

CHAPTER 77

Wave Runup on Beaches

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Abstract

Runup on beaches is an important topic in coastal engineering because it defines the upper limit of direct wave influence on land. It is also a phenomenon that has proved very difficult to analyze and characterize in a quantitative manner. The reason for this difficulty is the more general problem of understanding the response of beaches to waves. This paper develops formulas to estimate the approximate upper limit of wave runup on sand and gravel beaches. The relationship between these formulas and recent progress in quantifying beach profile response to waves and the morphodynamics of beaches is discussed. It was found that estimates of wave runup on sand beaches could be improved if sediment sizes in both swash zone and surf zone are known.

Introduction

This paper develops equations to predict the approximate upper limit of wave runup on sand and gravel beaches and shows the connection between runup and beach morphology. Runup on beaches is an important topic in coastal engineering because it defines the limit of direct wave influence on land. Specifically, these concerns relate to beach and dune erosion and coastal flooding.

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Background and Approach

A logical starting point for the analysis of wave runup is Hunt's (1959) formula, i.e.\

$$R/H = C[\tan\theta/\sqrt{(H/L_o)}] = C\xi \quad \text{Eq. 1}$$

where R is the vertical limit of wave uprush above the still water level, H is the incident wave height, L_o is the deep water wave length, $\tan\theta$ is the tangent of the slope of the structure or beach, C is a dimensionless coefficient, ξ is the Iribarron Number (Battjes 1974) and R/H is referred to as relative runup. Hunt found that Eq.1 worked very well for monochromatic breaking wave conditions on plane, smooth slopes. Subsequent research has confirmed the usefulness of Eq.1 for irregular wave conditions and rough slopes (van der Meer and Stam 1992).

Eq. 1 presents some problems for use with beaches. It is not clear if the submerged beach slope that the wave is propagating over or the beach face slope should be used in Eq. 1. The submerged slope presents the additional problem of being dynamic and not being plane, even in an approximate way. To circumvent the problem, beach slope can be treated as a dependent variable and Eq. 1 can be rewritten as:

$$R_2/\sqrt{(H_{so}L_o)} = f(X_1, X_2, \text{etc.}) \quad \text{Eq. 2}$$

where R_2 is the elevation above the still water level exceeded by two percent of the runups and H_{so} is the deep water significant wave height. The left hand side of Eq. 2 is referred to as the runup intensity and the equation indicates that it is a function of one or possibly more variables that act approximately as surrogates for beach slope. Recent research, including this study, suggests some possible simple variables for predicting beach slope, they are:

$$N_o = H_{so}/wT \text{ -----> beach face slope, Kriebel, et al. (1991), and}$$

$$F_o = w/\sqrt{(gH_{so})} \text{ -----> beach slope in surf zone, Dean (1991) and appendix,}$$

where T is a characteristic wave period, g is the acceleration of gravity and w is the sediment fall velocity. H_{so}/wT is a fall speed parameter sometimes referred to as the Dean Number and $w/\sqrt{(gH_{so})}$ is a surf zone Froude-type number, Kraus, et. al (1991). Equations developed in this study are formulated in terms of deep water wave conditions because of the wider range of applicability and utility for users.

The approach sketched out above converges with findings from a very extensive study of beach profiles and beach evolution that show erosion/accretion profiles, bar size, bar depth, and bar movement are all

functions of wave steepness and the fall speed parameter, Larson and Kraus (1989, 1992). There is also convergence with research on beach morphology by Wright and Short (1984). Wright et al. (1985) shows that reflective beaches are associated with small values of the fall speed parameter, dissipative beaches are associated with large values and beaches with a variety of bar and trough configurations are associated with intermediate values. Wright also notes that runup is high on reflective beaches and low on dissipative beaches.

Sources of Data

Data on the height of the berm crest of gravel beaches was collected in the laboratory by van Hijum and Pilarczyk (1982) and Ward and Ahrens (1992). The primary source of the two percent runup was Nielsen and Hanslow (1991), collected on six sand beaches in New South Wales, Australia. Additional two percent runup data was collected on the sand beach at the Field Research Facility of the Corps of Engineers at Duck, North Carolina, Holman (1986) and Douglass (1990).

Analysis and Development of Equations

Swash Zone Fall Speed Parameter

Fig. 1 shows the runup intensity versus fall speed parameter calculated using the fall speed for sediment in the swash zone. The data shown is from the two laboratory studies of gravel beaches, van Hijum and Pilarczyk (1982) and Ward and Ahrens (1992), and the six sand beaches in New South Wales, Nielsen and Hanslow (1991). From a morphological perspective the gravel beaches can be regarded as adsorbitive beaches and the sand beaches fall into the reflective, dissipative, or intermediate categories of Wright and Short (1984). For the gravel beaches the two percent runup has been assumed to be equal to the berm crest height. This assumption is supported by laboratory and field research of Powell (1988), who found the berm crest formed after 3000 waves was overtopped by less than 3% of the waves, with a mean probability of being overtopped of 0.015 ± 0.011 . A data trend curve is shown which is given by the equation

$$R_2/\sqrt{H_{s0}L_0} = 0.27\exp[-0.26(H_{s0}/d)]/(1.0+6.3\exp[-5.6/N_0]) \quad \text{Eq. 3}$$

where, d , is the depth of water at the toe of the gravel beach. The trend curve in Fig. 1 does not fit the gravel beach data well, i.e. $N_0 < 1$, because the figure does not account for the influence of relative wave height which is assumed to be zero, i.e. $H_{s0}/d = 0$, for plotting in Fig. 1.

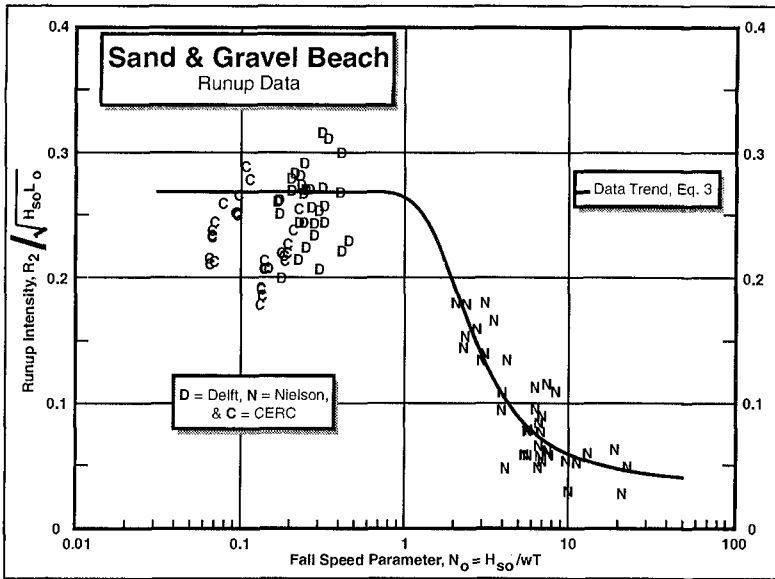


Fig. 1 Runup intensity as a function of the swash zone fall speed parameter, for three studies.

Fig. 2 shows the runup intensity versus the relative wave height for just the gravel beach data. The figure shows that runup intensity decreases as relative wave height increases. This trend is interpreted as being due to the truncation of the wave height distribution in shallow water; it is the larger waves which build and maintain the berm crest. Eq. 3 follows this trend reasonably well, which is more pronounced for CERC data, Ward and Ahrens (1992), because of the relatively shallow water at the toe of the gravel beach in that study as compared to the Delft data, van Hijum and Pilarczyk (1982).

Eq. 3 approaches a limit of 0.27 for runup intensity on gravel beaches which appears logical from Fig. 2, albeit there is considerable data scatter. For dissipative beaches the limiting value of Eq. 3 for runup intensity is 0.037 which seems reasonable from Fig. 1 and is consistent with a limiting value of 0.035 found by Yamamoto, et al. (1994). The analysis based on the swash zone fall speed parameter and relative wave height worked moderately well, but has at least two limitations for sand beaches: 1.) It is not clear what is the meaning of the water depth, d , (possibly the depth over a bar ?) and 2.) The lack of information about the surf zone. In order to predict runup on a sand beach in terms of deep water wave conditions, clearly some information about what is going on between the two locations is required.

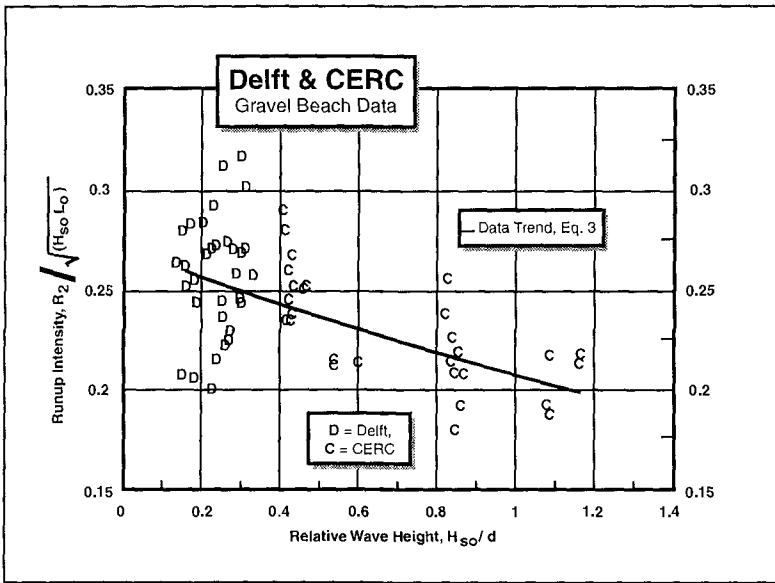


Fig. 2 Runup intensity as a function of relative wave height, for gravel beaches.

Surf Zone Froude Number

At the beginning of this study the authors did not know the sediment characteristics in the surf zone of the New South Wales beaches, Nielsen and Hanslow (1991). After receiving this information (Nielsen 1996) the authors realized that information about the surf zone was probably more important for predicting sand beach runup than swash zone data. The gravel beach studies (van Hijum and Pilarczyk 1982, Ward and Ahrens 1992) were collected in the laboratory with fixed beds offshore so no surf zone analysis was conducted with this data.

The surf zone Froude Number, F_o , addresses limitations noted in the above section in using the swash zone fall speed parameter to predict wave runup on sand beaches. Fig. 3 shows there is a surprisingly strong linear relationship between runup intensity and F_o on sand beaches, Nielsen and Hanslow (1991). Regression analysis was used to quantify the data trend shown in Fig. 3, i.e.,

$$R_2 / \sqrt{(H_{s0} L_0)} = 11.6 [w_{sr} / \sqrt{(g H_{s0})}] = 11.6 F_o \quad \text{Eq. 4}$$

As shown in the appendix F_o is proportional to the submerged beach slope in the surf zone. An attempt was made to use the information given in Nielsen and Hanslow to classify beaches using categories proposed by Wright and Short (1984) and shown by symbols in Fig. 3.

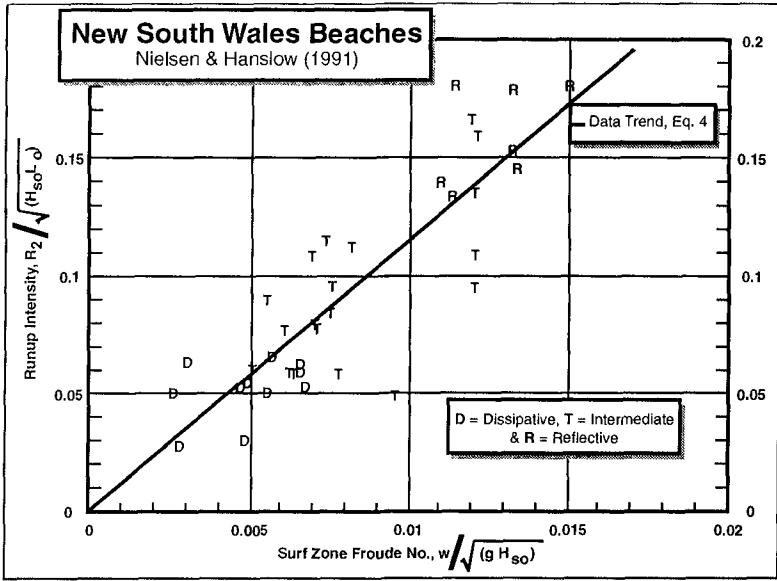


Fig. 3 Runup intensity as a function of surf zone Froude Number, for sand beaches in New South Wales.

Beach Diversity Correction

In examining the New South Wales data (Nielsen and Hanslow 1991) it was observed that there was a tendency for runup intensity to increase with increasing diversity in the sediment sizes on the beach profile. Beach diversity is defined as the ratio of median sediment size in the swash zone, d_{sw} , to median size in the surf zone, d_{sr} . Typically this ratio is greater than one and for the New South Wales beaches it was in the range, $1.0 \leq d_{sw}/d_{sr} \leq 1.6$.

Considering the influence of the surf zone Froude No. and beach diversity on runup intensity, regression analysis was used to help develop the following prediction equation for sand beaches:

$$R_2 / \sqrt{(H_{s0}L_0)} = 10.4 \sqrt{(d_{sw}/d_{sr})} [w_{sr} / \sqrt{(gH_{s0})}] \tag{Eq. 5}$$

The right hand side of Eq. 5 can be thought of as the $C[\tan\theta]$ terms in Hunt's Eq., Eq. 1, in the sense of a compound slope approach to predicting beach runup. Median diameter of the swash zone is used in Eq. 5 rather than fall speed of the sediment because beach permeability is so strongly dependent on sediment size. Permeability and wave conditions are the most important variables influencing the slope of the beach face. Therefore, the form of Eq. 5 helps identify the physical processes influencing runup even though it may seem anomalous to use sediment size and fall speed in the same equation. Eq. 5 explains 77.3 %

of the variance in the dependent variable compared to 74.6 % for Eq. 4.

For wave runup the value of a prediction method can best be judged by how well it predicts the dimensional runup rather than a dimensionless runup parameter. Fig. 4 shows the predicted two percent runup using Eq. 5 versus the observed two percent runup. Eq. 5 explains about 80% of the variance in the data.

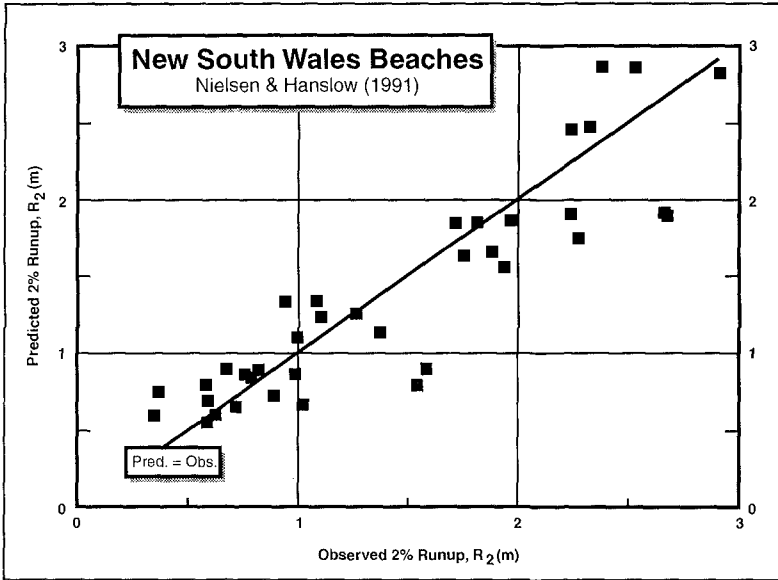


Fig. 4 Predicted two percent runup versus the observed two percent runup for sand beaches in New South Wales.

Thought Experiment

It is useful to use a thought experiment approach to illustrate the implications of Eq. 5. Imagine two beaches both having fine sand in the surf zone but one also having fine sand in the swash zone, i.e. a uniform beach, and the other having coarse sand in the swash zone, i.e. a diverse beach. Consider the response of the two beaches to both mild and storm wave conditions. For the same wave conditions the runup intensity is always greater on the beach with coarse sand in the swash zone. In going from mild to storm wave conditions runup intensity decreases on both beaches because the slope of the surf zone gets flatter or the surf zone gets wider. Interestingly, the difference in runup intensity between the two beaches is less during storm conditions because the increased width of the surf zone has made runup intensity less sensitive to conditions on the beach face, see thought experiment sketch, Fig. 5. The effects noted in the experiment seem consistent with current understanding of runup on beaches.

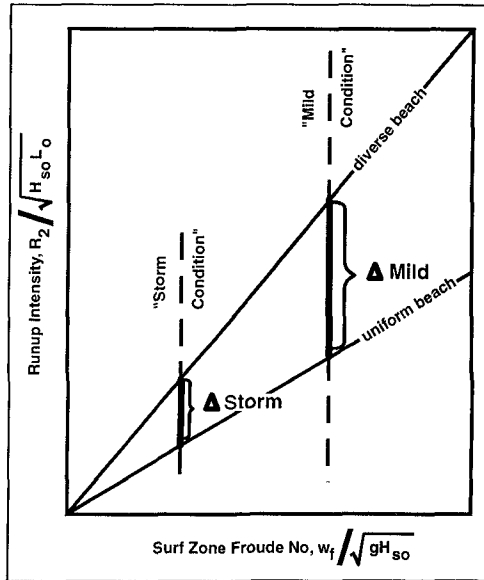


Fig. 5 Thought experiment figure to illustrate implications of runup equation, Eq. 5.

Duck 1982 Data

During the initial stages of the analysis the runup data from the beach at Duck, N.C. seemed to be anomalously high (Holman 1986 and Douglass 1990). However, after seeing the influence of sediment diversity the Duck data looked more logical. The beach at Duck has sediment with a median diameter of about 0.75mm in the swash zone and about 0.20mm in the surf zone (Mason et al. 1984) for a diversity ratio of 3.75, much higher than any of the New South Wales beaches.

Fig. 6 shows the predicted two percent runup versus the observed two percent runup for the data collected on the beach at Duck, N.C. Some data has been omitted to eliminate bimodal spectra using the approach of Resio (1987). Eq. 5 explains about 51% of the variance in the data.

There is relatively high alongshore variability in runup on the beach at Duck. This is what would be expected based on the morphological categorization of Wright and Short (1984) which puts the beach in intermediated categories that are dominated by a variety of offshore bars and troughs. Normally some of these features are three dimensional. There is one particularly long subset of data from Duck that shows runup variability along 500m of beach for storm wave conditions. For 13 stations the mean two percent runup was 2.163m with

a standard deviation of 0.215m or a mean percent deviation of about 10%. Conditions were unusual for an East Coast storm with $H_{s0} = 2.51\text{m}$ and $T_p = 12.2\text{sec}$. Eq. 5 gives a predicted $R_2 = 2.26\text{m}$ which is somewhat higher than the observed mean value of the two percent runup but well within one standard deviation of the mean.

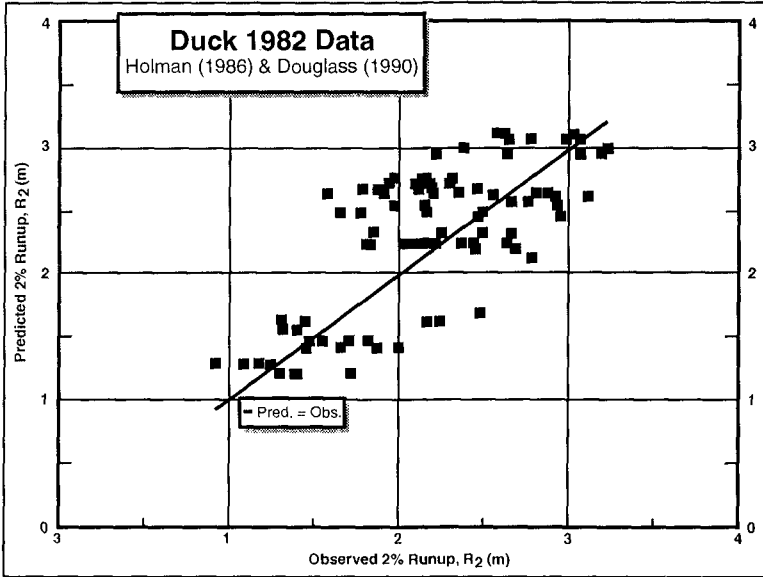


Fig. 6 Predicted two percent runup versus the observed two percent runup for the sand beach at Duck, N.C.

Surf Zone Fall Speed Parameter and Morphological Connection

Surprisingly, Eq. 5 can be rewritten in a form which helps suggest the processes involved and using the surf zone fall speed parameter provides a connection to beach morphology, i.e.

$$\begin{aligned}
 R_2/H_{s0} &= C[\tan\theta]/\sqrt{(H_{s0}/L_0)} \\
 &= 10.4\sqrt{(d_{sw}/d_{sr})}[w_{sr}\sqrt{(gH_{s0})}]/\sqrt{(H_{s0}/L_0)} \\
 &\quad \begin{array}{c} <-----> <-----> <-----> \\ \text{swash} & \text{surf} & \text{deep-water} \end{array} \\
 &= \{10.4/\sqrt{(2\pi)}\}\sqrt{(d_{sw}/d_{sr})}/N_0 = 4.1\sqrt{(d_{sw}/d_{sr})}/N_0
 \end{aligned}
 \tag{Eq. 6}$$

The fall speed parameter, N_o , used in the final line of Eq. 6 is similar to the parameter used by Wright and Short (1984) to categorize the morphology of beaches. The difference between the parameters is that Wright and Short used the breaker height while the deep water significant height is used in this study.

Eq. 6 also provides a link to the beach profile study of Kraus, et al. (1991) who found that erosional or accretionary profiles could be predicted using $w_{sr}/\sqrt{(gH_{s0})}$ and H_{s0}/L_0 . Since these two variables can be used to form a surrogate for the surf similarity parameter or Iribarron No., i.e. $\xi \approx 3.4 [w_{sr}/\sqrt{(gH_{s0})}] / \sqrt{(H_{s0}/L_0)} = 1.3/N_o$, they should be able to predict breaker characteristics near the shoreline.

In Fig. 7 the relative runup is shown as a function of the surf zone fall speed parameter, as suggested by Eq. 6. Data points are shown in Fig. 7 using letters to distinguish morphopological conditions of dissipative, reflective, or intermediate beaches as defined by Wright and Short (1984). The figure shows that both relative runup and beach morphology are strongly correlated to the surf zone fall velocity parameter.

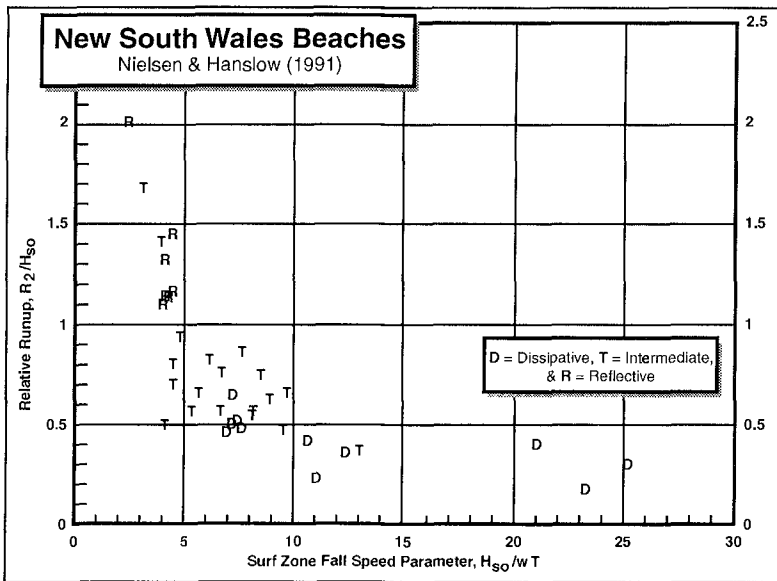


Fig. 7 Relative runup as a function of the surf zone fall speed parameter for sand beaches in New South Wales.

Surf Beat and Edge Waves

A variety of oscillations are present in the surf zone at periods greater than the period of incident wind waves. These long waves tend to modulate the conspicuous runup oscillations occurring at approximately the period of wind waves. Trapped edge waves at periods twice the incident wind waves are responsible for cusp formations on reflective beaches, Guza and Inman (1975). As morphological conditions change from reflective to dissipative beaches there is a corresponding attenuation of trapped edge waves and amplification of surf beat. Surf beat periods typically are in the range of one to three minutes.

Summary, Conclusions and Recommendations

Starting with Hunt's (1959) equation and replacing the beach slope with functional relationships, which provide an equivalent compound slope that includes both the slope of the beach in the surf zone and the slope of the beach face, produces an equation, Eq. 6, to calculate the approximate upper limit of runup on a beach. Eq. 6 makes reasonably good estimates of runup for a variety of beach and wave conditions and helps to show the connection between beach morphology and wave runup.

It was found that when estimating runup in terms of deep water wave conditions that the slope of the beach in the surf zone or the width of the surf zone, was generally more important than the slope of the beach face. In Eq.6 the beach face or swash zone characteristics are treated as a correction to the basic runup equation, Eq. 4. This correction is important when there is a big difference between the sediment size in the surf and swash zone.

This research shows that studies of beach morphology and beach runup should be coordinated. Data requirements include, but are not limited to: knowledge of the sediment size across the beach profile, reliable estimates of deep water wave conditions, the three dimensional characteristics of the submerged beach, and the alongshore variation in runup. In addition, a time history of beach states and wave characteristics as discussed by Wright et al. (1985) would almost certainly help make better estimates of beach runup.

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Appendix: Slope of Beach in Surf Zone

A standard beach equilibrium profile is given by:

$$h = Ax^{2/3} \text{ or } x = (h/A)^{3/2}, \quad \text{Dean (1991)}$$

therefore the tangent of the submerged slope from the still water line out to some depth, h , is given by $\tan\theta = h/x = A^{3/2}/\sqrt[3]{h}$,

and since $A = 2.25(w_f^2/g)^{1/3}$, good in the range $1.0 \leq w_f \leq 10$ cm/sec. and temperatures around 20°C, Kriebel, Kraus & Larson (1991), then

$$\tan\theta = 3.375w_f/\sqrt[3]{gh},$$

and if h is set equal to H_{s0} as suggested for biplaner slopes or composite slope analysis, de Waal and van der Meer (1992), then

$$\tan\theta = 3.375w_f/\sqrt[3]{gH_{s0}} = 3.375F_o.$$

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