CHAPTER 88

Fast methods for computing the shoaling of nonlinear waves

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Abstract

Accurate nonlinear numerical methods for wave propagation have existed for some years. Most of these are very demanding of computer resources as they use global means of approximation which usually requires the costly solution of a full matrix equation at each time step. It is the aim of the present paper to describe and to compare the features of two new methods for the two-dimensional propagation of nonlinear waves over varying topography. A method based on local polynomial approximation is presented, which was found to be efficient, cheap and accurate. A novel boundary integral method is also presented, which was capable of good accuracy even for waves which overturned. For practical purposes, the local polynomial approximation method is to be preferred and may have some useful contributions to make.

Introduction

In computing nonlinear wave evolution over topography, there has always been a trade-off between accuracy and efficiency. On one hand, boundary integral equation methods (BIEM) have been able to perform accurate potential flow calculations past the point of overturning, but their computational expense has traditionally been very high. On the other hand, Boussinesq-type methods have had a much smaller computational cost, especially for large domains, but are limited to mildly nonlinear, mildly dispersive waves. Recently, there have been many attempts both to increase the efficiency of BIEM (Wang et al., 1995, de Haas *et al.*, 1996) and to increase the accuracy of Boussinesq-type approximations (Madsen and Sørensen, 1992, Nwogu, 1993, Wei *et al.*, 1994) However, much room still remains for fast methods which can accurately predict wave evolution.

This paper presents details of and results from two new potential flow methods which have not yet been widely published, but combine excellent accuracy with a reasonable

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computational cost. The first is a finite depth technique which assumes that the flow field is represented locally by polynomial variation. Two variants are presented: one has excellent accuracy for highly nonlinear waves, while the other has greater efficiency at the cost of some accuracy for nonlinear waves. Both variants have a computational cost which is linearly proportional to the number of computational points, which makes them suitable for computations over large domains. The second method presented here is a BIEM which is formulated differently from those currently in use. This new form is simpler, potentially more accurate, and allows for the use of faster solution techniques than are currently standard. It is not as robust as the local polynomial approximation (LPA) method presented and does take longer to run, but unlike that method it can describe wave overturning. As with all boundary integral techniques, accuracy remains good up to overturning.

Governing Equations

For irrotational flow a velocity potential $\phi(x, y, t)$ exists such that the fluid velocity vector $(u, v) = (\partial\phi/\partial x, \partial\phi/\partial y)$, restricting consideration here to two dimensions. If the fluid is incompressible, the potential satisfies Laplace's equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0. \tag{1}$$

At any time t, this elliptic equation is governed by the conditions on the domain boundary. At all points along the free surface specified by $y = \eta(x, t)$:

$$\phi = \phi_s(x, t), \tag{2}$$

where ϕ_s is known. The kinematic boundary condition on the bed is

$$\frac{\partial\phi}{\partial y} - \frac{\partial h}{\partial x}\frac{\partial\phi}{\partial x} = 0 \quad \text{on} \quad y = h, \tag{3}$$

where $h(x)$ is the bed elevation. Along the left and right boundaries, the horizontal velocity is set to

$$\frac{\partial\phi}{\partial x} = f(y). \tag{4}$$

These equations completely specify the velocity potential, and (1) may then be used to solve for the flow field. To advance the solution in time, the free surface kinematic boundary condition is used to advance the free surface elevation:

$$\frac{\partial\eta}{\partial t} = \frac{\partial\phi}{\partial y} - \frac{\partial\eta}{\partial x}\frac{\partial\phi}{\partial x} \quad \text{on} \quad y = \eta, \tag{5}$$

and the unsteady form of Bernoulli's equation, modified here to compute the rate of change of at a surface point, is used to advance the free surface velocity potential ϕ_s:

$$\frac{\partial\phi_s}{\partial t} = C - g\eta - \frac{1}{2}\left(\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2\right) + \frac{\partial\phi}{\partial y}\frac{\partial\eta}{\partial t} \quad \text{on} \quad y = \eta, \tag{6}$$

where $C(t)$ is constant throughout space.

Slightly different versions of these equations are generally used for BIEM which do not assume a single valued free surface, and use Lagrangian methods to track moving surface particles.

Local Polynomial Approximation Methods

In finite depths, the representation of a velocity potential locally by a polynomial can provide an excellent approximation to the flow field. Local polynomial approximation (LPA) methods for one dimension in plan use this principle to model wave evolution over varying topography, with very good results. A preliminary version has been given by Kennedy and Fenton (1995). Two main variants are summarised here: a fully nonlinear model which can provide highly accurate results, and an model which uses Taylor expansions about the undisturbed surface to increase speed, but which sacrifices some accuracy for high waves. For both methods, the expense of solution at each time step is directly proportional to the number of computational subdomains, which allows wave evolution to be computed over relatively large regions with a reasonable computational cost.

Solution of Laplace's Equation

For both the fully nonlinear and expansion LPA methods for one dimension in plan, the basic method of solution for Laplace's equation is very similar. As shown in Figure 1, the computational domain is divided into subdomains extending vertically from the free surface to the bed. In any typical subdomain, m, the velocity potential ϕ_m at any point (x_m, y) is represented by the polynomial

$$\phi_m(x_m, y, t) = \left[A_{0R} + \mathrm{Re}\left(\begin{array}{c} \left(\sum_{j=1}^{n-1} (A_{jR} + iA_{jI})(x_m + iy)^j\right) + \\ (x_m + iy)^n \begin{cases} A_{nR}, & n \text{ odd} \\ iA_{nI}, & n \text{ even} \end{cases} \end{array} \right) \right]_m \tag{7}$$

where n is an integer ≥ 3 which controls the level of approximation, $i = \sqrt{-1}$, $\mathrm{Re}(\ldots)$ means taking the real part. The A coefficients are functions of time. For any given n, it is these A coefficients which must be chosen to best satisfy the boundary value problem. Because of the complex formulation used, Equation (1) is identically satisfied. With the introduction of subdomains, two additional constraints are introduced: the velocity potential, ϕ, and its normal derivative, $\partial\phi/\partial x$, must be continuous across subdomain boundaries.

The velocity potential ϕ may be made analytically continuous across subdomain boundaries through a transformation of basis functions which, in addition, almost halves the number of independent coefficients. However, $\partial\phi/\partial x$ will still be discontinuous across boundaries. (Details of the transformation may be found in Kennedy and Fenton, 1995.) In a domain with M subdomains, the revised basis functions may now be though of as having n independent coefficients defined at each internal boundary between subdomains, plus n coefficients at each of the left and right global

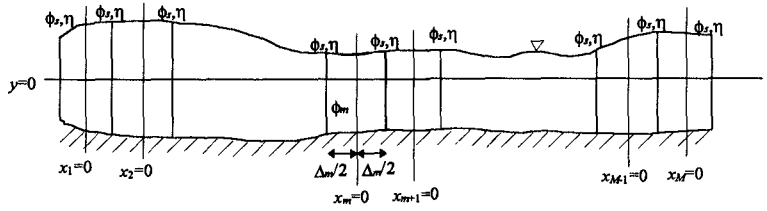

Figure 1: Definition sketch for local polynomial approximation

boundaries, for a total of $(M-1)n+2n=(M+1)n$ independent coefficients. The constraints on these are as follows. At each internal boundary between subdomains, the free surface velocity potential is set to the specified value, satisfying (2). Next, the bottom boundary condition (3) is imposed, using the average value of $\partial\phi/\partial x$ across the boundary. The remaining $n-2$ constraints at each internal boundary match the horizontal velocity, $\partial\phi/\partial x$, across the boundary at $n-2$ discrete points. For overall continuity, these collocation points are here set to the Gauss-Legendre points for level $N=n-2$, using the free surface (or still water level for the expansion method) and bed as limits. At each of the left and right global boundaries, (2) is also specified at the surface and (3) at the bed. However, instead of a velocity match as with the internal boundaries, the horizontal velocity at the boundary, $\partial\phi/\partial x$, is instead set to the known value at $n-2$ collocation points, satisfying (4).

All of these constraints result in a set of block banded linear equations. These may be solved using any banded or block banded matrix solver, both of which have a computational cost which is directly proportional to the number of subdomains, M. This allows for the computation of wave evolution over reasonably large areas without great expense. Traditional methods which use global approximation usually have a computational cost proportional to the second or third power of the number of computational points.

Linear Dispersion Characteristics

Here, as a test of the ability of polynomials to describe the flow field, we consider what results they give for the linear phase speed, compared with traditional approximation by periodic functions in x and hyperbolic functions in y. As subdomain lengths go to zero, a set of differential equations for the velocity potential results, which may be easily solved for the case of small amplitude waves over a level bed. Figure 2 shows the LPA small amplitude phase speed relative to the exact relationship for the levels $n=3,4,5,7$, with collocation points set to the Gauss-Legendre points for $N=n-2$. Accuracy for the level of approximation $n=3$ is poor in anything other than shallow water but increasing to $n=4$ gives usable small amplitude results past the nominal deep water limit of $kd=\pi$ $(L/d=2)$. The level of approximation $n=5$ (usually used with the LPA expansion method) has good dispersion characteristics even for

very short waves with $kd = 2\pi$ ($L/d = 1$), while with $n = 7$, phase speeds remain excellent past a dimensionless wavenumber of $kd = 3\pi$ ($L/d = 2/3$).

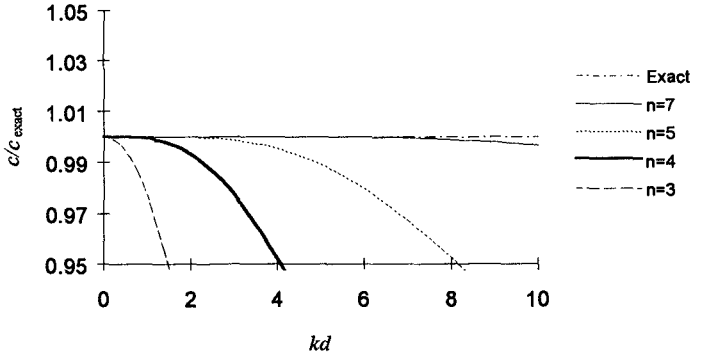

Figure 2: LPA linear phase speed

Fully Nonlinear LPA

The fully nonlinear version of LPA can provide excellent accuracy for finite depth potential flow calculations. There are two main steps: Laplace's equation is solved exactly as described earlier, and a time stepping method (here usually third or fourth order Adams-Bashforth) is then used to solve the evolution equations (5) and (6) to advance the solution to the next time step. If the Gauss-Legendre points of level $N = n - 2$ are used as collocation points, then the first $n - 3$ weighted moments of flow will be conserved between subdomains, as well as having velocity matches at the collocation points. For an accurate potential flow method, computations are also quite efficient. For a very large computational run with 900 subdomains and 4000 time steps, total run time for the level $n = 7$ would be about 3.5 hours on a Pentium 150 personal computer. Figure 3 shows the shoaling of a solitary wave of initial height $H/d = 0.15$ as it propagates onto a shelf of depth $0.5d$. The classical fissioning into multiple solitons is clearly evident, with the leading wave reaching a final dimensionless height on the shelf of 0.507. As an independent estimate of computational accuracy, relative energy fluctuations were less than 2×10^{-4}.

LPA Free Surface Expansion Method

The free surface expansion method is somewhat more complex, with two major differences from the fully nonlinear version. The first difference is that, instead of solving Laplace's equation using the free surface and the bed as limits, it is instead solved between the still water level and the bed. The mode coupling free surface expansion of Dommermuth and Yue (1987) is then used to relate the value of ϕ at the free surface to the value of ϕ at the still water level. The order of expansion may be easily changed to accommodate the level of nonlinearity of the problem considered.

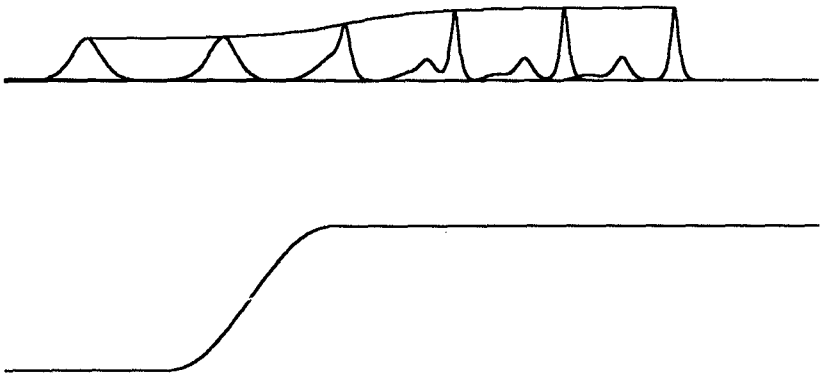

Figure 3: Results from the LPA method for the propagation of a wave over a shelf of depth 50%

Accuracy is still, of course, less than the fully nonlinear version for higher waves, but there is one major advantage: since the upper limit of the computational domain remains constant through time, a matrix equation must only be filled and decomposed once, rather than at each time step as with fully nonlinear LPA. This decomposed matrix is then solved with different right hand sides at each time step, which is much faster.

It is worthwhile to implement the second major change only if the computational domain is invariant with time, as is the case here. This involves another change of basis functions, so that there is only one independent variable per computational point. Details of this transformation may be found in Kennedy (1997). With the new basis functions, all conditions but (2) are automatically satisfied, so this constraint is used at every computational point to generate a new set of linear matrix equations for the LPA solution to Laplace's equation. The new matrix is purely banded and has both fewer variables and a smaller bandwidth than with the previous basis functions. Computational speeds are therefore further increased. A reasonable analogy may be made between the new basis functions and B-splines, as both are piecewise continuous polynomials which use a set of interpolation conditions to reduce the number of independent computational variables to one per computational point. These new basis functions could also be computed for the fully nonlinear version, but to retain full accuracy, they would have to be recomputed at each time step as the free surface moves. This would slow down computations, which is why they were not used.

The two parameters which control the accuracy of the LPA expansion method are the degree of the polynomials, n, and the order of free surface expansion, Q. While the fully nonlinear LPA was developed to calculate potential flow problems with very high accuracy, the LPA expansion method is viewed as a tool for more practical problems. To give reasonable nonlinear accuracy, which is mostly controlled by the order of free surface expansion, Q, and good frequency dispersion, which is only affected by the LPA level, n, the parameters $Q = 3$ and $n = 5$ were generally used.

For an example of the capabilities of the method, computations here will be compared with the experimental results of Beji and Battjes (1993) as reported by Ohyama *et al.* (1994). In this experiment regular waves were propagated over a two dimensional bar-trough setup and time series of surface elevations were taken at various points. Two wave trains were considered – both were initially of reasonably small amplitude in intermediate depths, but became significantly nonlinear over the bar.

The first wave train had an initial height of $H_0/d = 0.05$ and a period of $T\sqrt{g/d} = 9.903$. Figure 4 shows a comparison between computed and experimental values at Stations 1, 3, 5 and 7, which are, respectively, just before the bar, on the bar crest, on the downslope and in the trough. Agreement is quite good, with the LPA expansion model accurately predicting the steepening of the wave as it progresses up and sheds secondary waves on the bar, and its decomposition into higher harmonics on the downslope. As the wave progresses, computations begin to overestimate wave heights slightly due to the lack of dissipation in the model, and a small phase difference appears. However, similar differences were also noted in the fully nonlinear boundary element computations of Ohyama et al. (1994).

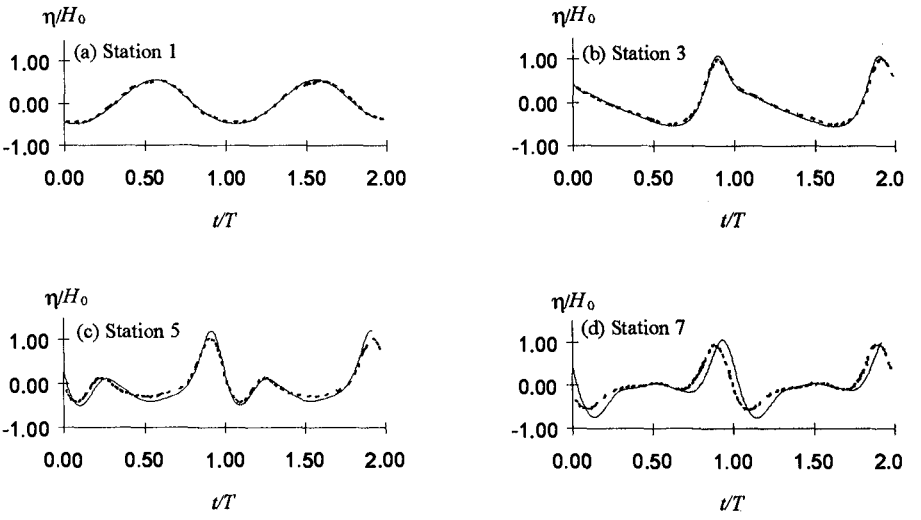

Figure 4: Measured and computed time series, $H_0/d = 0.05$, $T\sqrt{g/d} = 9.903$.

Figure 5 shows results for the second wave train which was shorter, with a period of $T\sqrt{g/d} = 6.189$ and an initial height of $H_0/d = 0.0625$. Due to its shorter length, this wave did not evolve as much passing over the bar, but the model still predicts the features of its evolution well. A small phase lag is visible at the last station. The fully nonlinear results of Ohyama *et al.* (1994) were similar to those here, although the phase lag at Station 7 was somewhat smaller. Overall, the expansion model predicts wave evolution quite well, and may be relied on to provide a good estimate of nonlinear wave evolution for a wide range of waves.

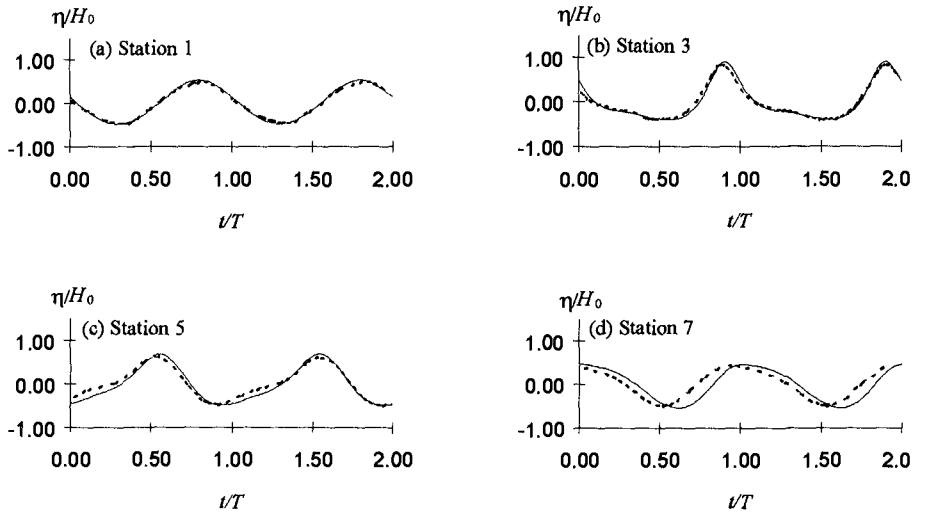

Figure 5: Measured and computed time series, $H/d = 0.0625$, $T\sqrt{g/d} = 6.189$. Solid lines – computations, dashed lines – experiment

Computations with the expansion method are quite fast. For example a computation with 900 subdomains and 4000 time steps would have a total run time on a Pentium 150 personal computer of about 10 minutes for the LPA level $n = 5$ and expansion level $Q = 3$, compared with 3.5 hours for the fully nonlinear version using $n = 7$.

A new boundary integral equation method

Introduction

The approach of using boundary integral equations for the study of waves in deep water was initiated by Longuet-Higgins and Cokelet (1976), who set up and solved a boundary integral equation with a logarithmic kernel. A different approach was introduced by Vinje and Brevig (1981), who used the Cauchy integral theorem in terms of a complex potential function. A number of powerful results have been

obtained in recent years for waves over topography, including Dold & Peregrine (1985), Grilli *et al.* (1989, 1994), and Subramanya & Grilli (1994). A brief history of various attempts using BIEM is described by Liu *et al.* (1992).

A quicker but less accurate approach was introduced by Leitão and Fernandes (1992), who took as the upper surface of the computational domain the undisturbed water surface, and used a second-order Taylor expansion there. As with the LPA expansion method described above, their computational domain was constant, and they only had to solve a matrix equation once rather than at each time step.

Fenton (1992 & 1996) has developed a method for the solution of Laplace's equation in two dimensions which has some advantages over traditional methods: it is simpler in theory and implementation, yet is more accurate. Of particular importance to the problem of shoaling waves, is that it allows the use of iterative methods for solution which are rapidly convergent because of the nature of the equations and because information from previous time steps can be incorporated. A preliminary study applying that method to shoaling waves has been published (Fenton, 1993), but in that work it was concluded that the method advocated, despite its high accuracy for fixed domains, was somewhat fragile for shoaling. Here, the method will be briefly described and the results of rather more robust computations will be described.

Theory

Consider a two-dimensional region such as that shown in Figure 1 containing an incompressible fluid which flows irrotationally, in which case a scalar potential function ϕ exists and satisfies Laplace's equation: $\nabla^2\phi = 0$. As ϕ is an harmonic function, another function ψ exists, related to ϕ by the Cauchy-Riemann equations: $\partial\phi/\partial x = \partial\psi/\partial y$ and $\partial\phi/\partial y = -\partial\psi/\partial x$. It can be shown that if these relations are satisfied, then the complex function $w = \phi + i\psi$, where $i = \sqrt{-1}$, has a unique derivative with respect to the complex variable $z = x + iy$, satisfies the integral equation

$$\oint \frac{w(z) - w(z_m)}{z - z_m}\, dz = 0, \tag{8}$$

for a reference point m. In this equation, unlike other formulations, the integrand is everywhere continuous, even at $z = z_m$, and its numerical approximation should be simpler and potentially more accurate. It will be shown below that using this form leads to a system of algebraic equations which are all nearly diagonally dominant, giving desirable numerical properties.

Boundary conditions: On the sea bed, assumed impermeable here, the condition that flow does not cross the boundary is $\psi = 0$. On the free surface, denoted by $y = \eta(x,t)$, the governing equations are nonlinear, partly because the location of the free surface also appears in them. There are two equations: one is the kinematic condition that the velocity of a particle on the surface is equal to the fluid velocity at that point. These have been described above. A slight difference here is that a Lagrangian description is used, like other BIEM, such that surface particles are followed and it is necessary to update the ϕ on the surface. This necessitates the

computation of the material derivative which can be shown to become

$$\frac{D\phi_m}{Dt} = -g\eta_m + \frac{1}{2}\left|\frac{dw}{dz}\right|_m^2, \tag{9}$$

a differential equation for ϕ_m at the free surface particle. This gives us a way of calculating ϕ as time evolves so that at any instant it is known at all points on the free surface, while we know that $\psi = 0$ on the sea bed. Hence we have enough boundary information to obtain a solution of equation (8) at each time step, namely to obtain the values of ϕ on the bottom and ψ on the free surface, so that $\omega = \phi + i\psi$ is known at all points, dw/dz can be calculated, the solution advanced, and so on.

Numerical scheme using periodicity around the contour

Around the boundary all variation is periodic, for in a second circumnavigation of the boundary the integrand is the same as in the first, and so on. This suggests the use of methods that exploit periodicity to gain handsomely in accuracy. A continuous co-ordinate j is introduced here, which is 0 at some reference point on the boundary, and after a complete circumnavigation of the boundary has a value N, which will be taken to be an integer. The integral in equation (8) can be written

$$\int_0^N \frac{w(z(j)) - w(z_m)}{z(j) - z_m}\frac{dz}{dj}\,dj = 0. \tag{10}$$

Now a numerical approximation is introduced to transform the integral equation into an algebraic one in terms of point values. The integral in equation (10) is replaced by the trapezoidal rule approximation:

$$\sum_{j=0}^{N-1} \frac{w(z_j) - w(z_m)}{z_j - z_m} z_j' = 0, \tag{11}$$

where $z_j = z(j)$ and $z_j' = dz(j)/dj$, but in which after the differentiation, j takes on only integer values. In this case the trapezoidal rule has reduced to the simple sum as the end contributions are from the same point, $z_0 = z_N$ because of the periodicity. This is a particularly simple scheme when compared with some such as Gaussian formulae which have been used to approximate boundary integrals. Where the integrand is periodic, as it is here, the trapezoidal rule is capable of very high accuracy indeed, a fact which is relatively little-known.

In the form of equation (11), the expression is not yet useful, as the point $j = m$ has to be considered. It is easily shown that in this limit, the integrand (and hence the summand) becomes $dw(m)/dm$, and extracting this term from the sum gives the expression with a "punctured sum" $j \neq m$:

$$\frac{dw}{dm}(m) + \sum_{j=0,\ j\neq m}^{N-1} \frac{w_j - w_m}{z_j - z_m} z_j' = 0, \tag{12}$$

for $m = 0,\ 1,\ 2, \ldots,\ N-1$, and where the obvious notation $w_j = w(j)$ etc. has been introduced. The notation $dw(m)/dm$ means differentiation with respect to the continuous variable m, evaluated at integer value m. It is convenient here to introduce the symbol Ω_{mj} for the geometric coefficients:

$$\Omega_{mj} = \alpha_{mj} + i\beta_{mj} = \frac{z'_j}{z_j - z_m}, \tag{13}$$

whose real and imaginary parts are the coefficients α_{mj} and β_{mj}. One is free to use either the real or imaginary part of the integral equation and of the sum which approximates it, equation (12). The two parts can be extracted to give

$$\frac{d\phi}{dm}(m) + \sum_{j=0,\ j\neq m}^{N-1} [\alpha_{mj}(\phi_j - \phi_m) - \beta_{mj}(\psi_j - \psi_m)] = 0 \tag{14}$$

and

$$\frac{d\psi}{dm}(m) + \sum_{j=0,\ j\neq m}^{N-1} [\alpha_{mj}(\psi_j - \psi_m) + \beta_{mj}(\phi_j - \phi_m)] = 0. \tag{15}$$

One of these equations can be used at each of the N computational points, provided either $d\phi/dm$ or $d\psi/dm$ is known that point, which can be done from the boundary conditions as described above. Each equation is written in terms of the $2N$ values of ϕ_j and ψ_j . If N of these are known, specified as boundary conditions, then there are enough linear algebraic equations and it should be possible to solve for all the remaining unknowns.

As equation (14) can be used on the free surface where $d\phi/dm$ can be evaluated and where ψ_m is the unknown and (15) on the sea bed where $d\psi/dm = 0$, and where ϕ_m is unknown, examination of the coefficients shows that the system of equations is nearly diagonally dominant, which suggests a certain computational robustness, and the possibility of iterative solution.

Distribution of computational points: The linear algebraic equations approximating the integral equations have been expressed relatively simply in terms of the coordinates of the computational points z_j and the derivative around the boundary, z'_j. The accuracy of the method depends on how continuous the latter are, and in Fenton (1992, 1996) some effort was spent in ensuring continuity across corners of the boundary. In fact it was found that even if no special spacing was used, the accuracy was still surprisingly high.

Numerical computation of coefficients: In problems of wave shoaling, the boundary of the computational region, including the sea bed and the free surface, is quite irregular. The periodicity around the boundary may be exploited to give a simple scheme for computing the necessary derivatives around the boundary. The main problem is to compute values of the z'_j. Also, it is convenient to be able to use a means of interpolation between the computational points for plotting purposes which

has the same accuracy as the underlying numerical method. Both can be accomplished simply and economically using Fourier approximation, once again exploiting periodicity as the boundary is traversed. If fast Fourier transform programs are available, the z'_j may be computed easily and accurately in $O(N \log N)$ operations, where there are N boundary points.

Set-up and solution of system of equations: When the z'_j have been obtained, the coefficients $\Omega_{mj} = \alpha_{mj} + i\beta_{mj}$ can be calculated and used in expressions (14) and (15), one for each point at which an unknown exists. As the equations are nearly diagonally dominant, however, it should be possible to exploit the simple Gauss-Seidel iterative procedure, particularly for timestepping problems such as those for wave propagation, and in practice this was found to work very well indeed. The computational effort is $O(N^2)$ per iteration, and the happy result was found in the present work, that as all boundary points are interpreted as Lagrangian particles, and carry the geometry of the problem with them, then the coefficients are very slowly varying, and a forward extrapolation of previous results gave such an accurate initial estimate that typically only five iterations were necessary each time step. Much programming detail can be avoided if the step of assembling into a matrix is bypassed. In this case, equations (14) and (15) may simply be rewritten: for points on the free surface to give an equation for ψ_m, and for points on the sea bed an equation for ϕ_m.

In practice, a procedure of over-relaxation can be adopted to give faster convergence. It was found convenient in the present work where the coefficients changed slowly, not to store all the coefficients α_{mj} etc., as this requires storage of $O(N^2)$, but to generate the coefficients necessary for each equation every time it had to be evaluated such that the storage was $O(N)$, and large numbers of points could be used. Overall, the implementation of the scheme in the form described here was particularly simple.

Results

The only results reported here are for a wave height $H/d = 0.25$ and a length 25 times that of the depth, rather higher than that used in Figure 3 above. The initial conditions were computed using an accurate Fourier method. The wave was allowed to propagate across a shelf with a cosine profile, which shoaled to $1/4$ the depth in a distance of roughly half the horizontal length scale of the wave, a rather more abrupt case than Figure 3, and corresponding to the shoaling of a wave on a coral reef. Results are shown in Figure 6, and they show some of the interesting phenomena associated with this nonlinear problem. After the wave travelled almost right across the shelf, quite quickly it started to grow in height, travelling over water of constant shallower depth, and the large feature of a shelf developed behind the wave, which seemed to be in the process of separating from the main wave and possibly becoming part of an oscillatory tail. At the final stage a sharp crest began to form, which turned over as shown, the surface particles in this latter stage experiencing very large accelerations.

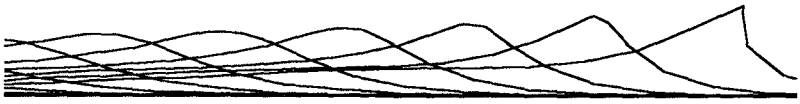

Figure 6: High wave on a steeply-shelving beach showing overturning

Conclusions

The local polynomial methods were found to be robust, accurate, and efficient, and may provide a useful practical means of computing nonlinear wave propagation over extended regions and times. The boundary integral method can be used to simulate shoaling considerably faster than other similar methods, and it has the potential to be more accurate than them. It can describe wave overturning with relatively few points, but for some problems it was found to be not as robust as had been hoped. Its most appropriate application might be to methods such as those of Leitão and Fernandes (1992) which use such a fixed domain with approximate boundary conditions.

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