Assuming the irrotational motion of the fluid and introducing a discharge velocity potential, φ, we can rewrite Eqs.(10) and (11) as

$$\nabla_h^2 \varphi + \varphi_{zz} = 0 \quad ; \quad -(h + h_s) \le z \le -h \tag{12}$$

$$p^{(\mathrm{II})} = -\rho\left(\frac{\tau}{n}\varphi_t + gz + f\frac{\omega}{n}\varphi\right) \quad ; \quad -(h + h_s) \le z \le -h \tag{13}$$

The boundary condition at the upper face of the porous layer is

$$\varphi_z = -\nabla_h h \cdot \nabla_h \varphi + \nabla_h \cdot (\delta \nabla_h \varphi) + w^{(\mathrm{II})} \quad ; \quad z = -h \tag{14}$$

and the boundary condition at the bottom of the porous layer is

$$\varphi_z = -\nabla_h (h + h_s) \cdot \nabla_h \varphi \quad ; \quad z = -(h + h_s) \tag{15}$$

where $w^{(\mathrm{II})}$ is the vertical discharge velocity at the interface between the region (I) and (II).

At the interface, the pressure and the vertical discharge velocity should be continuous:

$$p^{(\mathrm{I})} = p^{(\mathrm{II})} \quad ; \quad z = -h \tag{16}$$

$$w^{(\mathrm{I})} = w^{(\mathrm{II})} \quad ; \quad z = -h \tag{17}$$

The solutions of velocity potentials, ϕ and φ, may be expressed as

$$\phi(\boldsymbol{x}, z, t) = f^{(\mathrm{I})}(\boldsymbol{x}, z)\ \tilde{\phi}(\boldsymbol{x}, t) + (\text{non} - \text{propagating modes}) \tag{18}$$

$$\varphi(\boldsymbol{x}, z, t) = f^{(\mathrm{II})}(\boldsymbol{x}, z)\ \tilde{\varphi}(\boldsymbol{x}, t) + (\text{non} - \text{propagating modes}) \tag{19}$$

Under the condition of horizontal bottom ($\nabla_h h = \nabla_h h_s = 0$), we can obtain the vertical distribution functions of $f^{(\mathrm{I})}$ and $f^{(\mathrm{II})}$ as follows:

$$f^{(\mathrm{I})} = \frac{1}{D}\{\cosh kh_s \cosh k(h+z) + \gamma \sinh kh_s \sinh k(h+z)\} \tag{20}$$

$$f^{(\mathrm{II})} = \frac{1}{D}\gamma \cosh k(h + h_s + z) \tag{21}$$

where

$$D = \cosh kh_s \cosh kh \left(1 + \gamma \tanh kh_s \tanh kh\right) \tag{22}$$

$$\gamma = n/(\tau + if) \tag{23}$$

The dispersion relation is given by

$$\omega^2 = gk \frac{\tanh kh + \gamma \tanh kh_s}{1 + \gamma \tanh kh \tanh kh_s} \tag{24}$$

For the case of mild slope bottom, the velocity potentials could be described by Eqs.(18) and (19) with Eqs.(20) and (21). Substituting Eqs.(18) and (19) into the matching condition of pressure yields

$$\tilde{\phi} = \tilde{\varphi} \tag{25}$$

Following Smith and Sprinks (1975) and Kirby (1986), we employ Green's second identity to the propagating component of ϕ and $f^{(\mathrm{I})}$:

$$\int_{-h}^{0} f^{(\mathrm{I})} \phi_{zz}\, dz - \int_{-h}^{0} \phi f_{zz}^{(\mathrm{I})}\, dz = \left[f^{(\mathrm{I})} \phi_z - \phi f_z^{(\mathrm{I})} \right]_{-h}^{0} \tag{26}$$

Integrating the above equation yields

$$\begin{aligned} &-\nabla_h \cdot \int_{-h}^{0} \nabla_h \tilde{\phi} f^{(\mathrm{I})2}\, dz - \int_{-h}^{0} k^2 \tilde{\phi} f^{(\mathrm{I})2}\, dz \\ &= -\frac{1}{g}\left(\tilde{\phi}_{tt} f^{(\mathrm{I})2} \right)\Big|_0 - \frac{\omega^2}{g}\left(\tilde{\phi} f^{(\mathrm{I})2} \right)\Big|_0 - \nabla_h \cdot \left(\delta \nabla_h \tilde{\phi} \right) f^{(\mathrm{I})2} \Big|_{-h} \\ &\quad - w^{(\mathrm{I})} f^{(\mathrm{I})} \Big|_{-h} + \tilde{\phi} f^{(\mathrm{I})} f_z^{(\mathrm{I})} \Big|_{-h} + \text{ high order terms} \end{aligned} \tag{27}$$

Green's second identity to the propagating component of φ and $f^{(\mathrm{II})}$ is described by

$$\int_{-(h+h_s)}^{-h} f^{(\mathrm{II})}\varphi_{zz}\,dz-\int_{-(h+h_s)}^{-h}\varphi f_{zz}^{(\mathrm{II})}dz=\left[f^{(\mathrm{II})}\varphi_z-\varphi f_z^{(\mathrm{II})}\right]_{-(h+h_s)}^{-h} \tag{28}$$

and the integration yields

$$-\nabla_h\cdot\int_{-(h+h_s)}^{-h}\nabla_h\tilde{\phi} f^{(\mathrm{II})^2}dz-\int_{-(h+h_s)}^{-h}k^2\tilde{\phi} f^{(\mathrm{II})^2}dz$$

$$=\nabla_h\cdot\left(\delta\nabla_h\tilde{\phi}\right)f^{(\mathrm{II})^2}\Big|_{-h}+w^{(\mathrm{II})}f^{(\mathrm{II})}\Big|_{-h}$$

$$-\tilde{\phi}f^{(\mathrm{II})}f_z^{(\mathrm{II})}\Big|_{-h}+\text{ high order terms} \tag{29}$$

Eliminating $w^{(\mathrm{I})}$ and $w^{(\mathrm{II})}$ (actually $w^{(\mathrm{I})}=w^{(\mathrm{II})}$) from Eqs.(27) and (29) and using $\tilde{\phi}=\tilde{\varphi}$, we obtain

$$\frac{1}{g}\left(\tilde{\phi}_{tt}+\omega^2\tilde{\phi}\right)-\nabla_h\cdot\left(\alpha\nabla_h\tilde{\phi}\right)-k^2\alpha\tilde{\phi}+\frac{\cosh^2 kh_s}{D^2}(1-\gamma)\nabla_h\left(\delta\nabla_h\tilde{\phi}\right)=0 \tag{30}$$

where

$$\alpha=p+q/\gamma \tag{31}$$

$$p=\int_{-h}^{0}f^{(\mathrm{I})^2}dz=\frac{1}{4kD^2}\{\cosh^2 kh_s\sinh 2kh(1+2kh/\sinh 2kh)$$

$$+\gamma\sinh 2kh_s(\cosh 2kh-1)+\gamma^2\sinh^2 kh_s\sinh 2kh$$

$$\times(1-2kh/\sinh 2kh)\} \tag{32}$$

$$q=\int_{-(h+h_s)}^{-h}f^{(\mathrm{II})^2}dz=\frac{1}{4kD^2}\left\{\gamma^2\sinh 2kh_s\sinh 2kh\left(1+2kh_s/\sinh 2kh_s\right)\right\} \tag{33}$$

Eq.(30) is the time-dependent wave equation. Factoring the time out of $\tilde{\phi}$ as

$$\tilde{\phi}=\hat{\phi}\,e^{-i\omega t} \tag{34}$$

we can transform Eq.(30) into

$$\nabla_h\cdot\left(\alpha\nabla_h\hat{\phi}\right)+\alpha k^2\hat{\phi}-\frac{\cosh^2 kh_s}{D^2}(1-\gamma)\nabla_h\cdot\left(\delta\nabla_h\hat{\phi}\right)=0 \tag{35}$$

The effects of the porous medium are taken into account through the complex wavenumber k given by Eq.(24) and the complex coefficients of α and γ.

Relation to Existing Theories

Case of $h_S = 0$ and $\delta = 0$

In the case that there are not porous layer and rapid undulation, Eq.(35) reduces to the mild slope equations derived by Berkhoff (1972):

$$\nabla_h \cdot \left(CC_g \nabla_h \hat{\phi}\right) + k^2 CC_g \hat{\phi} = 0 \tag{36}$$

Case of $h_S = 0$ and $\delta \neq 0$

In the case that there is not porous layer and there exists rapid undulation, Eq.(35) reduces to the general wave equation over rippled bed derived by Kirby (1986):

$$\nabla_h \cdot \left(CC_g \nabla_h \hat{\phi}\right) + k^2 CC_g \hat{\phi} - \frac{g}{\cosh^2 kh} \nabla_h \cdot \left(\delta \nabla_h \hat{\phi}\right) = 0 \tag{37}$$

Case of very small permeability of porous layer

The case of very small permeability is treated as a mathematical limit of very small porosity ($n \to 0$) and very large friction factor ($f \to \infty$). In this condition, Eq.(35) reduces to the general wave equation expressed as Eq.(37).

Case of very large permeability of porous layer

Since $n \to 1$ and $f \to 0$, so $\gamma \to 1$. The resultant wave equation becomes the same as the mild slope equation expressed as Eq.(36) where the phase velocity C and the group velocity C_g are defined by using the water depth of $h+h_S$.

Numerical Calculations of Bragg Scattering

Boundary condition

At the seaward boundary condition, the following condition has to be satisfied:

$$\hat{\phi}_x = -ik(\hat{\phi} - 2\hat{\phi}_I) \tag{38}$$

where $\hat{\phi}_I$ is the incident wave potential amplitude. At the downstream boundary condition, the following transmitted condition was needed:

$$\hat{\phi}_x = ik\hat{\phi} \tag{39}$$

Numerical conditions

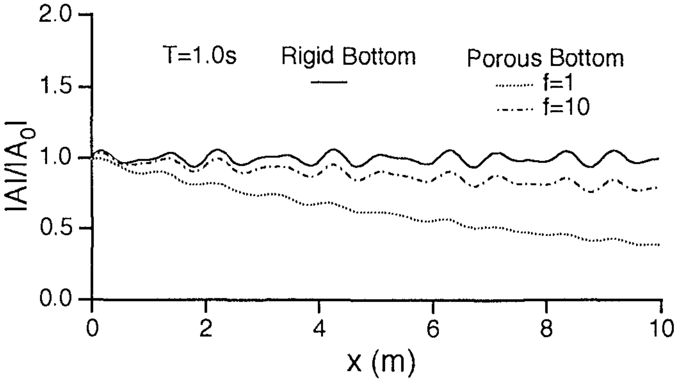
Numerical conditions followed the experimental ones of Davies and Heathershaw (1984). The water depth h and the thickness of porous layer h_S were constant, and the undulation δ was given by

$$\delta = D\sin(\lambda x) \ ; \quad 0 \le x \le ml \tag{40}$$

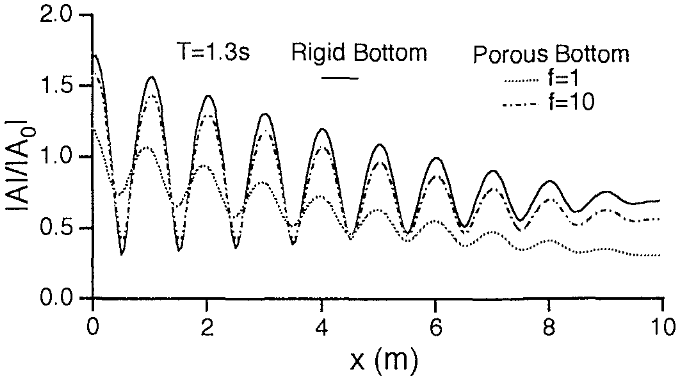
where m, λ, l, and D are the number, the wavenumber, the wavelength and the amplitude of the ripples, respectively. Two cases of Case 1 ($m = 10$ and $D/h = 0.16$) and Case 2 ($m = 4$ and $D/h = 0.32$) were employed. Actual values of D, l and h_S in the experiments of Davies and Heathershaw (1984) were 5 cm, 1.0 m and 0 m, respectively. Here we changed h_S from 0 m to 0.2 m to examine the effects of permeability on the Bragg scattering.

Calculated results and discussion

Figure 2 shows the spatial distributions of wave amplitudes of wave period 1.0 s and 1.3 s, where the solid line corresponds to the case of impermeable rigid bottom, the dotted and the dash-dotted lines correspond to the case of porous bottom of $f = 1$ and $f = 10$, respectively, with $\tau = 1.0$, $n = 0.4$ and $h_S = 0.2$ m of Case 1. In Fig.2 (a), the reflection coefficient is less than 0.1; and, in Fig.2(b), the Bragg reflection condition is nearly satisfied, and the variations of the amplitudes are remarkable. It is seen from Fig.2 that when the bottom is permeable, the amplitudes and their variations become small.



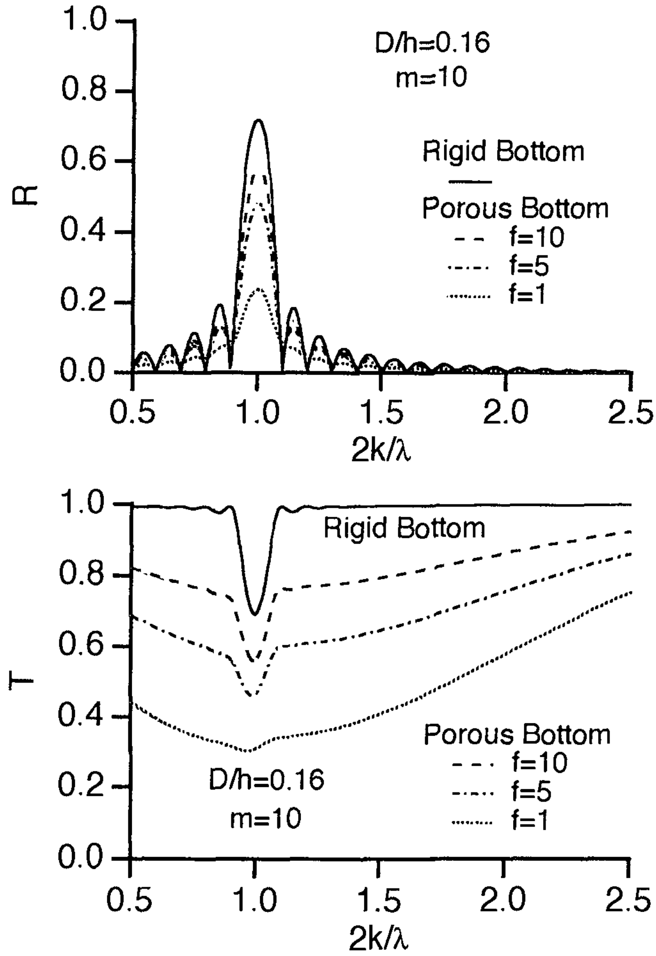
(a)



(b)

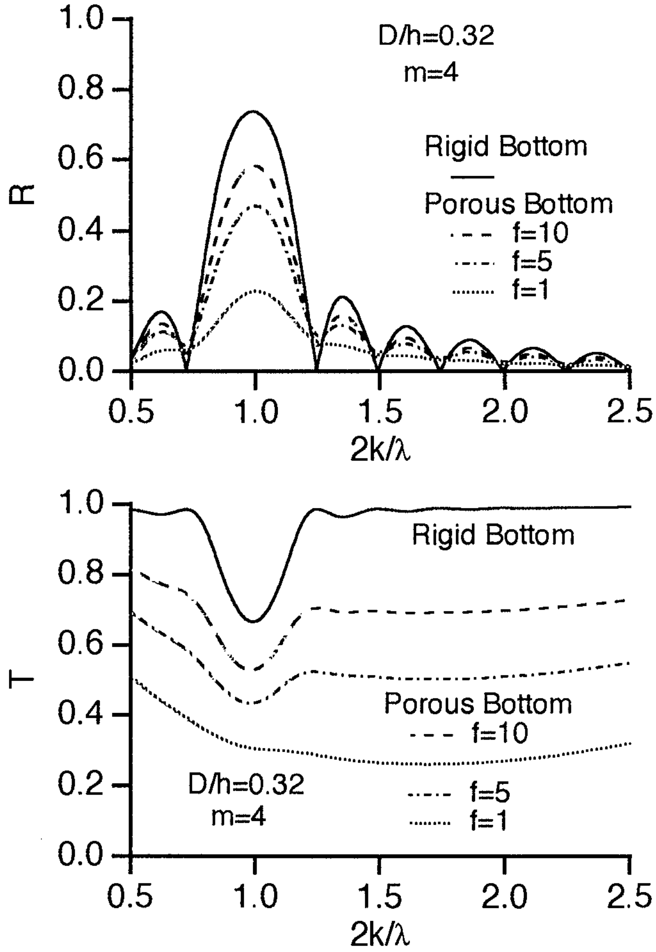
Fig.2 Spatial distributions of wave amplitudes: (a) non-resonant condition; (b) resonant condition

Figure 3 shows the reflection coefficient, R, and the transmission coefficient, T, against the ratio of wavenumbers, $2k/\lambda$, where the k is taken as the real part for the case of permeable ripples. In the figures, the linearized friction factor was changed by 10, 5, and 1, keeping $\tau = 1.0$, $n = 0.4$ and $h_S = 0.2$ m. The calculated results for impermeable rigid bottom, shown by the solid lines, were obtained by setting $n = 0$. In the range of $1 \leq f \leq 10$, the reflection and transmission



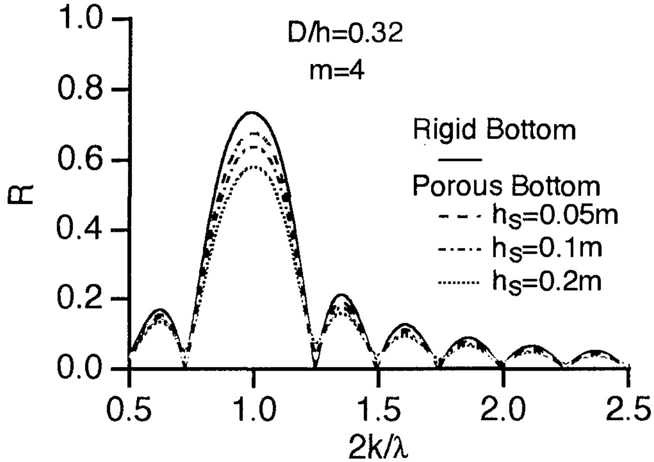
(a)

Fig.3 Reflection and transmission coefficients: (a) Case 1; (b) Case 2



(b)

Fig.3 (Continued)



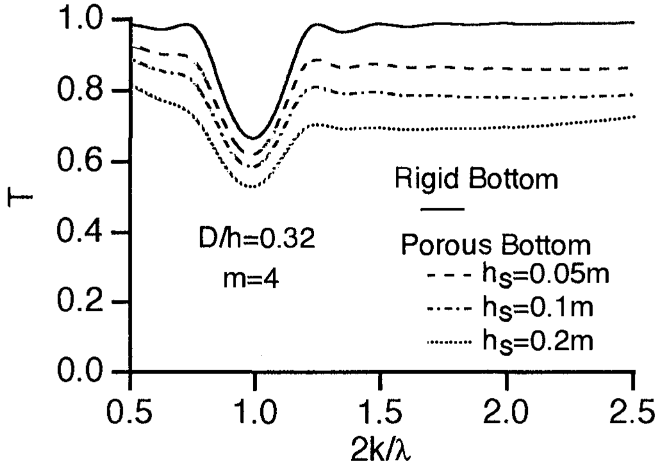


Fig.4 Effect of thickness of porous layer on reflection and transmission

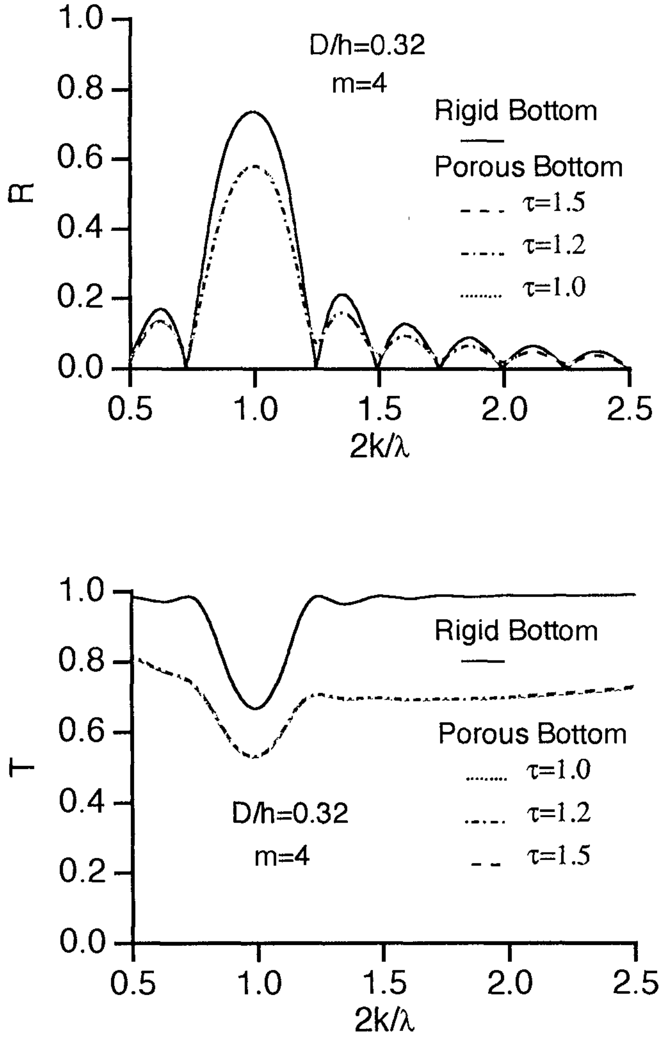


Fig.5 Effect of inertia coefficient of porous layer on reflection and transmission

coefficients become small with decrease in the linearized friction factor.

The effect of thickness of the porous layer on the Bragg scattering is shown in Fig.4, by changing h_S, with $\tau = 1.0$, $n = 0.4$ and $f = 10$. The reflection and transmission coefficients become small with increase in the porous layer thickness, as easily expected.

Figure 5 shows the effect of the inertia coefficient by changing τ, with $f = 10$, $n = 0.4$ and $h_S = 0.2$ m. It is seen from this figure that there is little effect of τ.

It should be noted that though the parameters of n, f, and τ were changed independently for the convenience in the calculations so far, these parameters are dependent each other. The determination of the parameters is difficult for arbitrary porous medium.

Conclusions

In order to deal with wave transformations over a permeable seabed with rapidly varying undulations, we developed the time-dependent and time-independent wave equations taking account of the effects of porous medium.

In the case that there are not porous layer and rapid undulation, the time-independent equation reduces to the mild slope equation derived by Berkhoff (1972). In the case that there is not porous layer and there exists rapid undulation, the time-independent equation reduces to the wave equation over rippled bed derived by Kirby (1986). When the permeability is very small, the time-independent equation reduces to the Kirby's equation. On the other hand, when the permeability is very large, the time-independent equation reduces to the Berkhoff's equation where the water depth is defined by a sum of the water depth and the thickness of porous layer.

Numerical examples of the Bragg scattering were shown in one-dimensional case. The reflected and transmitted waves became small due to the permeability of seabed bottom. The reflection and transmission coefficients were influenced by the friction factor, the thickness of porous layer, and the porosity; however, there was little effect of inertia coefficient.

Acknowledgments

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