CHAPTER 199

Wave impact beneath a horizontal surface.

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Abstract

Many coastal structures and natural coasts have openings, overhangs and projections which are open to impact by incident water waves. The sudden impact of a wave on a rigid surface leads to a rapid rise of pressure and consequent violent water motions. We consider the wave impact on the underside of a projecting surface. The example discussed is that of a flat deck close to the mean water level. A pressure-impulse approach is used, which has the advantage that given a solution for one problem it is possible to select pressure-impulse contours which give the solution to related problems. The pressure gradient on the underside of the deck is especially strong near the seaward edge of the impact region, so this is a region where any projections on the structure's surface may be subject to strong shearing forces. On the other hand the maximum pressure-impulse is at the landward end of the impact zone, it is here that the deck is most likely to be 'blown' upward.

Introduction

There are a number of circumstances in which the effect of the upward impact of a wave beneath a rigid horizontal surface needs to be estimated. For offshore oilrigs the lack of good estimates of such upward impacts leads to designs where the main platform of rigs is built to be out of reach of 'green water'. This may not be an option for some coastal structures, including piers and jetties, and temporary works in inter-tidal zones. Here we present pressure-impulse calculations for an impact on a horizontal surface near the surface of water of finite depth. For convenience we refer to the rigid surface as a deck.

In studies of wave impact on a wall Bagnold (1939) was the first to note that although pressure measurements show great variability between nominally identical wave impacts the integral of pressure over the duration of the impact,

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the pressure-impulse, is a more consistent measure of wave impact. This has been exploited theoretically by Cooker and Peregrine (1990 a,b, 1992, 1995) who show that the pressure-impulse and its distribution is not sensitive to the shape of the impacting water except in a region very close to the structure. Chan (1994) and Losada, Martin, and Medina (1995) show good agreement with experiment for wave impact on a wall.

Mathematical model

The geometrical simplifications we make may be seen in figure 1. The water is taken to be of finite depth CD= a, and to impact the horizontal deck BC of length L with an upward velocity V. The free surface outside the deck is taken to be flat, as BA, and to stretch to infinity. However, as indicated below alternative surface shapes are easily found by choosing different contours of pressure-impulse. The boundary conditions on CD given in figure 1 indicate that the problem can be reflected in the vertical plane of CD, corresponding to impact on a horizontal surface of length $2L$ with a central plane of symmetry.

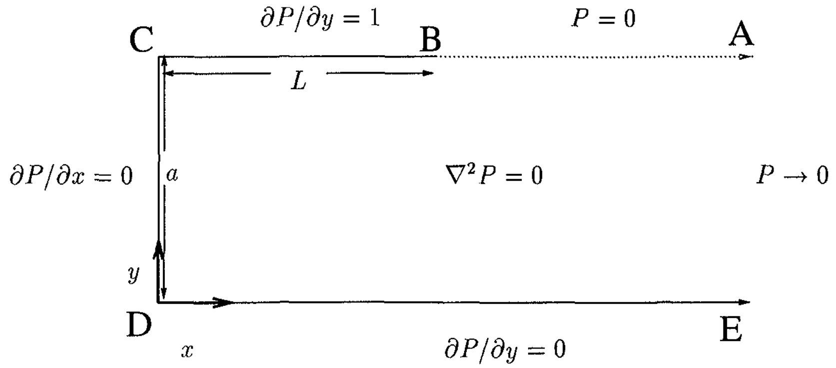

Figure 1: The problem to be solved.

Let the pressure be $p(x, y, t)$, then the pressure-impulse is

$$P(x, y) = \int_{t_b}^{t_a} p(x, y, t)dt \tag{1}$$

where t_a and t_b are the times after and before impact respectively, such that the time interval (t_b, t_a) is short compared with all other time scales in the problem. The main approximation is that during this short time the fluid motion changes so rapidly that the equation of motion may be approximated by

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{1}{\rho}\nabla p \tag{2}$$

where $\mathbf{u}(x, y, t)$ is the velocity field, and ρ the density which is assumed to be constant and uniform. The neglect of the convective terms $(\mathbf{u}.\nabla)\mathbf{u}$ is consistent except in any small region near the impact where jets may form.

Integration of (2), with respect to time, yields

$$\mathbf{u}_a - \mathbf{u}_b = -\frac{1}{\rho}\nabla P \tag{3}$$

Incompressibility gives $\nabla.\mathbf{u}_a = \nabla.\mathbf{u}_b = 0$. Therefore we find

$$\nabla^2 P = 0 \tag{4}$$

in the region of the water.

The boundary condition at the free surface is that the pressure must be constant and continuous therefore $P = 0$. At the walls and on the bed, the normal velocity must be zero before and after impact, therefore using equation (3), $\partial P/\partial n = 0$, where n is the normal direction. As the water meets the deck BC, the water has vertical velocity V, which could be any function of x, and after impact the water has zero vertical velocity. Therefore, again using equation (3), we have $\partial P/\partial n = V$. For simplicity, we choose V to be constant. We make the problem dimensionless by choosing units for which $V = 1$ and $L = 1$.

Infinite depth solution

The problem of a wave hitting upwards under a deck jutting out from a wall, is mathematically equivalent to a plate dropping onto a body of water and setting the water in motion. Also when considering solving Laplace's equation we can use the direct analogy with the velocity potential of irrotational flow. If we consider the complex potential for a flow past a plate then we just need a change of reference frame to find the complex potential of a moving plate in a stationary fluid. With a complex potential $w = \phi + \mathrm{i}\psi$, then $\partial\phi/\partial x = 0$ on $x = 0$, and $\partial\phi/\partial y = 1$ along the plate. These are the conditions that are required by P, and so the lines of constant pressure-impulse are given by lines of constant ϕ. The solution may be found in Lamb (1932, section 71), and in Milne-Thompson(1962, section 6.3), for a fluid flowing past an ellipse. If we allow one of the semi-axes to shrink to zero then we have a plate instead of an ellipse in the flow. Finally choosing the plate to be perpendicular to the flow, the length of the plate to be 2, and the velocity 1, we get an expression for the complex potential of a stream flowing past a plate:

$$w = -\sqrt{1 - z^2} \tag{5}$$

where the origin is taken to be the centre of the plate.

If we subtract the complex potential for a stream from this expression we have the potential for a moving plate. As the velocity of the stream is $(0, -1, 0)$, we must therefore subtract $\mathrm{i}z$ to get:

$$w = -\mathrm{i}z - \sqrt{1 - z^2} \tag{6}$$

This solution is symmetric about the centre of the plate. This means that we can consider a line drawn perpendicular to the plate from the centre of the plate, to be a wall, bringing us back to the original problem of the water hitting a deck jutting out from a wall. Hence we have an expression for the pressure-impulse:

$$P = \mathrm{Re}(-\mathrm{i}z - \sqrt{1 - z^2}) \tag{7}$$

This is the infinite depth solution. Figure 2 shows contours of pressure-impulse. The total impulse on the deck is $\pi/4$, in dimensional terms $\pi\rho VL/4$.

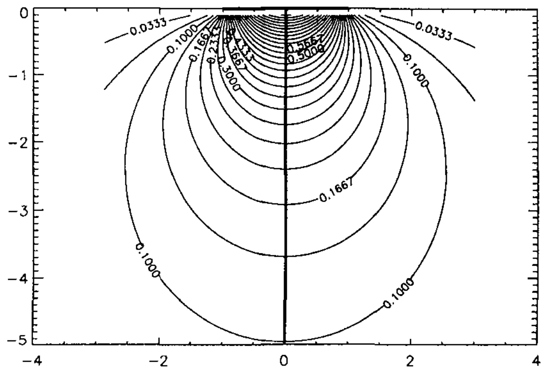

Figure 2: Infinite depth solution. Total impulse on deck (0,1) is $\pi/4$.

Infinitely long deck.

As a becomes small the effect of the free surface on the solution under the plate becomes small. This means it is possible to solve in that region by neglecting the condition at the free surface. Hence we solve Laplace's equation on a strip where $\partial P/\partial y = 1$ along the top, $\partial P/\partial n = 0$, where n is the normal direction, along the left-hand edge and bottom.

The solution is given by:

$$P = \frac{1}{2a}\left[y^2 - x^2\right] + K \tag{8}$$

where K is a constant which depends on the behaviour of P near $x = 0$, where this approximation fails. Figure 3 shows the case when $a = 0.1$, and K is set to zero.

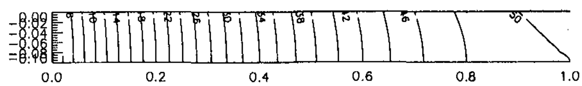

Figure 3: Analytic solution when a is small.($a = 0.1$,$K = 0$)

In practice the 'filling flow' solution of Peregrine and Kalliadasis (1995) may be more relevant to this case.

More general solution

Consideration of the boundary conditions in Figure 1, or the solution (7) shows that at B there is a square root singularity. This singularity causes problems for many numerical solution methods. However, one way to eliminate the problem of the singularity is to map the original problem using conformal maps as follows. First map to a half-space, then use another conformal map to perform a shift and stretch so that by using a final conformal map we can bend the problem back to a semi-infinite strip but with the boundary conditions shifted round to a convenient position, i.e. shift the boundary conditions on the deck round to the vertical wall.

Let the original plane in which the problem is posed be the z plane. The first map we need is $w = u + iv = \cosh(\pi z/a)$. This gives the problem shown in figure 4. As we only use conformal maps P continues to satisfy Laplace's equation throughout.

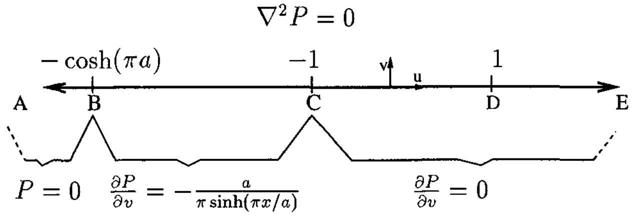

Figure 4: The problem in the w-plane after the first complex map.

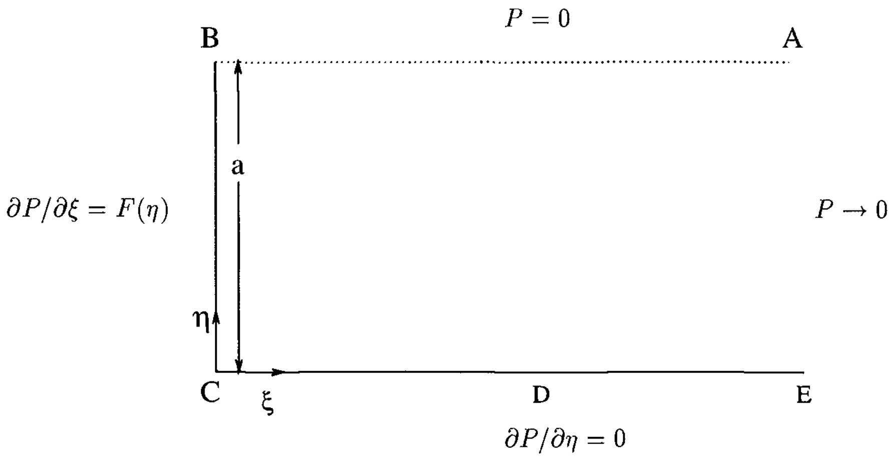

Figure 5. The final problem to be solved in the ζ-plane, where $F(\eta) = -\sin(\pi\eta/a)/(M\sqrt{b^2-1})$ with $b = [\cos(\pi\eta) - N]/M$.

We then use a translation and magnification to shift B to -1, and C to 1. The map required is $h = f + \mathrm{i}g = Mw + N$ where $M = 2/(\cosh(\pi/a) - 1)$ and $N = M + 1$. The last step is to map this problem back to the strip. The final map required is $\zeta = \xi + \mathrm{i}\eta = a\cosh^{-1}(h)/\pi$. This gives the problem as shown in figure 5, in a form with the singularty eliminated by being placed at a corner.

We solve Laplace's equation in this region by separation of variables. Let $P = f(\eta)g(\xi)$, giving $f'' = -\alpha^2 f$ and $g'' = \alpha^2 g$, where α is a constant. Solving for f, using the boundary condition that $f = 0$ at $\eta = a$, and $\partial f/\partial\eta = 0$ at $\eta = 0$, gives $f = A\cos(\alpha_n\eta)$ where $\alpha_n = (n + 1/2)\pi/a$. We now solve for g, using the condition that $P \to 0$ as $\xi \to \infty$. This gives $g = Re^{-\alpha_n\xi}$. Hence we have an expression for the pressure-impulse:

$$P = \sum_n A_n e^{-\alpha_n\xi}\cos(\alpha_n\eta) \tag{9}$$

Finally we use the condition that $\partial P/\partial\xi = -\sin(\pi\eta/a)/(M\sqrt{b^2 - 1})$, where $b = [\cos(\pi\eta) - N]/M$ along $\xi = 0$ to get expressions for the A_n. Using this condition we get:

$$-\sum_n A_n\alpha_n\cos(\alpha_n\eta) = -\frac{\sin(\pi\eta/a)}{M\sqrt{b^2-1}} \tag{10}$$

The final step is to multiply both sides by $\cos(\alpha_m\eta)$, and integrate along the line $\xi = 0$ to get:

$$A_m = \frac{2}{\alpha_m a}\int_0^a \frac{1}{M}\frac{\sin(\pi\eta/a)\cos(\alpha_m\eta)}{\sqrt{b^2-1}}d\eta \tag{11}$$

Similar results can be found for any velocity distribution $V = V(x)$.

Results and discussion

The integral in (11) is evaluated by using a numerical routine. For the cases of $a = 0.5$ and $a = 2.0$ taking thirty terms in the sum, gives an accuracy of 4 and 12 decimal places respectively. The distribution of pressure-impulse in the water beneath the deck is shown for deck width to depth ratios of 0.5, 1.0 and 2.0 in figures 6,7 and 8 respectively. The values of the total impulse on the deck and on the wall beneath each deck are given in each caption.

In figures 6,7 and 8 note the differing contour intervals, and the increasing impulse on the deck as the water depth a is decreased. The value of total impulse on the deck is given as a function of a in figure 9. This trend is for the impulse from impact of a given velocity and area to increase as the body of impacting water becomes more confined. The same trend is described by Cooker and Peregrine (1995) for impact on an interior wall of a rectangular box and by Topliss (1994) for impact within a circular cylinder. Consideration of flow in the most confined circumstances, as a becomes small, has given the concept of 'filling flows' (Peregrine and Kalliadasis, 1995). Further, an estimate of how the compressibility of dispersed air bubbles, such as those entrained in waves during breaking, may soften wave impact is given in Peregrine and Thais (1996).

The results are in dimensionless units, for practical use the dimensional pressure-impulse is needed; that is

$$P^*(x^*, y^*) = \rho VLP(Lx, Ly), \tag{12}$$

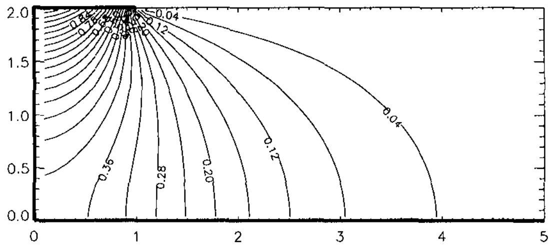

Figure 6: Pressure-impulse contours with $a = 2.0$. Total pressure-impulse on the deck and wall respectively are 0.81 and 1.02

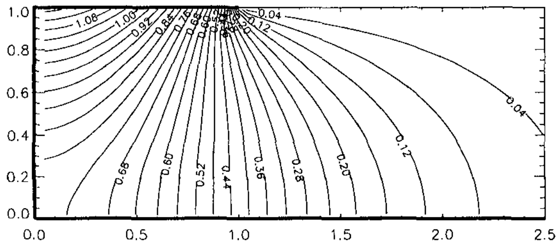

Figure 7: Pressure-impulse contours with $a = 1.0$. Total pressure-impulse on the deck and wall respectively are 0.92 and 0.74.

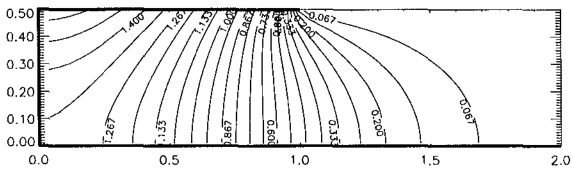

Figure 8: Pressure-impulse contours with $a = 0.5$. Total pressure-impulse on the deck and wall respectively are 1.193 and 0.44.

where * denotes the dimensional quantities. Whilst ρ and L will generally be known, V the vertical velocity of impact will need to be estimated. A simple method of estimating V is to first estimate how high a wave would be in the absence of the deck. Suppose it would have a height ΔH above the deck level. In simple projection of a particle this would require a velocity of $\sqrt{2g\Delta H}$. This is a reasonable, somewhat conservative, estimate for V.

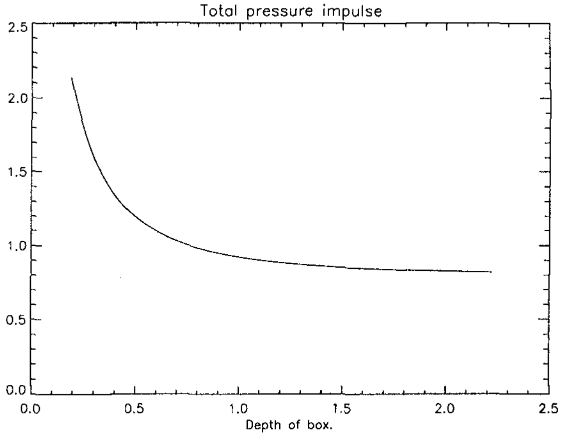

Figure 9: Total impulse on deck against depth a.

Note, the above solutions are not appropriate for impact from jets, e.g. see Cooker and Peregrine (1995), where the semi-infinite rectangular impact on a wall is equivalent to half of a plane jet and section 3.5 gives the solution for a circular jet. However, the solutions can be used for waves which are not nearly level with the deck as the figures indicate. By subtracting the appropriate constant from P, any of the contours of P can be chosen as an alternative free surface. Although such a surface tends downward rather than towards a horizontal level, this is not of great significance as long as the shape within roughly unit distance is appropriate. See Cooker and Peregrine (1995) for some examples.

Clearly the results presented here can be used to estimate the impulse and the spatial distribution of a wave impact. In addition, as illustrated by Cooker and Peregrine (1992) it is possible to estimate the impulse on smaller bodies on and near the wave impact area. The impulse may be derived from the local pressure-impulse gradient. Figure 10 shows the local gradient along the surface of the deck, and figures 11 and 12 show the gradient down the wall and along the bed respectively for a selection of values of a. On the wall and the bed the pressure gradient is tangential since $\partial P/\partial n = 0$. However, on the deck where the impact occurs $\partial P/\partial n \neq 0$ so that there is also a component of impulse perpendicular to the deck and downward.

Consideration of the gradient of pressure-impulse gradient near the edge of the deck shows alarmingly high values because the mathematical solution has a singularity at the edge of the deck. Clearly a better approximation is needed

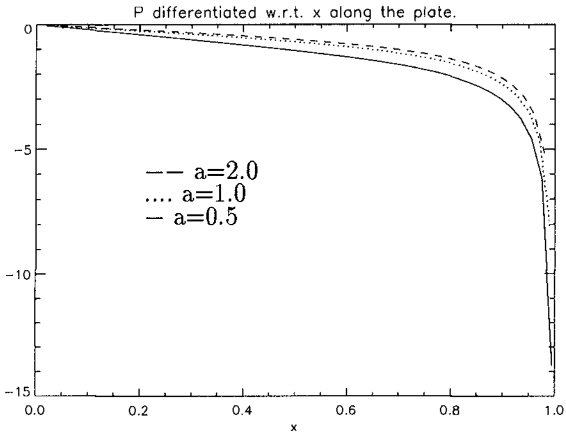

Figure 10: $\frac{\partial P}{\partial x}$ along the deck.

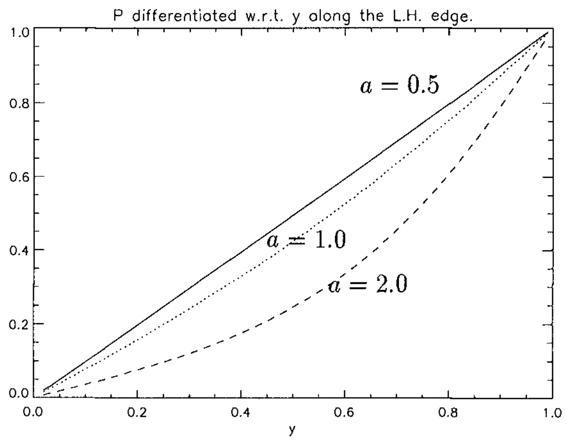

Figure 11: $\frac{\partial P}{\partial y}$ along the left hand wall.

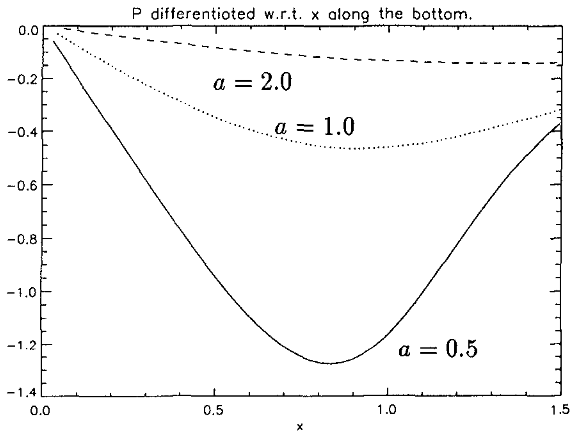

Figure 12: $\frac{\partial P}{\partial x}$ along the bottom.

there. One simple way of obtaining more realistic values is to consider how the solution is obtained for the infinite-depth case, $a = \infty$. There, the solution for the flow past a plate is used. This solution is a limit of flow past an ellipse. Thus a somewhat better solution could be obtained from the flow past a slender ellipse. In any case, it seems reasonable to conclude that attachments beneath a deck are vulnerable to especially large impact forces if they are near the edge of the deck, or the edge of the impact zone.

Three-dimensional effects

All the above work assumes uniformity perpendicular to the (x, y) plane, or some rigid boundaries parallel to that plane. In practice this is unlikely, and three-dimensional effects may be important. That is the impact area on the deck, rather than being a long strip of finite width L, should be taken as a finite area of an appropriate shape. This aspect of the problem is under study. For the present, we just note that for infinite depth solutions, a solution for impact on an elliptic area can readily be found from the potential flow round an ellipsoid.

Conclusions

A readily evaluated solution is presented for the pressure-impulse from waves hitting a deck from below. It is found that the impulse is greater if the water is shallow.

The same results may be useful for estimating the effects of upward impact by liquid confined within a container.

Acknowledgements

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