CHAPTER 203

Suspended Sediment Concentration Profiles under Non-breaking and Breaking Waves

Rattanapitikon Winyu and Tomoya Shibayama [1]

Abstract

Simple formulas to predict time-averaged suspended sediment concentration are formulated using steady diffusion equation. Empirical formulas are developed to compute reference concentration (boundary condition) and diffusion coefficient. For suspended sediment concentration in the field, the same formulas with regular wave condition can be applied by using root mean square wave height and average wave period. Total 139 data sets are used for calibration of empirical formulas and 175 data sets are used for verification.

Introduction

The need for reliable prediction of sediment transport and beach profile change is increasing due to an increasing of human activity on the coast. In order to predict the suspended sediment transport rate, it is necessary to predict the vertical distribution of sediment concentration and fluid velocity accurately. This study focuses an attention on the sediment concentration distribution. Sediment concentration is important not only for computing sediment transport rate but also for significant effect on the water quality for domestic and industrial use. From the last few decades, a number of models and experimental investigation have been performed to draw a clearer picture of suspended sediment concentration. Considerable amount of knowledge has been accumulated so far. However, it has not reached a satisfactory level. This is resulted from the unclear knowledge of movable sand layer and diffusion coefficient. At the present stage of knowledge, any type of model must be based on empirical or semi-empirical formula calibrated from the experimental results. For the prac-

[1]Dept. of Civil Eng., Yokohama National University, Hodogaya-ku, Yokohama 240, Japan

tical purposes, simple formulas may be more suitable than the complex ones (if it yields the accuracy in the acceptable range).

Based on wide range of wave and sediment conditions, Shibayama and Winyu (1993) proposed simple formulas to predict time-averaged concentration profiles under both non-breaking and breaking waves. The present study mainly focusses on the application and verification of the formulas. The measured surf zone concentration profile in prototype scale wave flume of Kajima et al. (1983) and in the field of Nielsen (1984) are used in this study. For some background information, a brief introduction of the formulas is also presented.

Governing Equation

The vertical distribution of suspended sediment is calculated by the diffusion equation. By considering time averaged value of concentration and neglecting convection and horizontal diffusion, the diffusion equation can be written as:

$$cw_s + \epsilon_s \frac{\partial c}{\partial z} = 0 \tag{1}$$

where c is the time averaged sediment concentration; w_s is the falling velocity; ϵ_s is the diffusion coefficient; and z is the vertical coordinate. To solve the diffusion equation, concentration at reference level should be given as a boundary condition, and the diffusion coefficient should also be known.

Reference Concentration under Non-breaking and Breaking Waves

The reference concentration is defined at the level where the concentration can be measured without disturbance to the bed formation. The formula for predicting reference concentration is derived by applying transport rate formula of Watanabe (1982) and dimensional analysis (for more details, see Shibayama and Winyu, 1993). Total thirteen sources of published experimental results, totally 139 experiments, are used to calibrate the formulas. Table 3.1 shows the experiments which are used in formula calibration. As a result, the formula for computing reference concentration is given as:

$$c_r = \frac{10}{3} \frac{(\psi - 0.05)\nu}{r\sqrt{(s-1)gd}} \tag{2}$$

where c_r is the reference concentration at $z = r$; r is the reference level; The reference level is equal to half of ripple height above the ripple crest for vortex ripple case and equal to a hundred times of sand diameter above the mean bed for flat bed (i.e., breaking wave case); ψ is the grain Shield parameter; s is the relative density of sediment; d is the sand diameter and ν is the fluid kinematic viscosity. The comparison between measured and computed reference concentration, c_r, is shown in Fig. 1. Figure 1 shows that about 80 percent of the predicted ones are within factor two.

Table 1: Experiments for suspended sediment concentration study.

No	Sources	Total No.	Legend	Wave condition
1	Bosman and Steetzel (1986)	3	B	regular, non-breaking
2	Deigaard et al. (1986)	6	G	regular, breaking
3	Dette and Uliczka (1986)	8	D	regular, breaking
	Dette and Uliczka (1986)	3	U	irregular, breaking
4	Hayakawa et al. (1983)	4	Y	regular, non-breaking
5	Horikawa et al. (1982)	7	H	regular, non-breaking
6	Irie et al. (1985)	27	I	regular, non-breaking
7	Nakato et al. (1977)	3	N	regular, non-breaking
8	Nielsen (1979)	44	P	regular, non-breaking
9	Sato et al. (1990)	14	T	regular, breaking
10	Sawamoto et al. (1981)	4	S	regular, non-breaking
11	Skafel and Krishnappan (1984)	8	K	regular, non-breaking
12	Sleath (1982)	4	L	regular, non-breaking
13	Vongvisessomjai (1986)	4	V	regular, non-breaking

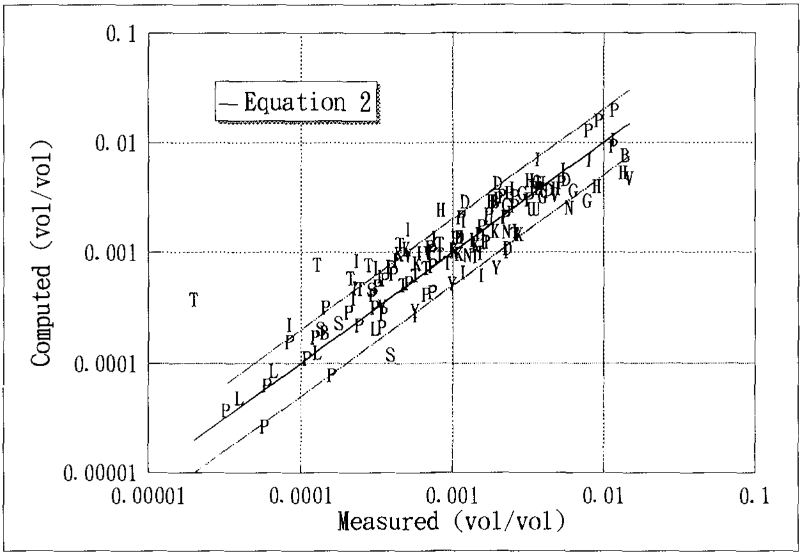

Figure 1: Comparison between measured and computed reference concentration under non-breaking and breaking waves, c_r, (legend see Table 1).

Concentration Profiles under Non-breaking Waves

Under non-breaking waves, high concentration areas of suspended sand are usually confined within the thin layer above the bottom, the thickness of which is usually about three ripple-heights or a few hundred times of grain diameter above the bed in laboratory tests and field measurements. Most of the previous experimental results show that if very small concentrations are neglected, the time averaged concentration profile fits well with the exponential form which is derived from steady diffusion equation (Eq. 1) under the assumption that the diffusion coefficient, ϵ_s, is independent of vertical coordinate, z. After the integration of Eq. 1, the analytical solution is expressed as:

$$c(z) = c_r \exp\{-w_s(z-r)/\epsilon_s\} \tag{3}$$

Equation 3 shows that the relation between $\ln(c)$ and z is the straight line with the gradient of $-w_s/\epsilon_s$.

At present, the knowledge of diffusion coefficient, ϵ_s, is very limited, an application of empirical formula cannot be avoided. The difficulty of this approach is to find out the way to relate the diffusion coefficient with flow and sediment properties. From the previous empirical formulas of Sleath (1982), Skafel and Krishnappan (1986), and Nielsen (1988), we may assume that the diffusion coefficient ϵ_s is a function of the following quantities:

$$\epsilon_s = f(u_b, A_b, \nu, w_s, d, s, f_w, \eta) \tag{4}$$

where f is the function; u_b is maximum orbital velocity; A_b is the orbital amplitude; f_w is the wave friction factor; η is the ripple height.

Experimental data of non-breaking wave cases, which shown in Table 1 (totally 108 cases), are used to calibrate the empirical formula. From dimensional analysis, using the experimental data of non-breaking wave cases in Table 1, the following formula is fitted well with the measured ones (see Fig. 2).

$$\epsilon_s = 0.21 u_* A_b \left(\frac{w_s}{u_*}\right)^2 \left(\frac{\eta}{d}\right)^{0.5} d_*^{-1.5} \tag{5}$$

where u_* is the maximum bed shear velocity; and $d_* = d(sg/\nu^2)^{1/3}$ is the particle parameter. Eq. 5 is kept in the dimensionless form as in the analysis. Since u_* can be canceled out, ϵ_s is not reverse propotional to u_*.

Figure 2 shows that about 80 percent of predicted diffusion coefficient, ϵ_s, are within factor 1.5. Examples of measured and computed concentration profiles under non-breaking waves are shown in Fig. 3.

It should be mentioned that the exponential form of suspended concentration profile is valid within the thin layer (about few hundred times of sand diameter above the bed). As a results, Eq. 3 can be used to predict only the high concentration near the bed.

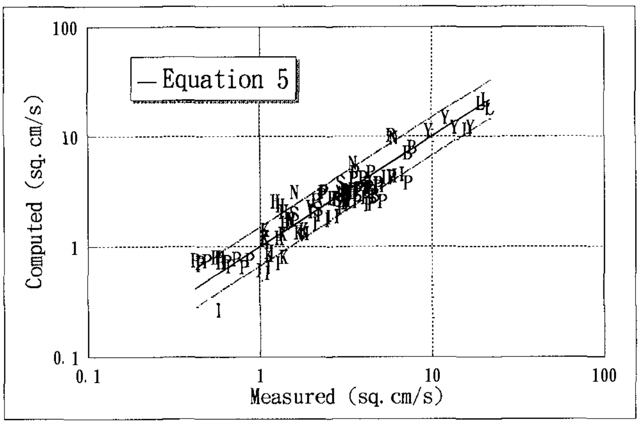

Figure 2: Comparison between measured and computed diffusion coefficient for non-breaking wave cases (legend see Table 1).

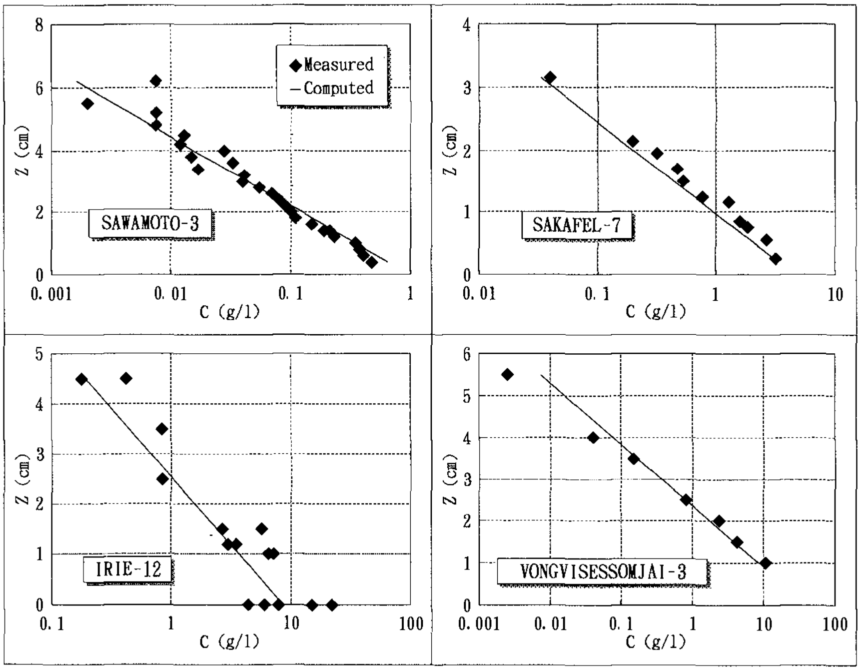

Figure 3: Examples of measured and computed concentration profiles under non-breaking waves of each data source.

Concentration Profiles under Breaking Waves

The turbulence generated by the breaking waves causes a significant increase in the amount of suspended sediments compare to non-breaking waves of the same wave conditions and water depth. Moreover, the effect of turbulence due to breaking waves changes the shape of concentration profile (see Nielsen, 1978, Fig. 5) and the exponential form of concentration profile is not found. Thus, in breaking wave cases, we can not assume that the diffusion coefficient is constant as we assumed in non-breaking wave cases.

According to the experimental results, Okayasu (1989) suggested the linear distribution of eddy viscosity, ϵ, as the function of the rate of energy dissipation due to wave breaking, D_B. By assuming diffusion coefficient proportional to the eddy viscosity of the flow and incorporating the diffusion coefficient caused by shear in wave field, the total diffusion coefficient, ϵ_{sb}, is expressed as:

$$\epsilon_{sb} = \left[k_a u_* + k_b (D_B/\rho)^{1/3}\right] z \tag{6}$$

where k_a and k_b are the constants and D_B is the rate of energy dissipation. From the bore model, we can set $D_B = \rho H^3 g/(4Th)$.

After integration of the diffusion equation (Eq. 1), the analytical solution of concentration profile can be written in the following form:

$$c(z) = c_r \left(\frac{r}{z}\right)^M \tag{7}$$

$$M = \frac{w}{\left[k_a u_* + k_b (D_B/\rho)^{1/3}\right]} \tag{8}$$

From the best-fit technique, using breaking wave data in Table 1 (totally 29 data sets), the following constant parameters are recommended.

$$k_a = 0.04,$$
$$k_b = \begin{cases} 0.144 & \text{spilling breaker,} \\ 0.216 & \text{spilling-plunging transition breaker,} \\ 0.450 & \text{plunging breaker.} \end{cases}$$

Figure 4 shows the measured and computed parameter M. Examples of measured and computed concentration profiles under breaking waves are shown in Fig. 5.

It should be noted that Eq. 6 is valid if the turbulent due to the breaking waves can uplift the sediment throughout the water depth. Equation 5 should be used with care for coarse sand (i.e., $d > 0.55$ mm) and certainly it can not be used in gravel bed material. Also it can not be used to compute concentration at the bed ($z = 0$).

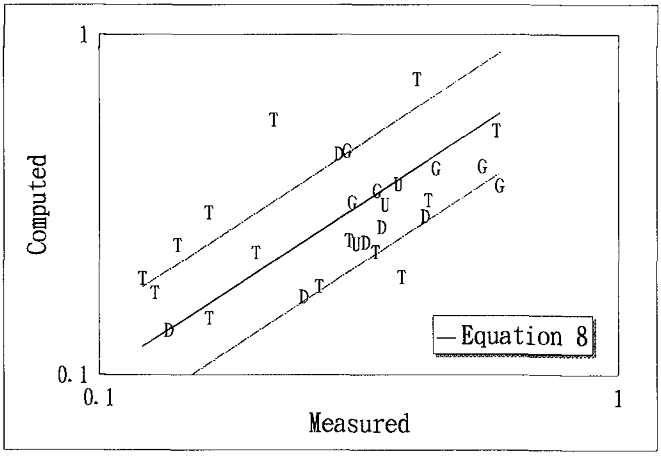

Figure 4: Comparison between measured and computed parameter M (legend see Table 1).

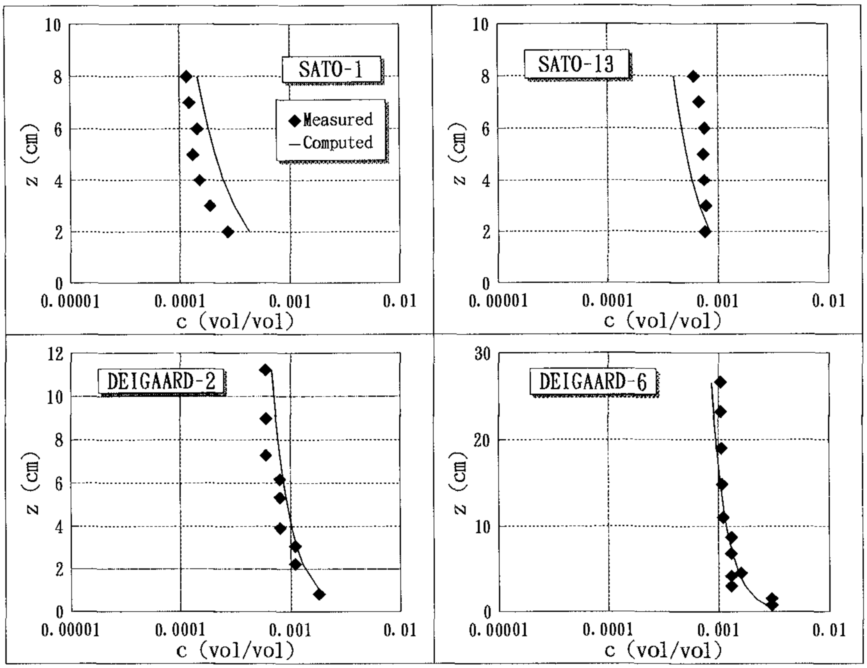

Figure 5: Predicted concentration profiles under breaking waves in comparison with the laboratory data of Deigaard et al. (1986), Sato et al. (1990).

Verification with Prototype Scale Measurements

In order to confirm an empirical formula, wide ranges of experimental results are necessary in the calibration or verification. The objective of this section is to modify and examine the validity of the Shibayama and Winyu (1993) formulas by using the measured surf zone suspended concentration of a prototype scale wave flume which were performed by Kajima et al. of Central Research Institute of Electric Power Industry (CRIEPI) in 1983. The CRIEPI experiments were carry out under regular wave motion in a large wave flume (205 m long, 3.4 m wide, and 6 m deep). The CRIEPI experimental conditions that used in this section are shown in Table 2. The suspended concentrations were measured, by suction type concentration meter, at various sections along the flume. However, not many points were measured in the vertical direction of each section. Also lowest measuring level is not very near to the movable bed. This may because of the difficulty to measure the concentration very near to the bed accurately. The data may difficult to be used in formula calibration but can be used for formula verification.

The proposed formula assumed that $\epsilon_{sb} \propto (D_B/\rho)^{1/3}$ and D_B is computed from the bore model which valid only inside the surf zone (full developed surface roller). Therefore if we use bore model to compute the rate of energy dissipation, D_B, near the recovery zone where the surface roller is not fully developed, the predicted ϵ_{sb} is expected to be over-estimated and also it will yield the over-estimation of suspended concentration at the level above the reference level. To avoid this problem, energy dissipation may be computed from the measured wave height transformation, based on linear wave theory, as:

$$D_B = -\frac{\partial(Ec_g)}{\partial x} \tag{9}$$

where $E = \rho g H^2/8$ is the wave energy; c_g is the group velocity; and x is the horizontal coordinate in wave direction.

Question may be asked that whether energy dissipation computed from bore model and from Eq. 9 are the same or not (in the case of no reformation zone). For verification, measured wave height transformation inside the surf zone (no reformation zone) of Hansen and Svendsen (1984), Okayasu et al. (1988), Sato et al. (1988 and 1989) are used. Figure 6 shows that energy dissipation computed from bore model and Eq. 9 give approximately the same results. Therefore the energy dissipation rate computed from the measured wave height, based on linear wave theory, will be used in the following analysis. However, due to the fluctuation of measured wave heights, the computed energy dissipation rates in some points are negative. So, in the present study, the fluctuated wave heights are smoothed before using for computing energy dissipation rates from linear wave theory.

For diffusion coefficient caused by breaking waves inside the transition zone, the eddy viscosity (or diffusion coefficient) at the breaking point cannot be incorporated with the same manner as in the inner zone (Okayasu, 1989). From the analysis of undertow data of Nadaoka et al. (1982) and Okayasu (1988) (based on eddy viscosity model), the coefficient of turbulent eddy viscosity, K, increases linearly from 0.3 at the breaking point to 1 at the transition point. By assuming no vortex ripple occur in the surf zone, the final formula for computing sediment concentration profile can be expressed as:

$$c(z) = \frac{10}{3} \frac{(\psi - 0.05)\nu}{100d\sqrt{(s-1)gd}} \left(\frac{100d}{z} \right)^{w/\left[0.04u_* + 0.144K(D_B/\rho)^{1/3}\right]} \tag{10}$$

where K is the coefficient varies linearly from 0.3 at breaking point to 1 at transition point; D_B is the energy dissipation computed from measured wave height based on linear wave theory (Eq. 9).

The verification results for all of the measuring points, totally 149 sets, 645 points, are shown in Fig. 7. The examples of topography, wave height and concentration profile variations along the cross-shore direction are shown in Figs. 8-11. Fig. 7 shows that about 80 percent of predicted concentrations are within the factor 3, which can be considered to be well estimation in the field of sediment concentration profile. Since the accuracy of computed reference concentration is within the factor 2 (see reference concentration section), about 2/3 of error in Fig. 7 is expected to be caused by the formula that used for computing reference concentration. The left 1/3 is expected to cause by the formula for computing the distribution of concentration (parameter M). Based on the above cosideration, for excellent prediction of concentration profiles, the formula for computing reference concentration should be the main target of improvement for the next step.

Table 2: Experimental conditions and number of data sets for verification.

Case	d_{50} (cm)	m_b	T (s)	H_i (cm)	h_i (cm)	No. of data set
3.1	0.027	5/100	9.1	107.0	450.0	10
3.2	0.027	5/100	6.0	105.0	450.0	11
3.3	0.027	5/100	12.0	81.0	450.0	11
3.4	0.027	5/100	3.1	154.0	450.0	13
4.1	0.027	3/100	3.5	31.0	350.0	7
4.2	0.027	3/100	4.5	97.0	400.0	9
4.3	0.027	3/100	3.1	151.0	400.0	26
5.2	0.027	2/100	3.1	74.0	350.0	25
6.1	0.027	10/100	5.0	166.0	400.0	27
6.2	0.027	10/100	7.5	112.0	450.0	10

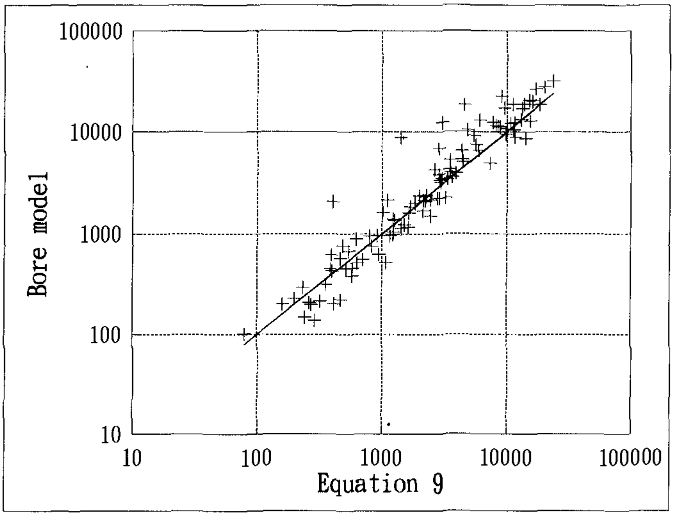

Figure 6: Comparison between computed enegy dissipation from bore model and Eq. 9 (laboratory data from Hansen and Svendsen, 1984; Okayasu et al., 1988; and Sato et al. 1988 and 1989).

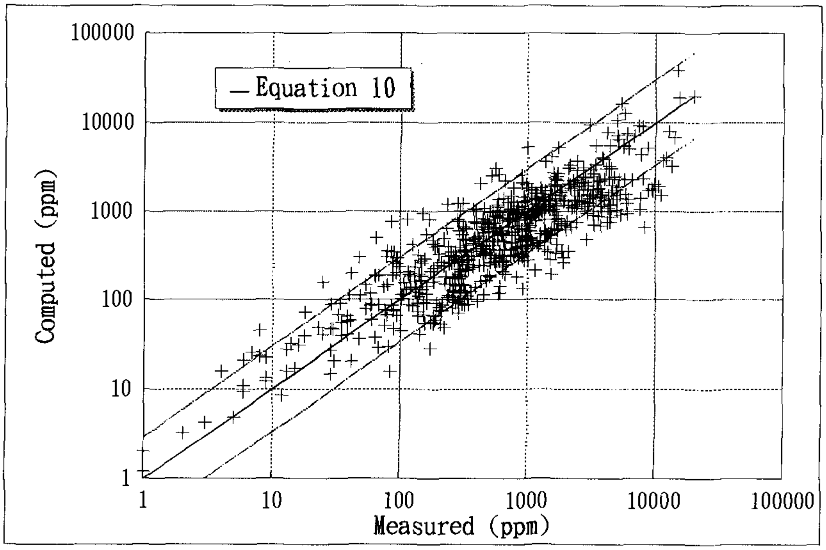

Figure 7: Computed and measured concentration for all of the measuring points (laboratory data from Kajima et al., 1983).

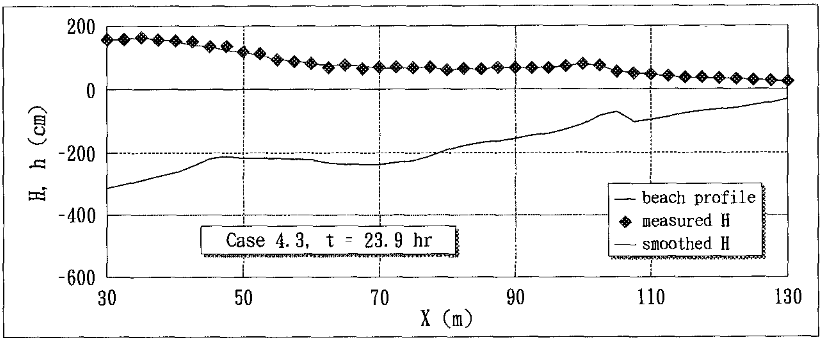

Figure 8: Measured bottom topography and wave height variation (laboratory data from Kajima et al., 1983, case 4.3, time = 23.9 hr).

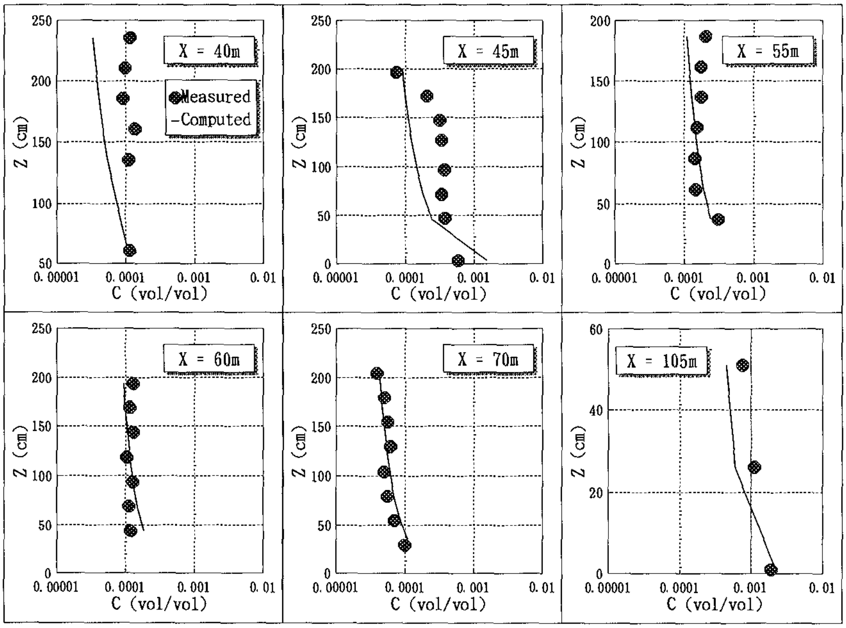

Figure 9: Computed and measured concentration profiles (laboratory data from Kajima et al., 1983, case 4.3, time = 23.9 hr).

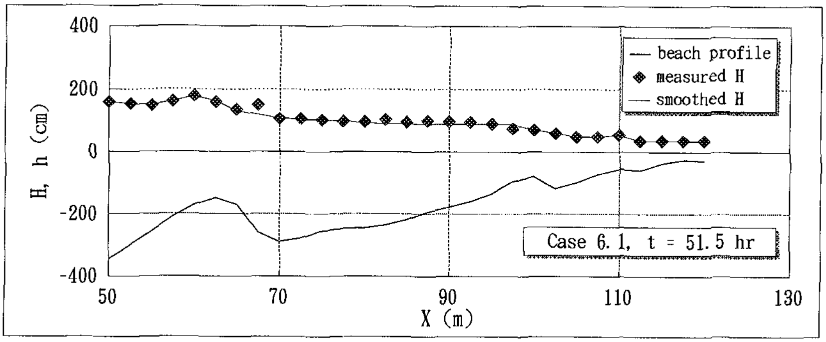

Figure 10: Measured bottom topography and wave height variation (laboratory data from Kajima et al., 1983, case 6.1, time = 51.5 hr).

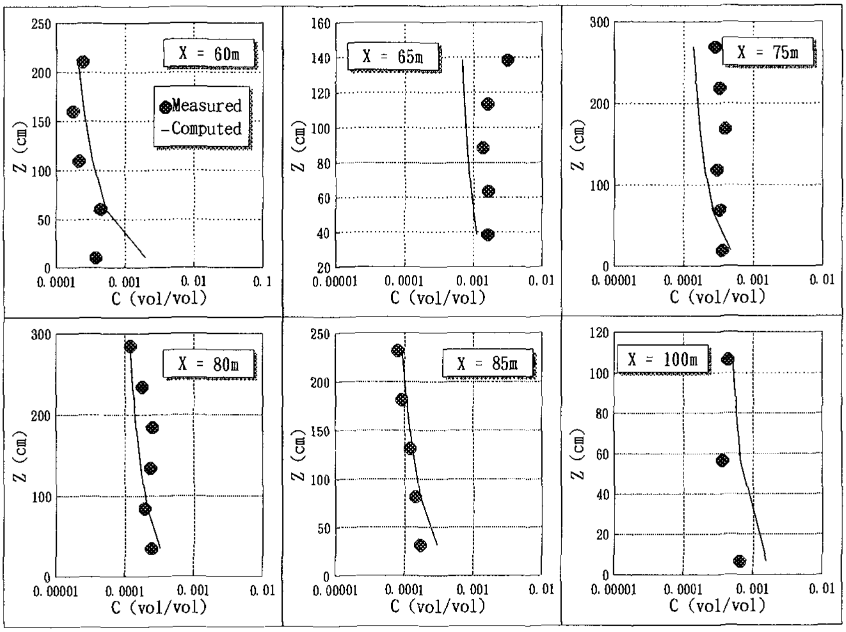

Figure 11: Computed and measured concentration profiles (laboratory data from Kajima et al., 1983, case 6.1, time = 51.5 hr).

Verification with Field Measurements

Although, the Shibayama and Winyu (1993) formula is developed from the data measured in laboratory, it may also be used to predict the sediment concentration in the natural beach. The objective of this section is to apply Shibayama and Winyu (1993) formula to predict surf zone sediment concentration in the natural beach. Nielsen (1984) measured sediment concentration and hydraulic properties on several beaches and locations in Australia. Total 26 concentration profiles are measured under breaking wave conditions. For each measuring location, 7 elevations suction type is used to measure sediment concentration and the sampling time is about 3 minutes. The selected working depth is about 1.5 m and the measurement started from about 1 cm above the bed. For more details of measured concentration, hydrodynamic and sediment data, please see Nielsen (1984).

By assuming all of the measured locations are within the inner surf zone and there is no vortex ripple, the formula for predicting sediment concentration profiles can be expressed as:

$$c(z) = \frac{10}{3}\frac{(\psi - 0.05)\nu}{(100d)\sqrt{(s-1)gd}}\left(\frac{100d}{z}\right)^{w/\left[0.04u_* + 0.144\{H^3 g/(4Th)\}^{1/3}\right]} \tag{11}$$

where all variables are calculated based on linear wave theory.

To compute concentration profiles, under irregular wave, from Eq. 11, representative wave height and wave period should be specified. By trial and error of various representative wave height and wave period, root mean square wave height (assuming Rayleigh distribution) and average wave period gives a good prediction on concentration profiles. The comparison between measured and computed sediment concentration for all of the measuring points (26 data sets) is shown in Fig. 12. Examples of predicted concentration profiles are shown in Fig. 13. Fig. 12 shows that about 70 percent of predicted concentrations are within the factor 3, which can be considered reasonably well estimation for the field measurements. However, the number of experimental results may not enough to confirm the ability of the present formula. In the further study, more field experimental results are required to check the ability of present formula.

Conclusion

Empirical formula of Shibayama and Winyu (1993) has been applied to compute surf zone concentration profiles in prototype scale wave flume experiments of Kajima et al. (1983) and in the field experiments of Nielsen (1984). The computed results show reasonable agreement with the experimental results.

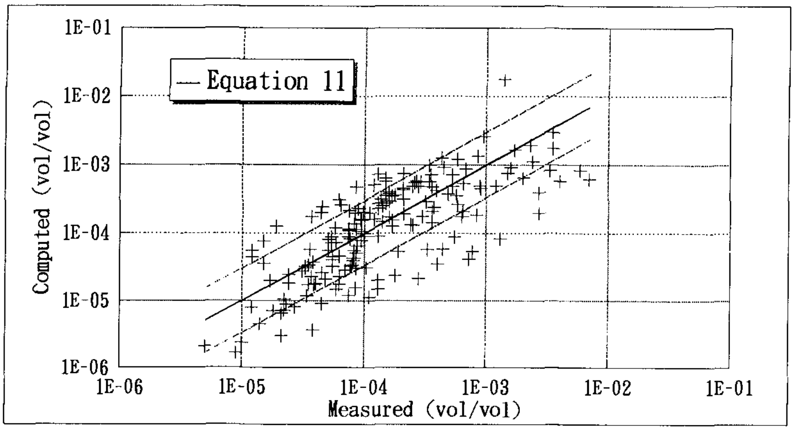

Figure 12: Computed and measured concentration for all of the measuring points (field data from Nielsen, 1984).

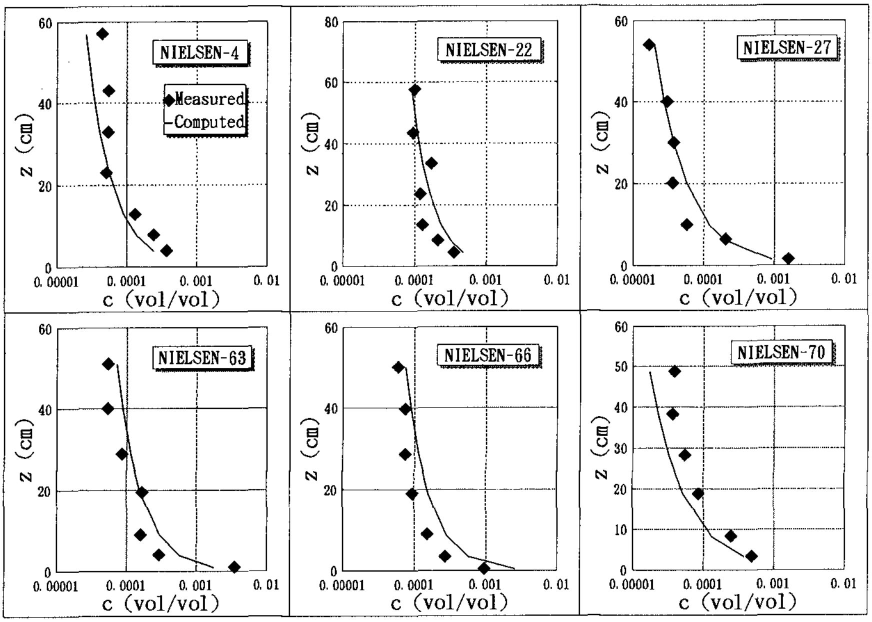

Figure 13: Examples of computed concentration profiles (field data from Nielsen, 1984).

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