CHAPTER 192

Calculation of Tombolo in Shoreline Numerical Model

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Abstract

An algorithm is developed that is capable of calculating the formation of a tombolo in the lee of an offshore breakwater in a shoreline numerical model by inversely applying the method proposed in 1985 by Hanson and Kraus for constraining the shoreline not to retreat landward of a seawall. The algorithm is linked to a shoreline numerical model which uses curvilinear coordinates that follow the shoreline. The project of six segmented breakwaters at Chippokes State Park, Virginia is reported. The developed model is applied to the simulation of the shoreline change during the first nine months after construction of the Chippokes breakwaters.

1 Introduction

Offshore breakwaters have been used as a means to protect beaches against severe erosion. The wave-induced longshore currents generated behind the breakwaters deposit sediments in the sheltered area to build a new morphological shape called a salient. Sometimes the salient grows until its apex reaches the breakwater to form a tombolo. A number of numerical models has been developed for predicting shoreline change in the vicinity of coastal structures. However, there are few numerical models that can simulate the formation of tombolos. Most of the numerical models stop computation when the computed shoreline reaches the breakwater or predict outgrowth of the salient

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seaward of the breakwater. Perlin (1979) has proposed a method for handling the formation of a tombolo. Unfortunately, however, he could not find a well-documented data set to test his model for the case of tombolo formation.

In the present study, a numerical shoreline evolution model is developed to permit the formation and growth (or decay) of tombolos in the lee of impermeable offshore breakwaters. Hanson and Kraus (1985) have proposed a method for constraining the shoreline for the case of a seawall in a shoreline numerical model, which corrects the longshore sediment transport rates so as not to allow the shoreline to retreat landward of a seawall. In the present model, the method of Hanson and Kraus is borrowed for the calculation of tombolo formation in the lee of offshore breakwaters, the inverse of the problem of a seawall. The project of six segmented breakwaters at Chippokes State Park, Virginia reported in Hardaway et al. (1988) is summarized, and the developed model is applied to the simulation of the shoreline response during the first nine months after construction of the breakwaters.

2 Numerical Model

Advanced knowledge of waves and currents and the resulting sediment transport, associated with the increased capacities of large high-speed computers and improved numerical modeling algorithms, has made it possible to apply such complex models as a multi-line model (Perlin and Dean 1985) or general 3-D topographical change models (Wang et al. 1975; Watanabe 1982) to numerical modeling of shoreline problems. However, due to the relatively accurate prediction of the shoreline change with less computational effort, shoreline models have been widely used. In this section, a shoreline numerical model is developed using the curvilinear coordinates that follow the shoreline, as done in LeBlond (1972), Uda (1983), and Kobayashi and Dalrymple (1986). The use of a curvilinear coordinate system may be more advantageous than the Cartesian coordinate system for the situation in which the shoreline orientation is deviated largely from the straight shoreline as on a tombolo behind an offshore breakwater.

2.1 Shoreline change model

The curvilinear coordinate system used in the present model is shown in Fig. 1 along with some other notations. The symbol s denotes the coordinate following the shoreline whose positive direction points to the right when facing seaward. The coordinate pair (x_s, y_s) gives the location of an arbitrary point on the curved shoreline in terms of a Cartesian coordinate system.

$$\vec{m} = \left(\frac{\partial x_s}{\partial s}, \frac{\partial y_s}{\partial s}\right) = (\cos\theta, \sin\theta) \tag{1}$$

is the unit tangential vector to the shoreline in the direction of increasing s,

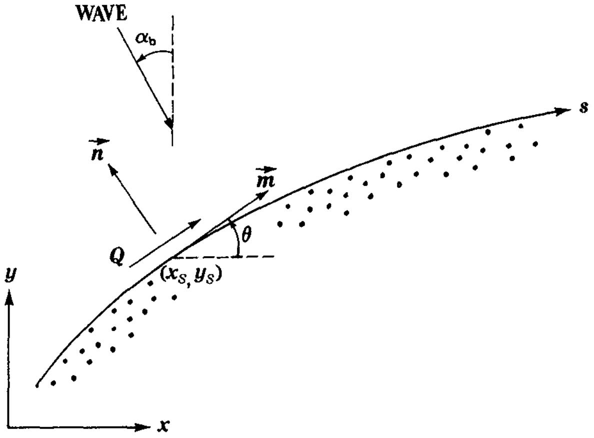


Fig. 1. Curvilinear coordinate system and definition of model variables.

$$\vec{n} = \left(-\frac{\partial y_s}{\partial s}, \frac{\partial x_s}{\partial s}\right) = (-\sin\theta, \cos\theta) \tag{2}$$

is the seaward unit normal vector to the shoreline, and θ is the angle between $\vec{m}$ and the x-axis which is measured counterclockwise from the positive x-direction. In the figure, Q is the volumetric longshore sediment transport rate, and α_b is the breaking wave angle between the wave crest and the x-axis which is measured counterclockwise from the positive x-direction.

Assume that the point, (x_s, y_s), moves perpendicular to the shoreline, so that

$$\left(\frac{\partial x_s}{\partial t}, \frac{\partial y_s}{\partial t}\right) = e\vec{n} \tag{3}$$

in which e is the rate of shore-normal movement of shoreline. Using the notation of complex variables,

$$z_s = x_s + iy_s \tag{4}$$

in which $i = \sqrt{-1}$, and expressing e as

$$e = -\frac{1}{D}\frac{\partial Q}{\partial s} \tag{5}$$

(3) can be written as

$$\frac{\partial z_s}{\partial t} = -\frac{1}{D}\frac{\partial Q}{\partial s}\exp\left[i\left(\theta + \frac{\pi}{2}\right)\right] \tag{6}$$

in which D is the depth of profile closure at which no measurable change in bottom elevation occurs. If θ is very small, the real part of this equation states that the shoreline does not move in the x-direction and the imaginary part reduces to the sediment continuity equation of traditional shoreline models using the Cartesian coordinate system.

A widely-used expression for the longshore sediment transport rate in places where diffraction dominates is that proposed by Ozasa and Brampton (1980):

$$Q = \Gamma H_b^{5/2}\left[K_1 \sin(2\delta_b) - K_2 \frac{\partial H_b}{\partial s}\cot\beta\cos\delta_b\right] \tag{7}$$

in which

$$\Gamma = \frac{\sqrt{g}}{16(s_s-1)(1-p)\sqrt{\kappa}} \tag{8}$$

and

$$\delta_b = \alpha_b - \theta \tag{9}$$

is the breaking wave angle relative to the shoreline under the assumption that the breakerline and the shoreline are locally parallel, and g = gravitational acceleration; s_s = specific gravity of sediment relative to fluid; p = porosity of sediment; κ = ratio of wave height to water depth at breaking; H_b = breaking wave height; $\tan\beta$ = beach slope; K_1, K_2 = empirical longshore sediment transport coefficients. The first term in (7) describes the sediment transport rate produced by obliquely incident waves, whereas the second term describes the transport due to a longshore gradient of breaking wave height, which has been found to be of great importance in cases where diffraction dominates. The beach slope is calculated by the empirical formula proposed by Sunamura (1984) for given breaking wave height, wave period and sand grain size.

A number of studies has been performed to determine the coefficient K_1. A value of $K_1 = 0.77$ was originally determined by Komar and Inman (1970), and a decrease from 0.77 to 0.58 was recommended by Kraus et al. (1982). The second transport coefficient, K_2, has not received great attention of researchers. Ozasa and Brampton (1980) used $K_2 = 3.24K_1$. Hanson and Kraus (1989), however, recommended that the value of K_2 is typically 0.5 to 1.0 times that of K_1.

One of the basic assumptions of shoreline numerical models is that the beach has a constant depth of profile closure throughout the model area, within which erosion or accretion of beach occurs. In this study, the expression proposed by Hallermeier (1983) is adopted with a deep water wave height, H_o, in place of the extreme wave height for an annual seaward limit of profile change:

$$D = \frac{2.9H_o}{(s_s-1)^{1/2}} \tag{10}$$

In the model, the greatest H_o during the simulation period is used for calculation of D.

2.2 Breaking wave model

The sediment continuity equation, (6), can be solved for the shoreline position, z_s, if the wave heights and angles along the breakerline are given. In the present model, the wave condition (height, period and angle of incidence) at the location of the breakwaters is given as input data and the wave deformation in the lee of the breakwaters is computed. It is assumed that the wave condition is constant along the line connecting the breakwaters and its longshore extensions to the lateral boundaries. This assumption will be appropriate if the offshore bottom topography does not deviate drastically from straight and parallel contours, the offshore distances of the breakwaters are almost constant, and the longshore distance of the model area is not too long.

The three major phenomena which alter the wave in the lee of the breakwaters are refraction, diffraction and shoaling. In the present model, it is assumed that all these wave phenomena occur independently, so that the breaking wave height, H_b, can be calculated by

$$H_b = K_D K_R K_S H_B \tag{11}$$

in which H_B = wave height at the location of breakwaters, and K_D, K_R, K_S = coefficients of diffraction, refraction, and shoaling. The diffraction analysis used in this model is based on the theory of Penney and Price (1952). The method of determining the refraction and shoaling coefficients and the breaking wave angle closely follows that of Kraus (1982).

2.3 Finite-difference equations

An explicit finite-difference method is used to solve (6), (7) and (9) numerically for the wave condition computed along the shoreline. The location and orientation of the shoreline and the breaking wave angle are defined at the nodal points, and the longshore transport rate is defined between the nodal points, as shown in Fig. 2. The sediment continuity equation, (6), at the ith point can be expressed as the following finite-difference form:

$$z'_{s_i} = z_{s_i} + \frac{\Delta t}{D}\left[\frac{Q_{i-1} - Q_i}{\frac{1}{2}(\Delta s_i + \Delta s_{i-1})}\right] \exp\left[i\left(\frac{\theta_i + \theta_{i-1}}{2} + \frac{\pi}{2}\right)\right] \tag{12}$$

in which Δt is the time step. The prime denotes the quantity being solved for the next time step and the unprimed quantities are known quantities at the present time step. The quantities, Δs_i, θ_i, and Q_i, are computed by

$$\Delta s_i = \left[(x_{s_{i+1}} - x_{s_i})^2 + (y_{s_{i+1}} - y_{s_i})^2\right]^{1/2} \tag{13}$$

$$\theta_i = \tan^{-1}\left(\frac{y_{s_{i+1}} - y_{s_i}}{x_{s_{i+1}} - x_{s_i}}\right) \tag{14}$$

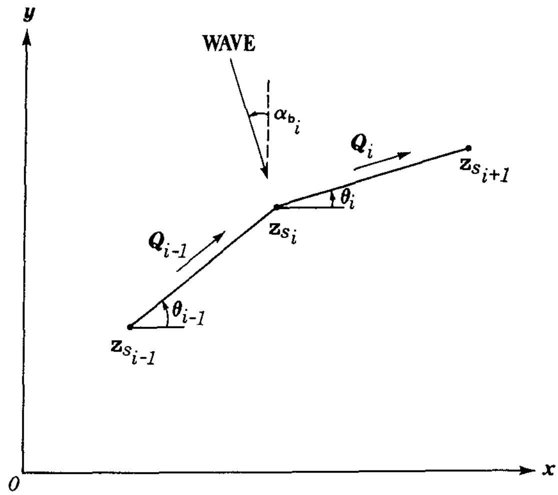


Fig. 2. Finite-difference representation of shoreline and associated transport around *i*th point.

and

$$Q_i = \Gamma\left(\frac{H_{b_i}+H_{b_{i+1}}}{2}\right)^{5/2} \cdot \left[K_1 \sin(2\delta_{b_i}) - K_2 \frac{H_{b_{i+1}}-H_{b_i}}{\Delta s_i}\left(\frac{\cot\beta_i+\cot\beta_{i+1}}{2}\right)\cos\delta_{b_i}\right] \quad (15)$$

in which

$$\delta_{b_i} = \frac{\alpha_{b_i}+\alpha_{b_{i+1}}}{2} - \theta_i \quad (16)$$

The explicit finite-difference scheme becomes unstable for a large time step Δt. As done in Kraus and Harikai (1983) for a Cartesian coordinate system, an approximate stability criterion for small δ_b can be obtained as

$$\Delta t \leq \frac{1}{2}\frac{(\Delta x_s)^2}{\epsilon} \quad (17)$$

in which $\epsilon = 2K_1\Gamma H_b^{5/2}/D$. This stability criterion can give the first approximation of the time step for stable solution. If the computed shoreline shows saw-tooth instability, a smaller time step will be needed. Such an instability can be suppressed by smoothing shoreline orientation, longshore transport rate, and shoreline position as done in Uda (1983), but the smoothing process of shoreline position introduces an error in the conservation of sand as Δx increases.

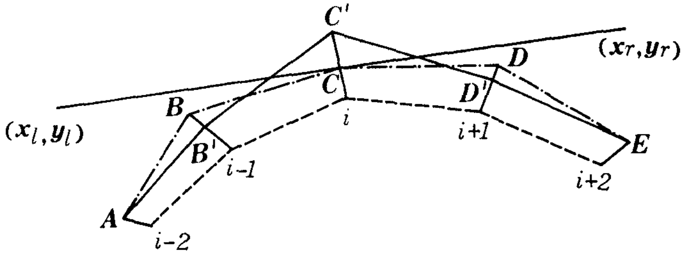


Fig. 3. Illustration of tombolo calculation.

2.4 Calculation of tombolo

When the shoreline reaches an impermeable offshore breakwater, its offshore movement should stop there. But a numerical model that does not recognize the presence of the breakwater may calculate shoreline positions located seaward of the breakwater. Thus, at each time step the calculated shoreline position should be examined to determine if any point has moved seaward of the breakwater. If so, the point should be pulled back to the location of the breakwater. Hanson and Kraus (1985) proposed a method to incorporate a seawall constraint in a shoreline numerical model. The method permits a grid point to move landward of the position of a seawall in the initial computation, and then adjusts the longshore transport rates near that point so as to pull it back to the location of the seawall while preserving sediment volume. In the present study their method is applied inversely for the calculation of tombolo formation in the lee of offshore breakwaters.

Consider the discretized shoreline position as shown in Fig. 3. The dashed line denotes the shoreline position at the present time step, and the solid line and dash-dotted line denote the uncorrected and corrected shoreline positions, respectively, at the next time step. In Fig. 3, the ith point moved seaward of the breakwater to reach the point $C'(x'_{s_i}, y'_{s_i})$. We want to pull it back to the point $C(x_{s_i}, y_{s_i})$, which is the intersection of the breakwater and the line passing the points C and C'. The equation of the breakwater whose tips are located at (x_l, y_l) and (x_r, y_r) is

$$y = (x - x_l)\tan\theta_B + y_l \tag{18}$$

in which θ_B is the angle between the breakwater and the x-axis which is measured counterclockwise from the positive x-direction. The equation of the line passing the points C and C' is

$$y = (x'_{s_i} - x)\cot\frac{\theta_{i-1} + \theta_i}{2} + y'_{s_i} \tag{19}$$

in which θ_i and θ_{i-1} are as defined in Fig. 2, and $-\cot\{(\theta_{i-1}+\theta_i)/2\}$ represents the slope of the line passing the points C and C'. The location of the point C is then calculated as the intersection of the lines expressed by (18) and (19).

The advance of the shoreline position from C to C' results in a fictitious (nonphysical) transport of sand from either one or both sides of the ith point. The fictitious transport rate of sand, ΔQ_{fic}, contributed to move the shoreline position from C to C' can be calculated by putting z_{s_i} for the point C and z'_{s_i} for the point C' in (12) as

$$\Delta Q_{fic} = \frac{D(\Delta s_i + \Delta s_{i-1})}{2\Delta t} \exp\left[-i\left(\frac{\theta_i + \theta_{i-1}}{2} - \frac{\pi}{2}\right)\right](z'_{s_i} - z_{s_i}) \tag{20}$$

The amount ΔQ_{fic} must be subtracted from the computed transport rates (Q_{i-1} and/or Q_i) depending on their directions as follows:

$$\left.\begin{array}{l} Q^c_{i-1} = Q_{i-1} - \Delta Q_{fic}\frac{Q_{i-1}}{Q_{i-1}-Q_i} \\ Q^c_i = Q_i - \Delta Q_{fic}\frac{Q_i}{Q_{i-1}-Q_i} \end{array}\right\} \text{ if } \; Q_{i-1} \geq 0 \text{ and } Q_i \leq 0 \tag{21}$$

$$Q^c_{i-1} = Q_{i-1} - \Delta Q_{fic} \;\text{ if } \; Q_{i-1} \geq 0 \text{ and } Q_i > 0 \tag{22}$$

$$Q^c_i = Q_i + \Delta Q_{fic} \;\text{ if } \; Q_{i-1} < 0 \text{ and } Q_i \leq 0 \tag{23}$$

The superscript c denotes the corrected transport rates. The new position of the point B and/or D is then calculated using the corrected transport rates. If the new point, B or D, is computed to move seaward of the breakwater, it is pulled back to the location of the breakwater using the same procedure as above. Finally, it should be mentioned that in the present model, if any two adjacent shoreline points touch the breakwater, the transport rate between these points is set to zero.

3 Chippokes State Park Breakwater Project

The six segmented breakwaters at Chippokes State Park are located on the southern shore of Cobham Bay in the James River, one of the tributary estuaries of Chesapeake Bay, Virginia (Fig. 4). Chippokes is a recreational and historic state park as well as a model farm. The site is characterized by high (12 m) eroding fastland banks composed of a lower unit of shelly, fossiliferous, fine to coarse sand overlain by an upper layer of slightly muddy, fine to medium sand.

The preconstruction beach at Chippokes was a curvilinear strand of sand about 7.5 m wide from MHW to the base of the bank. The beach itself consists of well sorted, shelly sand derived from the eroding bluff. The mean diameter of the beach sand averaged from six sediment samples is 0.44 mm (medium sand). The shoreline at Chippokes faces almost due north and has an average fetch of about 4.8 km. Long fetch exposures of 9.3 km and 14.8 km occur to NNE and NW directions, respectively. Net longshore transport here is eastward but with seasonal fluctuations and on-offshore movement. The seasonal wave climate favors northerly winds in winter and southwesterly winds in summer. Mean seasonal winds generate limited waves across the

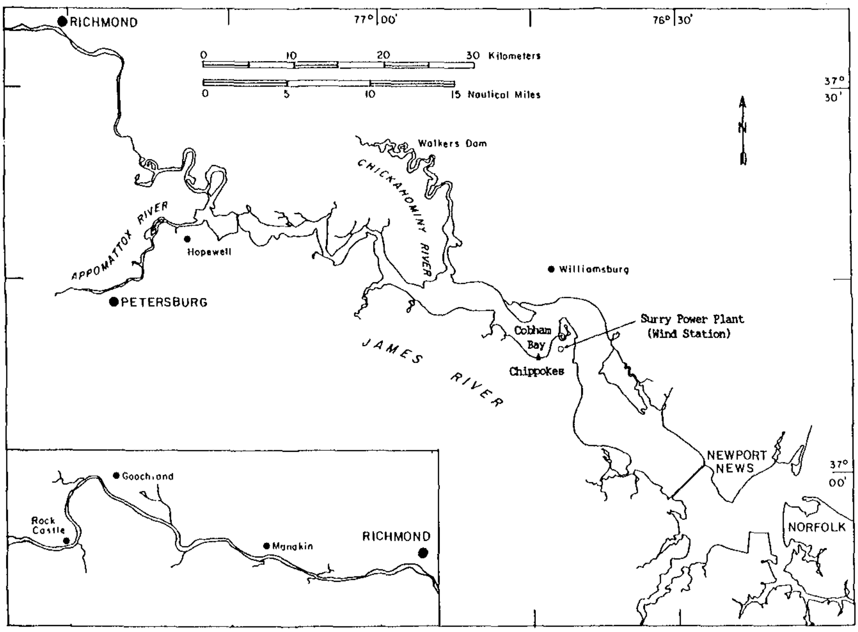


Fig. 4. Location map of Chippokes State Park Breakwaters, Virginia.

river. Extratropical and tropical storms with the associated storm surges are the main forces causing movement of beach sand and shoreline erosion. Mean tidal range is 58 cm.

The goal of the project was to design a system which would permit tombolos to form utilizing the existing volume of sand on the beach such that with time a stable backshore would develop and protect the base of the high banks. A system of six rubble mound breakwaters with a length to gap ratio of 1:1.5 was constructed in June 1987. The crest lengths are 15 m and gaps are 22.5 m. The centerline of the breakwaters is approximately 9 m from the initial MHW line. The water depth below MHW at the location of breakwaters varies between 0.75 and 0.87 m.

A baseline (x-axis in Fig. 5) was established using a transit. From this, 33 profile lines were determined and surveys performed using a rod and level. Profile measurements were made in July 28 and November 12, 1987 and February 23, 1988. Aerial photography was done every three months between September 2, 1987 and March 9, 1988. The photographs were used along with the profile data to create shoreline position maps.

Fig. 5 shows the measured shoreline change (dashed lines) from June 1987 to March 1988 along with the computed results. The position of MHW was used to track beach changes. Sand began accumulating and migrating toward each breakwater unit as cuspate spits formed almost immediately after construction. The characteristic double salients evolved behind each breakwater by September 1987. Sediment for the salients was derived from the adjacent embayments. By March 1988 all the bays showed signs of filling. Especially obvious are the accumulation of sand on the west end of the system and marked loss of sand on the east. One would infer a net west to east movement of sand along this portion of the reach.

4 Model Application

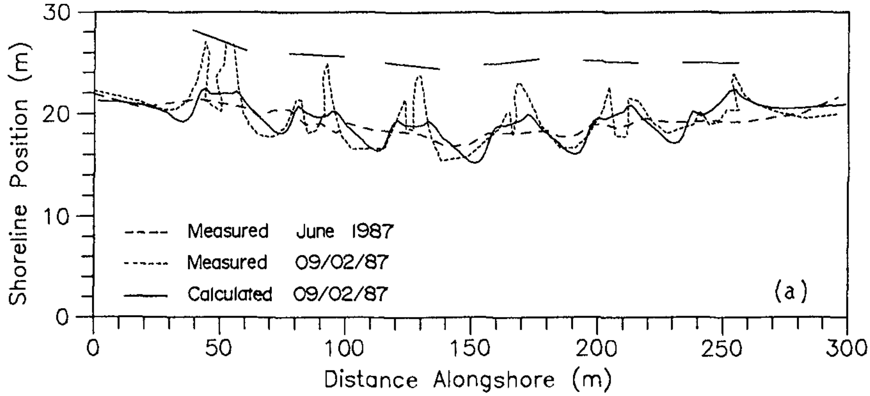
The developed numerical model was applied to the simulation of the shoreline change near the Chippokes State Park Breakwaters for the first nine months (from June 1987 to March 1988) after construction.

4.1 Wave hindcast

There are no available wave data near the project site for the simulation period. Therefore, input wave data at the location of the breakwaters were hindcasted from wind data measured at Surry power plant located about 2.8 km ENE of the project site (see Fig. 4). The wind data included speed and direction every one hour. Assuming that the wave direction corresponds to the wind direction, as seen in Fig. 4, the project site is affected by the wind blowing within the directional window fanning from 320^o to 20^o measured clockwise from the north. It is also assumed that in order to generate the wave field that affects the shoreline change, wind should blow for more than three hours (i.e., more than three consecutive observations) within the directional window. The average wind speed and direction for the period were calculated by vector-averaging the observations given every one hour. A constant wind field corresponding to the averaged wind speed and direction was assumed for that period.

The significant wave height and period at the location of the breakwaters were computed using the model of Kiley (1989), which is essentially a shallow water estuarine version of the quasi-empirical wind wave prediction model developed by Bretschneider (1966) and modified by Camfield (1977). The model includes variation in water depth, the effect of surrounding land forms on the computation of the effective fetch, wave growth due to wind stress and wave decay due to bottom friction and percolation. The wave angle at the location of the breakwaters is determined by Snell's law assuming that the offshore bottom contours are straight and parallel to the x-axis and the deep water wave direction (at the center of the river) is the same as the wind direction.

The weight-averaged (by duration of each wave condition) values of significant wave height and period, and wave angle at the location of breakwaters are 12.2 cm, 1.43 s, and 1.33°, respectively, indicating very short small waves and net eastward longshore transport rate. The highest wave during the simulation was 21 cm high with 2.0 s period. Waves of 0.5 to 0.7 m height with 2.5 to 3.0 s period have been observed on April, 13, 1988. In this area, these types of events occur every two to three years but slightly lesser events occur even more frequently. Total duration of the simulation is 568 hours so that the percent of calm is 90 %.



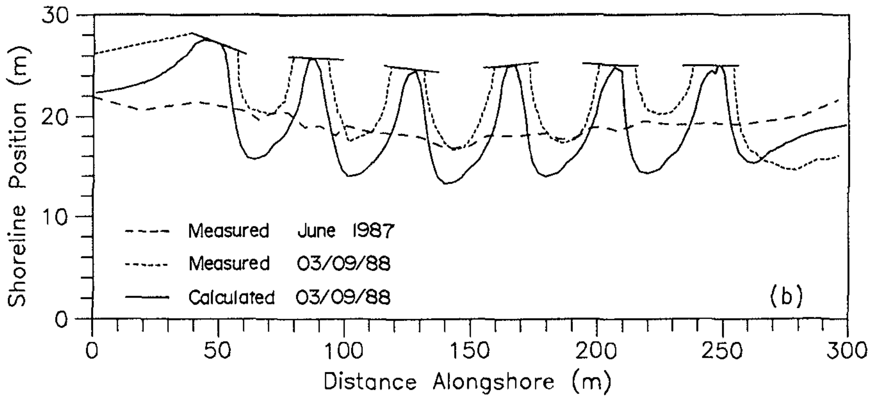


Fig. 5. Comparison between measurement and computation of shoreline change near Chippokes State Park Breakwaters, Virginia; (a) September 1987, (b) March 1988.

4.2 Simulation

The initial shoreline in June 1987 in Fig. 5 was discretized by $\Delta x = 1.524$ m (5 ft) to give about 10 points behind each breakwater and 195 points (296 m) for the shoreline reach shown in Fig. 5. $\Delta t = 90$ s was used. All the water depths and shoreline position in the model are with respect to MHW level, since the shoreline position in the report of Hardaway et al. (1988) is presented in terms of MHW line. The water level was assumed to be fixed at the MHW level. The coordinates of the breakwater tips were taken as the shoreward corners of the bases of the breakwaters.

The shoreline change in the areas far from the project site is not available. Therefore, the model area was extended to both sides by 100 m and fixed boundary conditions were used at both ends. The offshore distances of the initial shoreline in the extended areas were assumed to be the same as those at the end points of the initial shoreline in Fig. 5. The shoreline change in the extended areas is not shown in Fig. 5.

The longshore sediment transport coefficient $K_1 = 0.77$ was used as suggested by Komar and Inman (1970). $K_2 = K_1$ was used tentatively. The calculated depth of profile closure is 0.52 m.

Fig. 5 shows the computed (solid lines) shoreline changes in September 2, 1987 and March 9, 1988 along with the measurement (short-dashed lines). The initial shoreline position is also given by long-dashed lines. The formation of double salients in September 1987 is predicted by the model, even though it is not so clear as in the measurement. The model calculated smaller tombolos and more prominent erosion behind the gaps compared to those in the measurement in March 1988. This may be due to the addition of sediment to the system by runoff and bluff erosion as reported in Hardaway et al. (1988), which was not included in the present model that assumes zero on-offshore transport of sediment.

4.3 Decay of tombolos

Sometimes tombolos are built artificially as a means to protect beaches as in Elm's Beach, Maryland (Hardaway and Gunn 1989), for example. If the project is not designed properly, the tombolos can be reduced to salients. In order to test how the model works for such a situation, the model was run for 48 hours of $H_B = 7.9$ cm, $T = 1.16$ sec, and $\alpha_B = 0^o$ (the mildest wave condition used in the previous simulation) with the computed shoreline in March 9, 1988 as the initial condition. The result is shown in Fig. 6. For the mild wave condition, the tombolos reduced to salients and accretion occurred in the embayments.

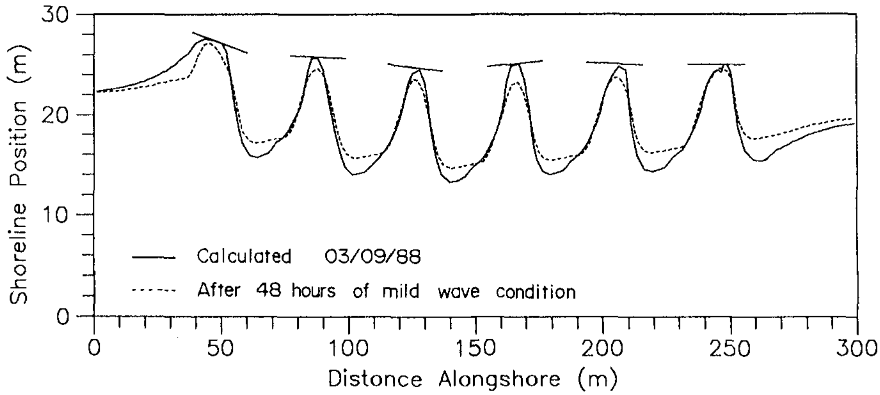


Fig. 6. Test for decay of tombolos.

5 Conclusion

The present study has proposed an algorithm that is capable of calculating the growth and decay of tombolos in the lee of offshore breakwaters in a shoreline numerical model. The algorithm permits a grid point to move seaward of an offshore breakwater in the initial computation, and then adjusts the longshore transport rates near that point so as to pull it back to the location of the breakwater while preserving sand volume. The neighboring shoreline positions are recalculated using the adjusted longshore transport rates. Even though in this study the algorithm was used along with a shoreline numerical model which uses curvilinear coordinates that follow the shoreline, it could be easily modified for the use in traditional shoreline models using Cartesian coordinate system.

The project of six segmented breakwaters at Chippokes State Park, Virginia has been reported. The developed model was applied to the simulation of the shoreline change during the first nine months after construction of the Chippokes breakwaters. The present model, which does not include cross-shore transport, predicted smaller tombolos and more embayment erosion compared to measurement at the Chippokes project site, where sediment was added to the system by runoff and bluff erosion. The model was applied for offshore breakwaters in an estuary with very short small waves, and applications in open coasts may be needed to fully examine the performance of the model.

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