CHAPTER 53

On a Method for Estimating Reflection Coefficient in Short-Crested Random Seas

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Abstract

A method for estimating the directional spectrum as well as the reflection coefficient in a incident and reflected wave field is developed for practical uses. In the method, the directional spectrum is assumed to be expressed by a circular normal distribution which includes several parameters, Then the parameters are estimated by the maximum likelihood method. The validity of the method is verified by numerical simulation.

1 Introduction

The randomness of sea waves has recently become accounted for in the design of coastal and ocean structures. Directional spectra have often been used to describe multi-directional random sea waves. However, to evaluate the reflection coefficient of structures, a theory for uni-directional random waves has usually been applied with a slight modification. This is because a theory for multi-directional random waves has not yet been established for practical uses.

The purpose of this paper is to derive a method to estimate the directional spectrum as well as the reflection coefficient of structures for multi-directional random waves and to examine its validity by applying it to simulated data.

In various methods to estimate the directional spectrum, the maximum likelihood method (MLM) has a high resolution (Capon, 1969). Isobe and Kondo

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(1984) proposed the modified maximum likelihood method (MMLM) to estimate the directional spectrum in a combined incident and reflected wave field, taking into account the fact that there is no phase difference between the incident and reflected waves at the reflective wall.

Recently for practical purposes, Isobe (1990) proposed a method to estimate the directional spectrum of a standard form in which the spectrum is expressed in terms of a few parameters. Yokoki *et al.* (1992) extended this method in order to estimate the directional spectrum in an incident and reflected wave field.

In this study, we modify the method to improve the numerical efficiency by assuming a circular normal distribution for the directional function.

2 Theory

2.1 Formulation of cross power spectrum

2.1.1 Definition of cross power spectrum

In a monochromatic wave field which consists of incident and reflected waves, the water surface fluctuation, $\eta(\boldsymbol{x}, t)$, at the position, $\boldsymbol{x}$, are represented by Eq. (1):

$$\eta(\boldsymbol{x}, t) = A(\boldsymbol{k}, \sigma)\left\{\cos(\boldsymbol{k}\boldsymbol{x} - \sigma t + \epsilon) + r\cos(\boldsymbol{k}_r\boldsymbol{x} - \sigma t + \epsilon)\right\} \tag{1}$$

The definitions of the variables in the above equation are given in **Table** 1. We integrate Eq. (1) with respect to $\boldsymbol{k}$ and σ, and get the expression of $\eta(\boldsymbol{x}, t)$ for multi-directional random waves as Eq. (2):

$$\eta(\boldsymbol{x}, t) = \int_{-\infty}^{\infty}\int_{\boldsymbol{k}} A(d\boldsymbol{k}, d\sigma) \times \left\{\exp\left[i(\boldsymbol{k}\boldsymbol{x} - \sigma t + \epsilon)\right] + r\exp\left[i(\boldsymbol{k}_r\boldsymbol{x} - \sigma t + \epsilon)\right]\right\} \tag{2}$$

where complex variables are introduced to represent the amplitude and phase. From Eq. (2), the directional spectrum (wavenumber-frequency spectrum), $S(\boldsymbol{k}, \sigma)$, is defined as Eq. (3):

$$S(\boldsymbol{k}, \sigma)d\boldsymbol{k}d\sigma = < A^*(d\boldsymbol{k}, d\sigma)A(d\boldsymbol{k}, d\sigma) > \tag{3}$$

Table 1: Definitions of variables in Eq. 1

$A(\boldsymbol{k}, \sigma)$	Amplitude (Eq. (1)),
$A(d\boldsymbol{k}, d\sigma)$	Complex amplitude (Eq. (2))
$\boldsymbol{k}$	Wave number vector of incident waves
$\boldsymbol{k}_r$	Wave number vector of reflected waves
σ	Angular frequency
ϵ	Phase

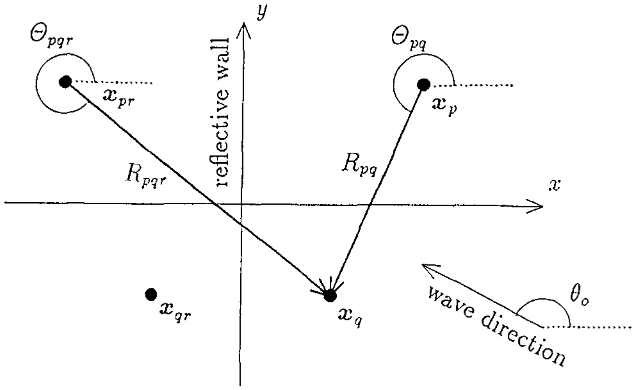

Figure 1: Definitions of variables

where < > represents the ensemble average and A^* the complex conjugate of A. Then, the cross power spectrum, $\Phi_{pq}(\sigma)$, can be defined by Eq. (4) (Horikawa, 1988, Isobe and Kondo, 1984):

$$\Phi_{pq}(\sigma) = \int_{\boldsymbol{k}} S(\boldsymbol{k}, \sigma) \times \{\exp(i\boldsymbol{k}\boldsymbol{x}_p) + r\exp(i\boldsymbol{k}\boldsymbol{x}_{pr})\}\{\exp(-i\boldsymbol{k}\boldsymbol{x}_q) + r\exp(-i\boldsymbol{k}\boldsymbol{x}_{qr})\}\, d\boldsymbol{k} \quad (4)$$

in which the variables are defined in **Fig.** 1.

We rewrite Eq. (4) by using the transformation of variables as follows. The definitions of variables are also indicated in **Fig.** 1.

$$\left.\begin{array}{lll} \boldsymbol{k} & = (k\cos\theta, & k\sin\theta \quad) \\ \boldsymbol{x}_p - \boldsymbol{x}_q & = (R_{pq}\cos\Theta_{pq}, & R_{pq}\sin\Theta_{pq} \quad) \\ \boldsymbol{x}_{pr} - \boldsymbol{x}_{qr} & = (R_{pq}\cos(\pi - \Theta_{pq}), & R_{pq}\sin(\pi - \Theta_{pq}) \;) \\ \boldsymbol{x}_{pr} - \boldsymbol{x}_q & = (R_{pqr}\cos\Theta_{pqr}, & R_{pqr}\sin\Theta_{pqr} \quad) \\ \boldsymbol{x}_p - \boldsymbol{x}_{qr} & = (R_{pqr}\cos(\pi - \Theta_{pqr}), & R_{pqr}\sin(\pi - \Theta_{pqr})) \end{array}\right\} \quad (5)$$

Then we get the expression of the cross power spectrum, $\Phi_{pq}(f)$:

$$\Phi_{pq}(f) = \int_0^{2\pi} S(f,\theta) \times [\exp\{ikR_{pq}\cos(\theta - \Theta_{pq})\} + r^2\exp\{ikR_{pq}\cos(\theta - \pi + \Theta_{pq})\} + r\exp\{ikR_{pqr}\cos(\theta - \Theta_{pqr})\} + r\exp\{ikR_{pqr}\cos(\theta - \pi + \Theta_{pqr})\}]\, d\theta \quad (6)$$

where f represents the frequency connected with the wave number vector, $\boldsymbol{k}$, by the dispersion relation, and r the reflection coefficient of the reflective wall (shown in **Fig.** 1).

2.1.2 Directional spectrum function

Three standard functions have been proposed to express the directional spectrum except the Fourier series.

The first one is the function proposed by Mitsuyasu *et al.* (1975) shown as:

$$G(\theta|f) \equiv \frac{S(f,\theta)}{P(f)} \propto \cos^{2s}\left(\frac{\theta-\theta_o}{2}\right) \tag{7}$$

where $P(f)$ indicates the frequency spectrum, s the degree of directional concentration, and θ_o the peak wave direction. Yokoki *et al.* (1992) used this standard directional function. However, it took fairly much time to calculate the cross power spectrum, Φ_{pq}.

The second one is the square of the hyperbolic secant function, which is proposed by Donelan *et al.* (1985):

$$G(\theta|f) \propto \mathrm{sech}^2\beta(\theta-\theta_o) \tag{8}$$

where β indicates the degree of directional concentration.

The last one is the circular normal distribution function proposed by Borgman (1969):

$$G(\theta|f) \propto \exp\{a\cos(\theta-\theta_o)\} \tag{9}$$

where a indicates the degree of directional concentration.

In the present study, we assume the directional spectrum, $S(f,\theta)$, to be expressed by using the circular normal distribution function, because we can obtain the cross power spectrum analytically to some extent and the numerical calculation becomes faster. Therefore, the directional spectrum, $S(f,\theta)$, is expressed by Eq. (10):

$$S(f,\theta) = P(f)\frac{1}{2\pi I_o(a)}\exp\{a\cos(\theta-\theta_o)\} \tag{10}$$

where I_o is the modified Bessel function.

2.1.3 Formulation of cross power spectrum

To rewrite Eq. (6) in a simpler form, we define a new function φ as follows:

$$\begin{aligned}&\varphi(a,\theta_o,R,\Theta|f)\\ &= \frac{1}{2\pi I_o(a)}\int_0^{2\pi}\exp\{-ikR\cos(\theta-\Theta)\}\exp\{a\cos(\theta-\theta_o)\}d\theta\end{aligned} \tag{11}$$

By using the above definition, Eq. (6) can be written as Eq. (12):

$$\Phi_{pq}(f) = \left\{\begin{array}{l} \varphi(\;a,\theta_o,R_{pq},\;\Theta_{pq}\;|f) \\ +r^2\varphi(\;a,\theta_o,R_{pq},\;\pi-\Theta_{pq}\;|f) \\ +r\varphi(\;a,\theta_o,R_{pqr},\Theta_{pqr}\;|f) \\ +r\varphi(\;a,\theta_o,R_{pqr},\pi-\Theta_{pqr}|f) \end{array}\right\} \times (1+\delta_{pq}\varepsilon_p)P(f) \tag{12}$$

Table 2: Directional spectrum parameters

a	Degree of directional concentration
θ_0	Peak wave direction
r	Reflection coefficient
$P(f)$	Frequency or power spectrum
ε_p	Ratio of the noise component to the power (at the p-th point; $p = 1 \sim 3$)

where ε_p indicates the ratio of the noise component to the power, $\Phi_{pp}(f)$. We finally get the expression of the cross power spectrum in terms of the seven parameters which are summarized in **Table** 2 and called the directional spectrum parameters in this paper.

The integral on the right-hand side of Eq. (11) can be expressed by using the integral expression of the Bessel function and the modified Bessel function (*e.g.* Abramowitz and Stegun, 1972):

$$\begin{aligned} \varphi(a, \theta_o, R, \Theta | f) &= J_o(kR) \\ &+ 2\sum_{m=1}^{\infty}(-1)^m \cos(2m\beta) J_{2m}(kR)\frac{I_{2m}(a)}{I_0(a)} \\ &+ 2i\sum_{m=0}^{\infty}(-1)^m \cos[(2m+1)\beta] J_{2m+1}(kR)\frac{I_{2m+1}(a)}{I_0(a)} \end{aligned} \tag{13}$$

In the present study, we use Eqs. (12) and (13) to calculate the cross power spectrum.

2.2 Maximum likelihood method

2.2.1 Definition of likelihood

The maximum likelihood method is used to get the most probable values of the directional spectrum parameters. The likelihood, L, is defined by Isobe (1990):

$$\begin{aligned} L(A^{[j]}; \Phi) &= \left\{ p(A^{[1]}) \times p(A^{[2]}) \times \cdots \times p(A^{[j]}) \times \cdots \times p(A^{[J]}) \right\}^{1/J} \\ &= \frac{1}{(2\pi \Delta f)^M |\Phi|} \exp\left(-\sum_{p=1}^{M}\sum_{q=1}^{M} \Phi_{pq}^{-1} \widehat{\Phi}_{qp} \right) \end{aligned} \tag{14}$$

where $p(A^{[j]})$ is a joint probability density function of the Fourier coefficients, $A^{[j]}$, of the time series data, Δf the frequency interval, and $|\Phi|$ the determinant of the matrix, Φ_{pq}. The quantity $\widehat{\Phi}_{qp}$ which is represented by Eq. (15) corresponds to the power spectrum ($q = p$) or the cross spectrum ($q \neq p$) with a use of rectanglar filter.

$$\widehat{\Phi}_{qp} = \frac{1}{2J\Delta f}\sum_{j=1}^{J}\overline{A}_q^{[j]}A_p^{[j]} \tag{15}$$

where $\overline{A}$ denotes the complex conjugate of A.

2.2.2 The most probable values of the parameters

In this Section, we show the procedure to estimate the directional spectrum parameters including the reflection coefficient by using the likelihood defined above.

The maximum likelihood method implies that the most probable values of λ_i are the solutions of the algebraic equation:

$$\frac{\partial L}{\partial \lambda_i} = \sum_{j=1}^{M}\sum_{l=1}^{M}\frac{\partial L}{\partial \Phi_{jl}}\frac{\partial \Phi_{jl}}{\partial \lambda_i} = 0 \tag{16}$$

From Eq. (14), Eq. (17) is obtained.

$$\begin{aligned}
\frac{\partial L}{\partial \Phi_{jl}} &= \frac{\partial}{\partial \Phi_{jl}}\left[\frac{1}{(2\pi\Delta f)^M|\Phi|}\exp\left(-\sum_{p=1}^{M}\sum_{q=1}^{M}\Phi_{pq}^{-1}\widehat{\Phi}_{qp}\right)\right] \\
&= -\frac{1}{(2\pi\Delta f)^M|\Phi|^2}\exp\left(-\sum_{p=1}^{M}\sum_{q=1}^{M}\Phi_{pq}^{-1}\widehat{\Phi}_{qp}\right)\frac{\partial|\Phi|}{\partial \Phi_{jl}} \\
&\quad + \frac{1}{(2\pi\Delta f)^M|\Phi|}\exp\left(-\sum_{p=1}^{M}\sum_{q=1}^{M}\Phi_{pq}^{-1}\widehat{\Phi}_{qp}\right) \\
&\quad \times\left(-\sum_{p=1}^{M}\sum_{q=1}^{M}\frac{\partial \Phi_{pq}^{-1}}{\partial \Phi_{jl}}\widehat{\Phi}_{qp}\right) \\
&= -L\times\left(\frac{1}{|\Phi|}\frac{\partial|\Phi|}{\partial \Phi_{jl}}\right) + L\times\left(-\sum_{p=1}^{M}\sum_{q=1}^{M}\frac{\partial \Phi_{pq}^{-1}}{\partial \Phi_{jl}}\widehat{\Phi}_{qp}\right)
\end{aligned} \tag{17}$$

Also, the following relations are obtained from the theorem of the matrices.

$$\frac{\partial|\Phi|}{\partial \Phi_{jl}} = |\Phi|\Phi_{lj}^{-1} \tag{18}$$

$$\frac{\partial \Phi_{pq}^{-1}}{\partial \Phi_{jl}} = -\Phi_{lq}^{-1}\Phi_{pj}^{-1} \tag{19}$$

By using Eqs. (18) and (19), Eq. (17) is rewritten as follows:

$$\frac{\partial L}{\partial \Phi_{jl}} = L\times\left\{-\Phi_{lj}^{-1} + \sum_{p=1}^{M}\sum_{q=1}^{M}\Phi_{lq}^{-1}\widehat{\Phi}_{qp}\Phi_{pj}^{-1}\right\} \tag{20}$$

Substituting Eq. (20) into Eq. (16) and considering that $L \neq 0$, we obtain

$$\sum_{j=1}^{M}\sum_{l=1}^{M}\left\{-\Phi_{lj}^{-1}+\sum_{p=1}^{M}\sum_{q=1}^{M}\Phi_{lq}^{-1}\widehat{\Phi}_{qp}\Phi_{pj}^{-1}\right\}\frac{\partial\Phi_{jl}}{\partial\lambda_i}=0 \tag{21}$$

The directional spectrum parameters, λ_i, which satisfy Eq. (21) for all i ($i = 1 \sim 7$) are the most probable values. Then the directional spectrum parameters, including the reflection coefficient, are estimated.

The solutions, λ_i, of Eq. (21) are obtained numerically by using the Newton-Raphson method. The left-hand side of Eq. (21) is first defined as a function of the directional spectrum parameters:

$$f_i(\lambda_{i'})=\sum_{j=1}^{M}\sum_{l=1}^{M}\left\{-\Phi_{lj}^{-1}+\sum_{p=1}^{M}\sum_{q=1}^{M}\Phi_{lq}^{-1}\widehat{\Phi}_{qp}\Phi_{pj}^{-1}\right\}\frac{\partial\Phi_{jl}}{\partial\lambda_i} \tag{22}$$

In the Newton-Raphson method, the values of $\lambda_i^{(j+1)}$ at the $(j+1)$-th iteration of the calculation is expressed in terms of the previous values, $\lambda_i^{(j)}$, as follows.

$$\lambda_i^{(j+1)}=\lambda_i^{(j)}-\left[\sum_{i'=1}^{I}\left[\frac{\partial f_i}{\partial\lambda_{i'}}\right]^{-1}f_{i'}\right]_{\lambda_i=\lambda_i^{(j)}} \tag{23}$$

where $\partial f_i/\partial\lambda_{i'}$ is expressed by Eq. (24):

$$\begin{aligned}\frac{\partial f_i}{\partial\lambda_{i'}}=&\sum_{j=1}^{M}\sum_{l=1}^{M}\left\{-\Phi_{lj}^{-1}+\sum_{p=1}^{M}\sum_{q=1}^{M}\Phi_{lq}^{-1}\widehat{\Phi}_{qp}\Phi_{pj}^{-1}\right\}\frac{\partial^2\Phi_{jl}}{\partial\lambda_{i'}\partial\lambda_i}\\&-\sum_{j'=1}^{M}\sum_{l'=1}^{M}\sum_{j=1}^{M}\sum_{l=1}^{M}\frac{\partial\Phi_{j'l'}}{\partial\lambda_{i'}}\frac{\partial\Phi_{jl}}{\partial\lambda_i}\\&\times\left[-\Phi_{l'j}^{-1}\Phi_{lj'}^{-1}+\left\{\begin{array}{c}\Phi_{lj'}^{-1}\sum_{p=1}^{M}\sum_{q=1}^{M}\Phi_{l'q}^{-1}\widehat{\Phi}_{qp}\Phi_{pj}^{-1}\\+\Phi_{l'j}^{-1}\sum_{p=1}^{M}\sum_{q=1}^{M}\Phi_{lq}^{-1}\widehat{\Phi}_{qp}\Phi_{pj'}^{-1}\end{array}\right\}\right]\end{aligned} \tag{24}$$

In calculations of the present method, it sometimes happened that we obtained the parameters, $\lambda_i^{(j+1)}$, which made the value of the likelihood, L, smaller than the preceding parameters, $\lambda_i^{(j)}$. This was because the present procedure sometimes finds the minimum likelihood in stead of the maximum likelihood. Therefore, we put the additional procedure that we replace $\Delta\lambda_i$ ($=\lambda_i^{(j+1)}-\lambda_i^{(j)}$) with $-\Delta\lambda_i$ if $(\partial L/\partial\lambda_i)\Delta\lambda_i<0$. With this procedure, we can obtain converged solutions of the paramters of Eq. (21) from various initial values of the parameters.

3 Numerical calculation

3.1 Simulated cross power spectrum

To verify the validity of the present method, we applied it to simulated data.

Table 3: Values of the directional parameters

Case No.	f(Hz)	s	θ_o(°)	$P(f)$ (m^2s)	r	ε_1	ε_2	ε_3
PA01	0.1	5.0	120	1.0	0.1	0.1	0.1	0.1
PA02	0.1	10.0	120	1.0	0.1	0.1	0.1	0.1
PA03	0.1	5.0	120	1.0	0.5	0.1	0.1	0.1
PA04	0.1	5.0	120	1.0	0.1	0.3	0.3	0.3
PA05	0.1	5.0	120	1.0	0.1	0.1	0.2	0.3
PA06	0.2	5.0	120	1.0	0.1	0.1	0.1	0.1

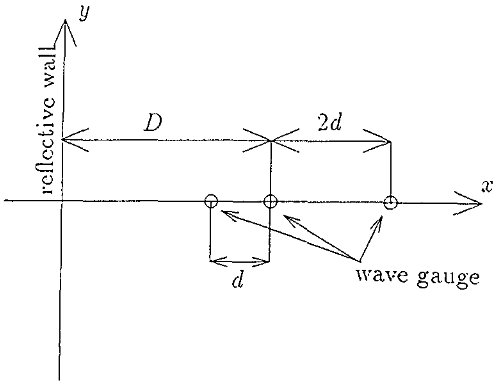

Figure 2: Arrangement of wave gauges

First, we created sets of $\widehat{\Phi}_{pq}$ by using of the directional spectrum proposed by Mitsuyasu *et al.* (1975) for each set of directional spectrum parameters given in **Table** 3.

The arrangement of wave gauges used to calculate $\widehat{\Phi}_{pq}$ is shown in **Fig.** 2. In the figure, we vary the distance between the reflective wall and the wave gauges, D, and that between the wave gauges, d, as shown in **Table** 4. We finally calculated $\widehat{\Phi}_{pq}$ for each combination of the arrangement of wave gauges and the set of the directional spectrum parameters. Then we applied the present method to each set of the cross power spectra, $\widehat{\Phi}_{pq}$.

3.2 Results of estimation

Figure 3 shows contour maps of likelihood in terms of the directional spectrum parameters, a, ε_p, r and θ_o. In these figures, the directional spectrum is calculated

Table 4: Arrangement of wave gauges

Case No.	D(m)	d(m)
WG01	20	5
WG03	40	5
WG05	40	15
WG07	60	5
WG09	60	15

from the set of the parameters, PA01, shown in **Table** 3. WG01, WG03 and WG05 indicates the arrangement of wave gauges as shown in **Table** 4.

From these figures, we can see that the likelihood becomes maximum at the points which correspond to the given values of the directional spectrum parameters.

Table 5 shows the results of estimations for all combinations of the directional spectrum parameters and the arrangements of wave gauges. For these directional spectrum parameters, estimated parameters agree quite well with the true values, as shown in **Table** 3.

However, for the directional spectrum parameters, PA06, the agreement is not good. This is because the distance between the wave gauges are too long compared to the wavelength.

4 Conclusion

By using a circular normal distribution as the directional function, a parametric method for estimating the directional spectrum as well as the reflection coefficient in an incident and reflected wave field is proposed. From the results of numerical simulations, the method is proved to be valid as long as the arrangement of wave gauges is appropriately designed.

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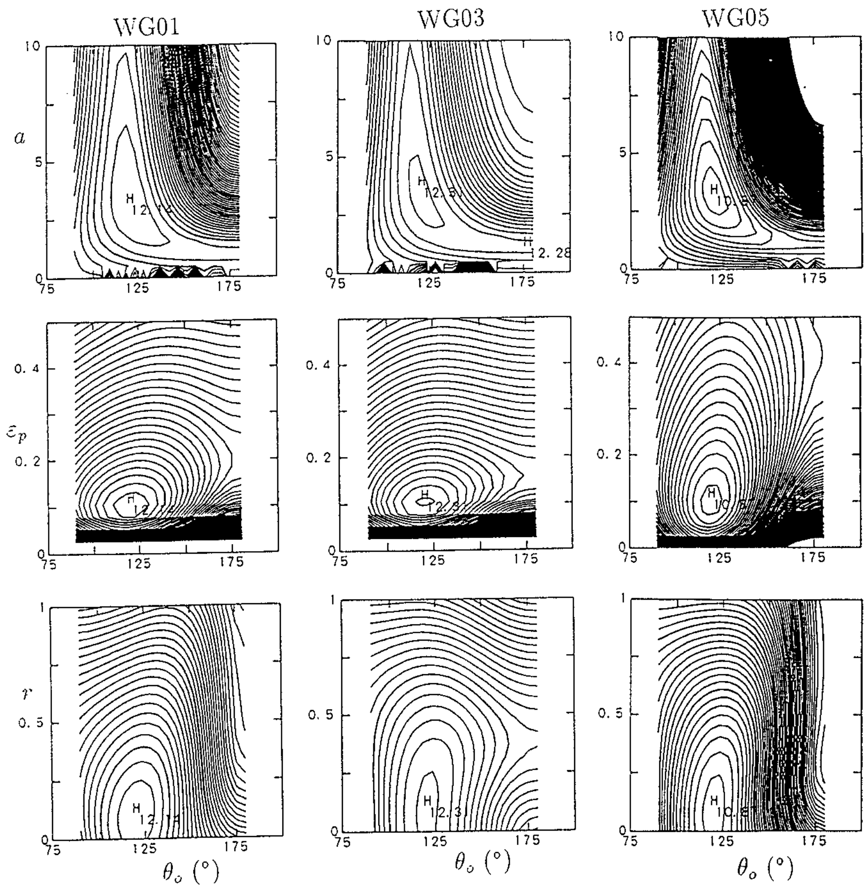

Figure 3: Contour map of the likelihood

Table 5: Results of estimation

Case No.	Case No.	f(Hz)	a	θ_o(°)	$P(f)$ (m^2s)	r	ε_1	ε_2	ε_3
PA01	WG01	0.1	3.22	120	1.00	0.098	0.10	0.099	0.10
	WG03	0.1	3.22	120	1.00	0.10	0.099	0.10	0.099
	WG05	0.1	3.01	121	1.01	0.11	0.096	0.088	0.096
	WG07	0.1	3.55	121	0.99	0.13	0.10	0.098	0.10
	WG09	0.1	3.08	120	1.01	0.081	0.095	0.091	0.10
PA02	WG01	0.1	5.65	120	1.00	0.098	0.10	0.10	0.099
	WG03	0.1	5.74	120	1.00	0.10	0.098	0.10	0.097
	WG05	0.1	5.58	120	1.00	0.092	0.10	0.096	0.096
	WG07	0.1	5.97	121	0.99	0.12	0.10	0.099	0.099
	WG09	0.1	5.62	120	1.00	0.094	0.099	0.098	0.098
PA03	WG01	0.1	3.04	120	1.01	0.48	0.10	0.099	0.10
	WG03	0.1	2.81	120	1.02	0.46	0.099	0.10	0.097
	WG05	0.1	2.82	119	1.03	0.45	0.11	0.097	0.093
	WG07	0.1	3.64	121	0.99	0.50	0.10	0.099	0.099
	WG09	0.1	2.75	118	1.04	0.44	0.095	0.098	0.10
PA04	WG01	0.1	3.13	121	1.00	0.095	0.30	0.30	0.30
	WG03	0.1	3.72	120	1.00	0.12	0.29	0.30	0.29
	WG05	0.1	3.07	120	1.01	0.10	0.29	0.28	0.29
	WG07	0.1	3.35	121	0.99	0.11	0.30	0.30	0.30
	WG09	0.1	3.05	120	1.01	0.085	0.30	0.29	0.30
PA05	WG01	0.1	3.07	120	1.01	0.096	0.099	0.20	0.30
	WG03	0.1	3.74	120	1.01	0.12	0.097	0.20	0.29
	WG05	0.1	3.08	120	1.01	0.11	0.096	0.19	0.29
	WG07	0.1	3.37	121	0.99	0.11	0.10	0.20	0.30
	WG09	0.1	3.07	120	1.01	0.081	0.096	0.19	0.30
PA06	WG01	0.2	3.17	122	0.98	0.19	0.13	0.063	0.11
	WG03	0.2	3.17	121	1.00	0.12	0.10	0.091	0.10
	WG05	0.2	2.89	120	1.02	0.10	0.071	0.068	0.077
	WG07	0.2	3.12	120	1.01	0.082	0.10	0.095	0.10
	WG09	0.2	2.81	120	1.04	0.12	0.056	0.048	0.060

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