CHAPTER 43

Computation of the near-bottom kinematics of shoaling waves

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Abstract

Two practical formulations from Isobe and Horikawa (1982) and Swart and Crowley (1988) are tested against laboratory measurements: seven cases with monochromatic waves and two cases with random waves. It appears that the Isobe and Horikawa formulation can be used with confidence over a large range of wave conditions except in the surf zone when monochromatic waves are considered. The covocoïdal theory from Swart and Crowley provides a more comprehensive description of wave properties but abnormal results have been noticed in a few cases. The use of one representative wave height and period in random waves may lead to an underestimation of velocity moments with low steepness waves.

1. Introduction

Shoaling waves (i.e. waves normally transforming over a sloping bottom) exhibit a more or less pronounced vertical asymmetry of the free surface elevation and orbital velocity when approaching the breaking point. Classical wave theories developed by assuming a flat bottom are able to predict the horizontal asymmetry of the waves in deep and intermediate water depths but not such a vertical asymmetry (Soulsby et al, 1993).

In this paper, we shall focus mainly on the prediction of near-bottom (i.e. outside the bottom boundary layer) kinematics of shoaling waves which is of special interest for the computation of sediment transport on the shereface and in the nearshore zone.

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Firstly, the main results obtained from published experimental investigations in laboratory will be reviewed. Secondly, two existing practical formulations able to predict the near-bottom orbital velocities of shoaling waves will be briefly described. Their ability and accurary will be then assessed against laboratory measurements in the third part of this paper.

2. A review of laboratory experiments

a) Monochromatic waves

A summary of the published laboratory studies reviewed for this paper is presented in table 1. Historically, many studies have focused on the prediction of the horizontal velocity under the crest which is one of the most important quantities used for design purposes (Dean and Perlin, 1986; Kirkgöz, 1986). The main result of interest is that **near the bottom**, the linear theory gives the best overall ajustment for flat and sloping beds up to the breaking point and within the surf-zone. It should be pointing out that this very flattering comparison cannot be extrapolated to the overall velocity profile (Hattori, 1986). In particular, the observed skewness of velocities (Nadaoka and Kondoh, 1982) cannot be predicted.

Table 1 Measurements of near bottom orbital velocities in laboratory (regular waves)

Authors	beach slope	n° of tests	type of measurements	measur. in surfzone
Iwagaki & Sakai (1970)	flat	22	profiles at crest and time series near bottom	No
Tsuchiya & Yamaguchi (1972)	1/100	3	at crest and trough above mid-depth	No
Stive (1984)	1/40	2	profiles at crest and trough	Yes
Flick et al. (1981)	1/35	2	time series near bottom	Yes
Isobe & Horikawa (1982)	1/10 & 1/20	21	time series near bottom	Yes
Nadaoka & Kondoh (1982)	1/20	12	near-bottom mean velocity and velocity skewness	Yes
Hattori (1986)	1/20	15	profiles at crest and trough	No
Sato et al. (1988)	1/20	3	time series near bottom (+ random waves)	Yes
Kirkgöz (1986)	1/4.45 to 1/12	17	profiles at crest	No

This skewness is peaking at the breaking point and tends to decrease significantly in the surfzone except for steep waves. It is a relevant indicator of the development of asymmetrics observed in shoaling waves which are of two kinds.

The horizontal asymmetry includes an increase of the velocity speed under the crest (u_c) which could become twice the velocity speed under the trough (u_t). This increase is accompanied by a shortening of the crest duration T_c (i.e. the duration of time when the velocity is in the direction of the wave propagation).

This is explained by the development of higher harmonics which are locked to the primary (sinusoïdal) wave. The ratio u_i/u_t was found to be maximum at the breaking point and then to decrease signficantly in the surf zone (Stive,1984).

The vertical asymmetry includes a steepening of the velocity (temporal) profile between a trough and the following crest producing a higher acceleration than between the crest and the following trough. This is explained by the development of phase differences between the harmonics and the primary wave (Flick et al., 1981) induced by the slope of the bottom and producing a sawtooth shape.

b) Random waves

When randomness of waves is considered, the situation becomes complicated by presence of bounded and free low frequency waves. In intermediate depths, bounded second-order wave velocities can be computed from second-order random theories (Dean and Perlin, 1986).

In the nearshore zone, partially standing long-waves are often present. Guza and Thornton (1985) have analysed velocities measured at Torrey Pines beach including the surfzone. They have shown that low frequency velocity variance increases monotonically as depth decreases. Cross-shore high frequency velocity moments appear to be near gaussian offshore, reach a maximum deviation from gaussian near the mean breaker location and trend back to gaussian as the shoreline is approached. Such an evolution cannot be predicted by a monochromatic or a linear gaussian model.

Roelvink and Stive (1989) carried out detailed near-bottom velocity measurements in a laboratory flume and also found that high frequency velocity moments were very significant in the nearshore region. They used a non-linear wave theory to successfully predict these moments.

3. Practical formulations tested

Existing formulations have been reviewed by Soulsby et al. (1993). Two of them have been chosen for the validation tests presented hereafter.

a) Isobe and Horikawa

Isobe and Horikawa (1982) proposed two series of empirical corrections of the linear velocity profile. As a first step, an estimation of the ratio $u_c/(u_c + u_t)$ is obtained for their equations (13) to (23). From these expressions, u_t could be predicted assuming that u_c is accurately computed with the linear theory.

Similarly, the ratio T_c/T could be computed. Two sinusoïdal profiles could then be adjusted to simulate the velocity profile under the crest and under the trough.

In a second step, the vertical asymmetry is accounted for by introducing two time lags in the above profile with the equations (24) to (28). To apply these expressions, one should correct the following typing errors:

1) The right side of equation (23) should read $0.5 - 0.018(T\sqrt{g/d})$.
2) In the second right side of equation (28), π is outside the brackets.

Please also note that in equation (28), the origin of time is at $0.5(T - T_c)$ following figure 9 of Isobe and Horikawa.

b) The covocoïdal theory

In order to provide a simple and accurate tool for engineering purposes, Swart and Loubster(1978) have presented a numerical method and a parametrized solution of the problem valid at any water depth for a flat bottom. They called it the vocoidal theory (vocoidal stands for **V**ariable **O**rder **C**osinus**OIDAL** function). It is based on basic assumptions concerning the form of the free-surface elevation, the orbital velocity and the celerity which are expressed as:

$$\eta(t) = H(\cos^{2P}(\pi \frac{t}{T}) - \eta_{*t}) \tag{1}$$

$$\frac{u(z,t)}{C} = k\eta \frac{\cosh(M(X)kz)}{\sinh(M(X)k(h+\eta))} \tag{2}$$

$$\frac{C^2}{gh} = \frac{1}{kh}\tanh(Nkh) \tag{3}$$

t is time, H and T are the wave height and period, X = t/T, P is the wave profile parameter, η_{*t} is the non-dimensional wave trough, k is the wave number, C the celerity, z the vertical coordinate, M(X) is the orbital velocity function and N is the wave celerity parameter. Equations (1) and (3) have been originally proposed by Van Hijum (1972) and equation (2) by Mejlhede(1975). A numerical method to solve this simplified problem has been established by Swart(1978).

Then, a parametrized form of this solution has been derived empirically from numerous computations in order to provide analytical equations very easy to use. The comprehensive derivation and validation of the method could be found in Swart (1978) and Swart and Loubster(1979).

This theory was then extended by Swart and Crowley(1988) to the case of a sloping bottom by modifying equations (1) and (2) in order to introduce a parametrized form of the vertical asymmetry. This new theory has been called the Covocoidal theory. The same numerical approach has been used and an analytical parametrized form of the solution was also given. This parametrization has then been slightly modified by Swart and Crowley(1989). This is this parametrized solution which has been tested here.

c) An unsuccessful test

Finally, it should be noted that Hattori and Katsurakawa (1990) followed the idea of Flick and al (1981) by proposing empirical formulae of phase lags. Such a method could be easily used with numerical wave models based on Fourier developments. Unfortunately, we have not been able to recompute the results presented in their paper (i.e. figure 5 of their paper from their equation (5)).

4. Validation tests

The two formulations have been tested against several laboratory data with monochromatic and random waves.

a) Monochromatic wave tests

As a first test, two velocity time-series of near-breaking shoaling waves measured by Isobe and Horikawa (1982) on a fixed slope of 1 in 20 have been used. The first one has been obtained with a deep-water steep wave ($H_0/L_0 = 0.059$) and the second with a low-steepness wave ($H_0/L_0 = 0.0067$). The measured and computed velocity profiles are shown on figure 1. For both cases the agreement is excellent with Isobe and Harikawa's formulation and reasonable with the covocoïdal theory. It is worth to note that in the first case, the ratio H/h is 0.88 far over the limiting value on a flat bottom. It means that very accurate high order Fourier wave theories cannot provide any result here.

A second test made use of the measurements of the skewness of the velocity from Nadaoka and Kondoh (1982) along a 1 in 20 slope. Two typical cases including as previously high and low steepness waves have been simulated. Figure 2 presents the results obtained. In order to perform the computations of the velocity, the wave height transformation has been computed with a finite amplitude wave model including the surfzone.

Outside the surfzone negative abnormal values are obtained with the covocoïdal theory in the first case. Furthermore, the skewness is peaking too much offshore in the second case. On the other hand, the Isobe and Horikawa formulation is providing smoother results in reasonable agreement with the measurements. In the surfzone, no formulation is able to predict the decrease of the skewness.

Because the covocoïdal theory provides a comprehensive wave description, further tests considering the free surface elevation profile have also been performed. Measurements of wave crests and troughs on a 1 in 12 slope reported by Bowen et al (1968) have been simulated. The results are shown in figure3. The agreement is quite good in the outer shoaling region. Further inshore the wave crest elevation is rather overestimated and the trough underestimated. This deviation from the measurements increases in the surfzone.

Finally, a last test concerning the shape factor (Bo defined as the ratio of the standard deviation of the surface elevation over the wave height) has been carried out using the data from Stive (1984). A good prediction of this factor is essential in surfzone models in order to accurately compute the radiation stresses and the set-up. Figure 4-up indicates that before the breakpoint, the use of the covocoïdal theory can improve the computation of Bo compared to a classical second-order cnoidal theory. This is no more the case in the surfzone. Results are even worse in the case of a low-steepness wave (figure 4-down).

b) Random wave tests

Results obtained with monochromatic waves show that both approaches can be used before the breaking point but not in the surfzone. If we now consider random waves which generate wider surfzones, it implies that a probabilistic approach considering the joint distribution of wave heights and periods to compute the velocity distribution will not work properly in the nearshore. This is the reason why an alternative approach to estimate the velocity moments in random waves is considered here. It consists of applying the two formulations with a representative wave height and period. Flume data at prototype scale collected in the Delta flume in the Netherlands (Arcilla et al., 1994) have been used to test that approach. Figure 5 presents the computation of the second order and the third order velocity moments (high frequency part only) obtained by using the root-mean-square wave height and the peak period of the waves. In the first case (test2A) the bottom slope is monotonic but in the second (test 1C) a bar has developed which explains the presence of two peaks. Furthermore, the low steepness of the waves in the second case makes it a more difficult case to simulate: a signficant underestimation of the third order velocity moment is clearly visible in figure5-down.

5. Conclusions

The validation tests of the two formulations indicate that the Isobe and Horikawa (1982) formulation can be used with confidence in a large range of wave conditions except inside the surfzone when monochromatic waves are considered. The covocoïdal theory provides a more comprehensive description of the waves properties but abnormal results have been noticed in a few cases. This is probably due to the limitations of the parametrization which is rather complex and should be carefully checked at both limits: « deep »-water and breaking point. The accuracy of the prediction is generally reasonable. More tests with random waves are needed to confirm the choice of the root-mean-square wave height as the most suitable input.

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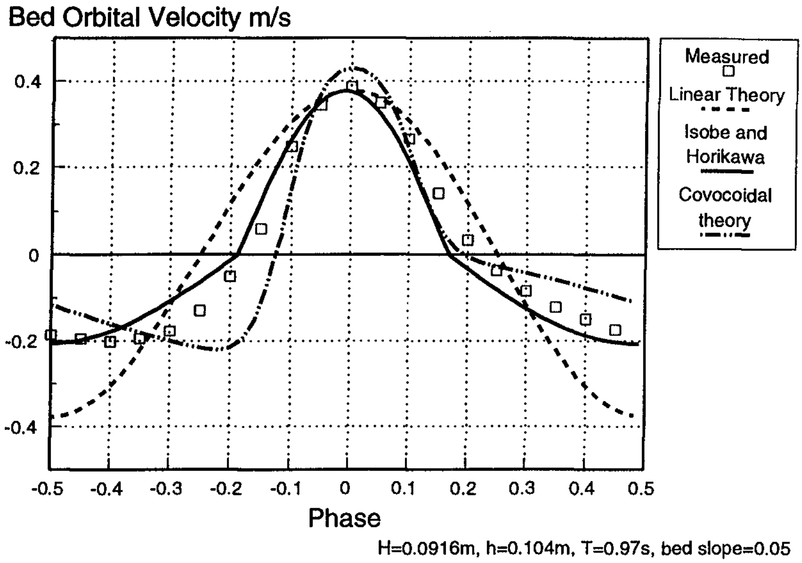


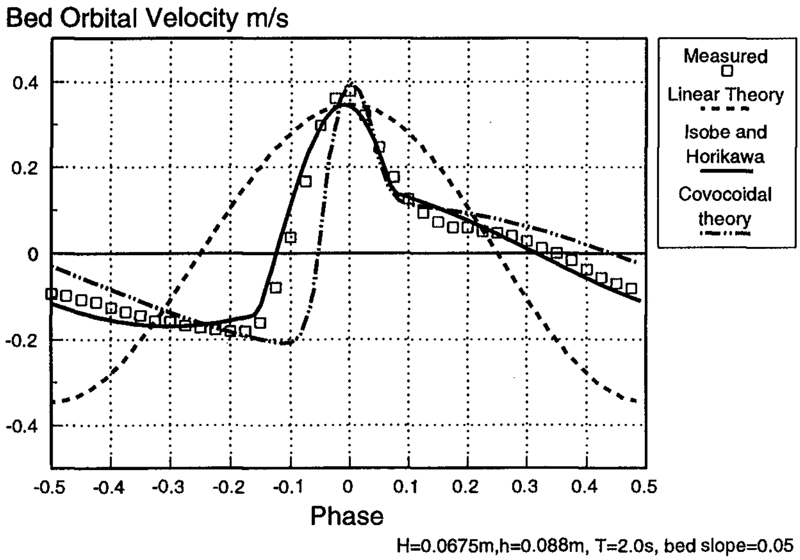


Figure 1. Validation test against bottom velocity measurements of Isobe and Horikawa (1982)
up: H = 0.0916 m, T = 0.97 s, h = 0.104 m, bed slope = 0.05
down: H = 0.0675 m, T = 2.0 s, h = 0.088 m, bed slope = 0.05

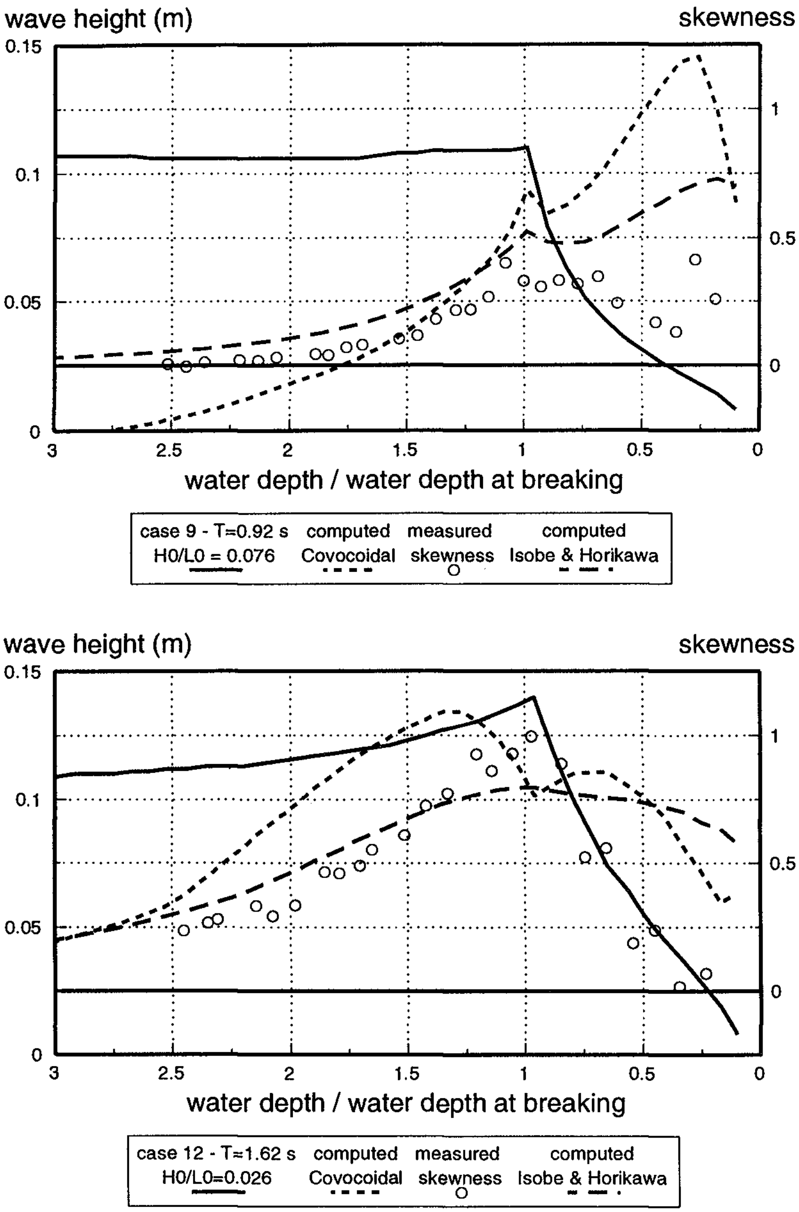


Figure 2. Validation test against bottom velocity skewness measurements of Nadaoka and Kondoh (1982)
up: case 9 - $H_0 = 0.10$ m, T = 0.92 s, bed slope = 0.05
down: case 12 - $H_0 = 0.108$ m, T = 1.62 s, bed slope = 0.05

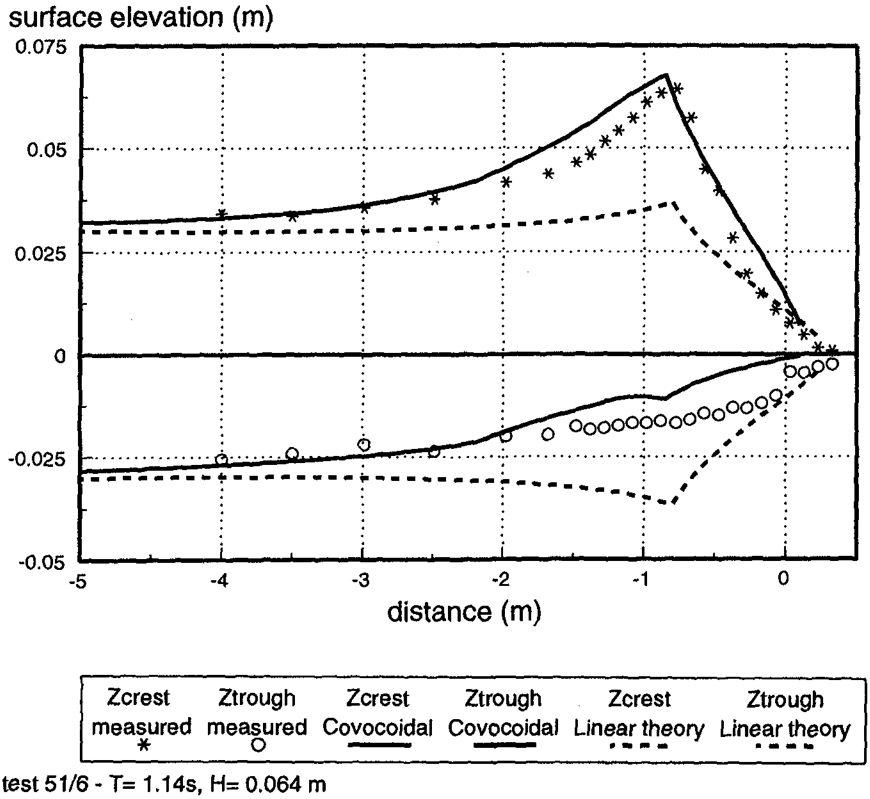
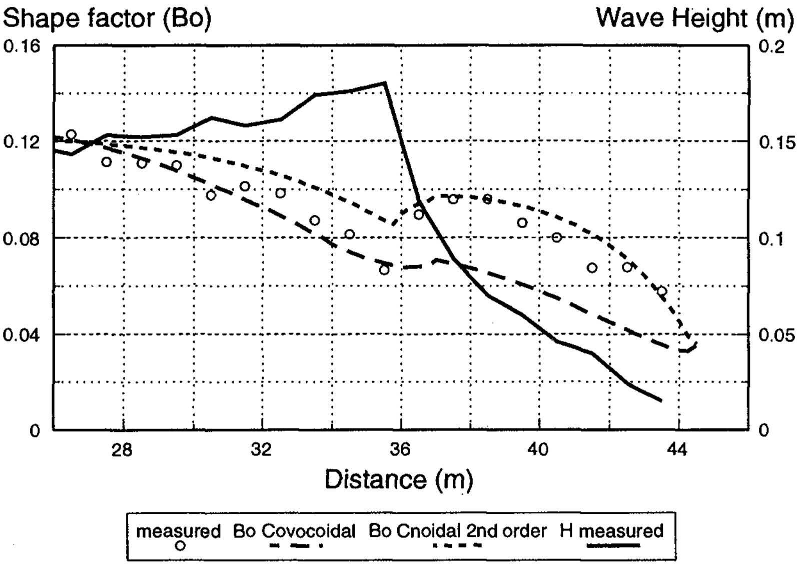
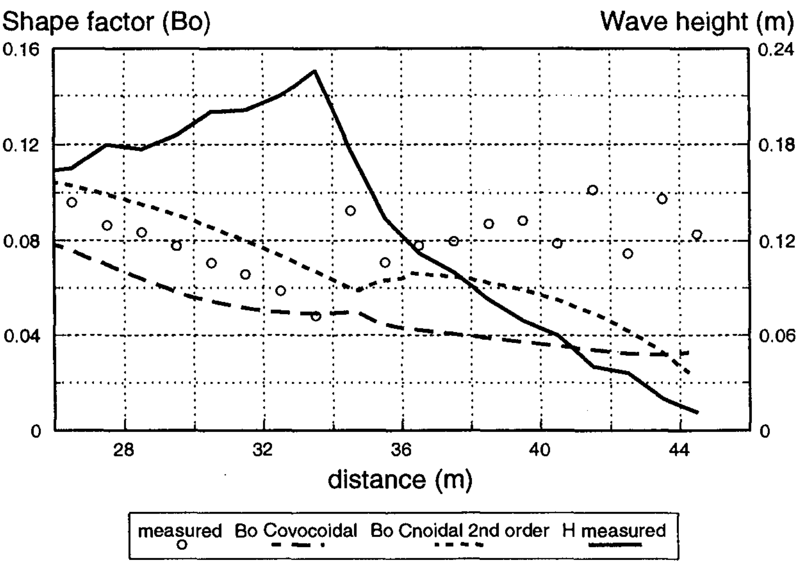


Figure 3. Validation test against surface elevation measurements of Bowen et al (1968)
test 51/6 - H_{inc} = 0.0645 m, T = 1.14 s, bed slope = 0.083

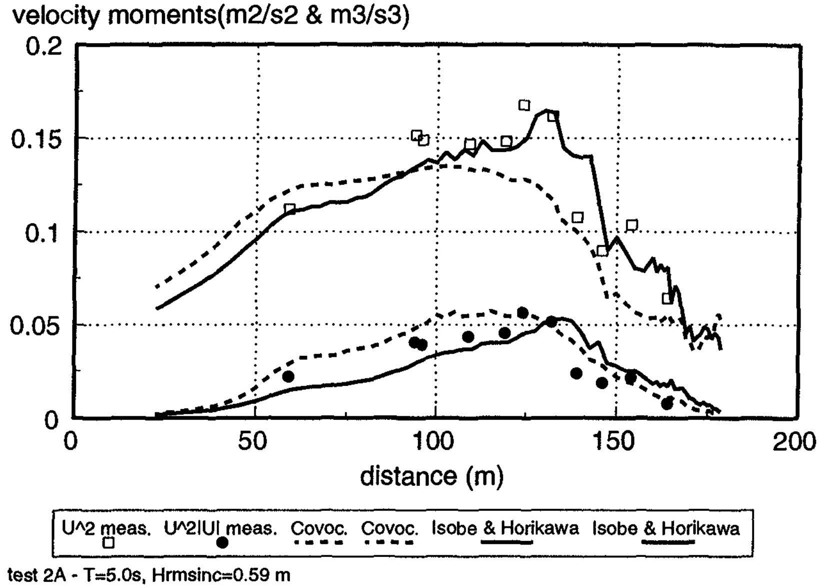


test 1 - T=1.79 s, H=0.145 m



test 2 - T=3.0 s, Hinc=0.145 m

Figure 4. Validation test against surface elevation shape factor measurements of Stive (1984)
up: test 1 - $H_{inc.}$ = 0.145 m, T = 1.79 s, bed slope = 0.025
down: test 2 - $H_{inc.}$ = 0.145 m, T = 3.0 s, bed slope = 0.025



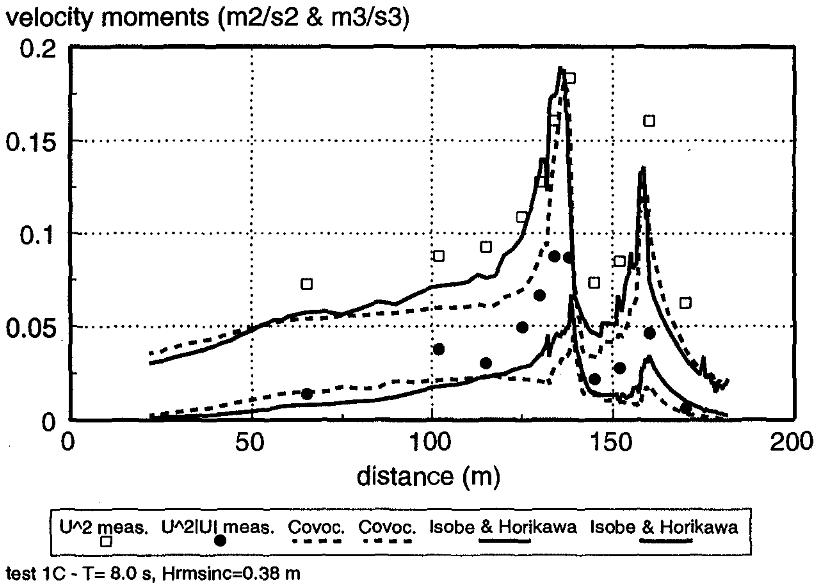


Figure 5. Validation test with random waves against Delta-Flume '93 experiments (Arcilla et al., 1994)
up: test 2A - $H_{sinc.} = 0.87$ m, $T_p = 5$ s, mean bed slope = 0.018
down: test 1C - $H_{sinc.} = 0.60$ m, $T_p = 8$ s, barred beach profile