CHAPTER 228

Mathematical and Physical Modeling of Beach Nourishment Projects

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Abstract

This paper evaluates the utility of mathematical models for prediction of the planform movement of beach nourishment projects. The results of one-line models (analytical and numerical) are compared to the evolution of nine laboratory-scale model beachfills. Several methods of planform modeling are evaluated, varying in complexity. Comparison of planform modeling suggests that, in some circumstances, using a simple analytical treatment of refraction may be a feasible alternative to using more rigorous numerical wave modeling for shoreline modeling purposes. The relative ability with which different equations for net longshore sediment transport rate reproduce observed beachfill evolution is discussed. Information on the effect of wave climate and beachfill geometry on beachfill lifetime is extracted from the laboratory study. The observed effect of lower wave height, greater beachfill length, and the tapering of a beachfill is a greater beachfill lifetime, in agreement with conventional wisdom. Increasing beach slope and decreasing wave period are both observed to cause a slight decrease in beachfill lifetime.

Introduction

A beach nourishment project is an attempt to widen the dry portion of a beach and offer protection from storms to existing nearby structures by adding large quantities of sediment to some portion of the beach. They are sometimes appropriately referred to as "sacrificial beachfills." This emphasizes the fact that beach nourishment projects are rarely expected to stay in place for more than a decade without losing a large portion of the placed material. Any improvement in the ability to predict this redistribution of beachfill material, through the use of better modeling techniques, allows for more cost effective design and more efficient coastal zone management.

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Laboratory Study

A laboratory study was conducted, with two primary purposes in mind: 1)to observe in detail longshore sediment transport patterns in the presence of beach nourishment projects, and 2) to provide prototype beachfills for validation of mathematical shoreline models. Similar studies have been conducted. Kamphuis et al. (1986), Kamphuis (1991), and Nielsen (1988) studied longshore sediment transport in a laboratory setting. Kamphuis and Meyer (1976) studied beachfills using physical models. Dean and Yoo (1994) conducted a laboratory study of beach nourishment projects in front of seawalls and compared the results to mathematical models.

Nine laboratory beachfills were monitored. A paddle-type wave-maker was used to generate nominally shore-normal, monochromatic waves. A set of parameters were compiled which were expected to have an effect on beachfill lifetime: project length, degree of beachfill tapering, breaking wave height, wave period, and beach slope. In a laboratory setting these parameters are easily controlled; each case was assigned a different set of values for these parameters. For each case, a beachfill "lifetime" was calculated based on the amount of beachfill remaining in place, by volume, after wave attack. Lifetime is defined as the time period required for a significant percentage of material to be eroded; loss of 5% of beachfill volume was used. Beachfill material which was dispersed in the longshore direction, outside the original location of the beachfill, was considered "lost" sediment volume. Based on these calculations, the effect of each of the important parameters on beachfill lifetime was inferred (see Table I). Comparison is made to mathematical models employing the "CERC equation" for longshore sediment transport rate (U.S. Army Corps of Engineers 1984).

Table I
Effect of Parameters on Beachfill Lifetime

Parameter	Mathematical Models (with CERC formula)	Physical Model Observations
H_b, wave height at breaking $\Uparrow$	Lifetime $\Downarrow$	Lifetime $\Downarrow$ (significant effect)
T, wave period $\Uparrow$	No effect	$\Uparrow$ (slight)
ℓ project length $\Uparrow$	$\Uparrow$	$\Uparrow$ (moderate)
m, beach slope $\Uparrow$	No effect	$\Downarrow$ (slight)
Tapering of fill	$\Uparrow$	$\Uparrow$ (moderate)

Figure 1 shows the location of two typical beach profiles from one of the laboratory cases. Figure 2 is a beach profile located at the "shoulder" of one of the beachfills; the profile experiences a net loss of sediment. Figure 3 shows a beach profile located in the adjacent, unnourished region; accretion occurs due to beachfill material deposited there via longshore transport.

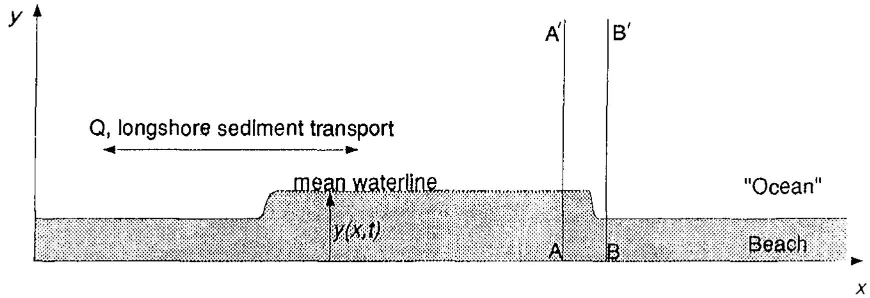

Figure 1. Locations of Laboratory Beach Profiles A and B

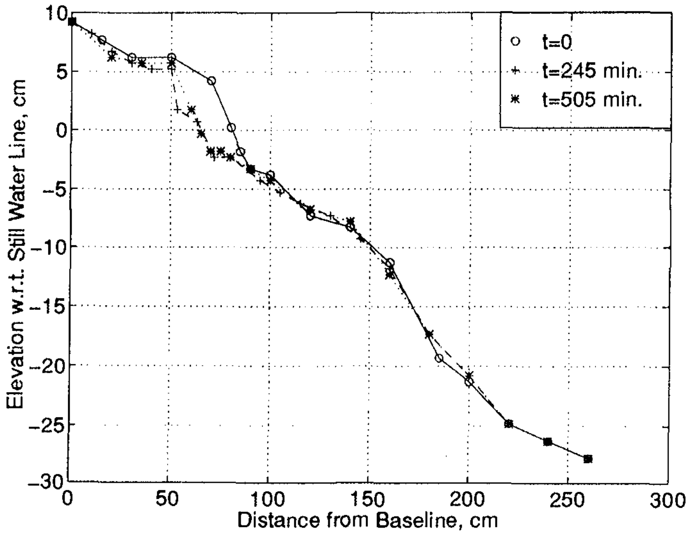

Figure 2. Laboratory Beach Profile A

With any physical model, scale effect is a major concern. Due to the relatively small scale of this laboratory study, it is unreasonable to expect accurate quantitative information. However, if the physical model behaves in a manner similar to full scale beachfills, useful qualitative information can be gained, like that shown in Table I. Figure 4 is a beach profile from the shoulder of a full-scale beach nourishment project (Perdido Key, FL). The similarity to Figure 2 is encouraging.

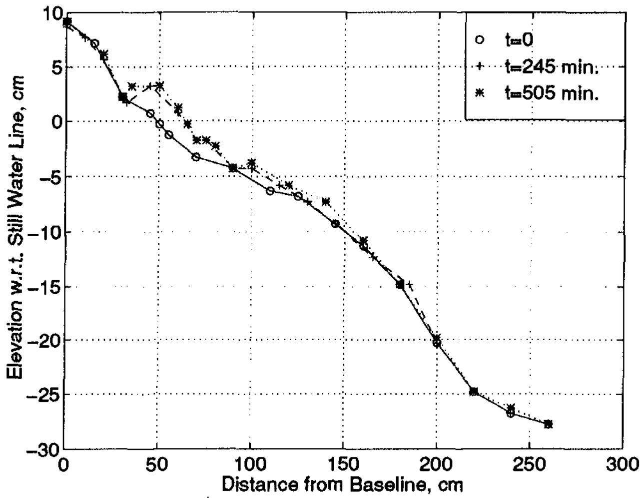

Figure 3. Laboratory Beach Profile B

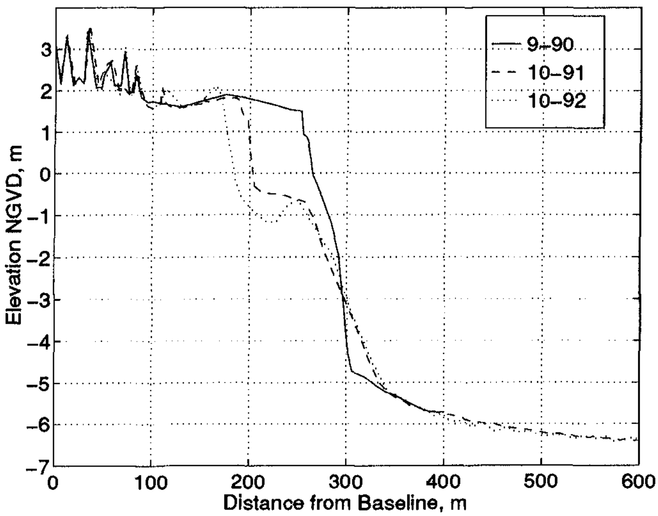

Figure 4. Beach Profile from Full-Scale Beachfill (Perdido, FL)

Shoreline Models

The shoreline model used in this study was a "one-line model", a tool which has been used extensively by researchers and engineers (e.g. Pelnard Considére 1956, Hansen and Kraus 1989). One-line models are used to predict the movement of a single elevation contour of a beach, typically the still water line. These models are governed by the sediment continuity equation,

$$\frac{\partial y}{\partial t}+\frac{1}{h_*+B}\frac{\partial Q}{\partial x}=0, \tag{1}$$

and an equation for longshore sediment transport. In equation (1), h_*+B defines the vertical extent of the active beach profile.

If a one-line model is used to provide the shoreline location, $y(x,t)$, the assumption must be made that the shape of a beach profile does not change, but that the profile only shifts onshore or offshore as erosion or accretion occurs (implying zero net cross-shore sediment transport over the time period of interest). However, a recently constructed beachfill typically has an overly steep beach slope, resulting in significant offshore sediment transport. Shoreline models were used to calculate longshore distributions of longshore sediment transport gradients, $\partial Q/\partial x$. A longshore sediment transport gradient can be directly compared to the time averaged rate of change of beach profile area $\partial A/\partial t$, which can be inferred from laboratory and field beach profile measurements:

$$\overline{\frac{\partial Q}{\partial x}}=-\overline{\frac{\partial A}{\partial t}}=-\frac{\Delta A}{\Delta t}. \tag{2}$$

Here, the overbar denotes time averaging. This method of comparison removes the effect of cross-shore sediment transport from calculations, provided that the entire "active" portion of the beach profile is included in area calculations.

Sediment Transport Equations used in Mathematical Shoreline Models

Three basic equations for longshore sediment transport were tested with the shoreline change models of the laboratory beachfills: the "CERC equation," the "Kamphuis equation" (Kamphuis 1991), and the "GENESIS equation" (Hansen and Kraus 1989). The CERC equation states that longshore sediment transport is proportional to the longshore component of wave energy flux:

$$Q=\frac{KH_{sb}^2 C_{gb}\sin 2\theta_b}{16(SG-1)g(1-p)}. \tag{3}$$

Here K is an empirical coefficient.

The GENESIS equation is a slight modification of the CERC formula, with an added term to account for longshore sediment transport driven by longshore gradients in wave height caused by the presence of structures or (less significantly) irregular bathymetry:

$$Q=(H^2C_g)_b[a_1\sin(2\theta_b)-a_2\cos\theta_b\frac{\partial H}{\partial x}]_b\ , \tag{4}$$

where

$$a_1=\frac{K_1}{16(SG-1)(1-p)(1.416)^{\frac{5}{2}}}\ , \tag{5}$$

and

$$a_2=\frac{K_2}{8m(SG-1)(1-p)(1.416)^{\frac{7}{2}}}\ . \tag{6}$$

The Kamphuis equation is an empirically-based equation, developed by a power fit analysis using data from lab and field studies:

$$Q=KH_{sb}^2T_p^{1.5}m_b^{0.75}D_{50}^{-0.25}\sin^{0.6}(2\theta_b). \tag{7}$$

Inclusion of Refraction in the Numerical Shoreline Model

Longshore sediment transport rate is generally accepted to be dependent on the angle between the wave orthogonal and the local shore normal. This suggests that the inclusion of refraction is requisite for any numerical modeling of irregular shorelines, such as a coastline with a trapezoidal beachfill. In most beach nourishment scenarios, inclusion of refraction will lead to a lower predicted rate of evolution, as it tends to decrease wave obliqueness at breaking.

The numerical wave transformation model REFRACT (Dalrymple 1988) was used to calculate breaking wave conditions for use in the shoreline model. This wave model includes shoaling and refraction (but not diffraction).

As an alternate method for including refraction in a numerical one-line model, a modified form of the CERC equation for longshore sediment transport (Work and Dean 1995) was employed. This equation is based on a two-line, analytical approach to wave transformation, which is much simpler than using a numerical wave model to supply breaking wave input data. The equation is a generalized form of an equation proposed by Dean and Yoo (1992). It can be stated as:

$$Q=K\frac{H_R^{2.4}g^{0.4}\cos^{1.2}(\beta_R-\alpha_R)}{8(SG-1)(1-p)\kappa^{0.4}}\frac{C_{G_R}^{1.2}}{C_*}\left[\frac{C_*\sin(\beta_R-\alpha_R)}{C_R}-\beta_t\left(1-\frac{C_*}{C_t}\right)+\beta_s-\beta_R\frac{C_*}{C_t}\right]. \tag{8}$$

The notation is illustrated in Figure 5. Subscript t denotes the location of the "toe" of the beach nourishment.

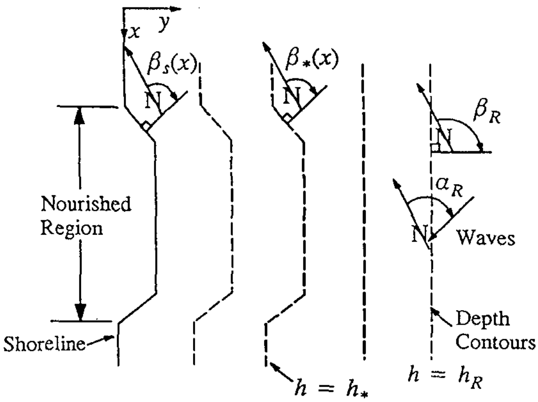

Figure 5. Notation used in Equation (8)

Inclusion of Diffraction in the Numerical Shoreline Model

Diffraction acts to reduce along-wave wave height gradients and thus tends to counter the effects of refraction. Without the inclusion of diffraction, a wave transformation model may tend to yield exaggerated focusing of wave energy near the "shoulders" of a beach nourishment project, which would compromise the predictive capability of the one-line model. The numerical wave model REF/DIF1 (Kirby and Dalrymple 1994) was used to generate breaking wave conditions (for use in the one-line model) including the effects of the shoaling, refraction, and diffraction of waves.

Comparison of One-Line Model Results to Laboratory Results

Laboratory data and numerical model results were compared based on longshore gradients of longshore sediment transport rate ($\partial Q/\partial x$), rather than shoreline evolution; thereby the effects of cross-shore sediment transport on shoreline movement were removed. Time-averaged longshore gradients of longshore sediment transport were inferred from beach profile measurements using equation (2).

Several shoreline modeling schemes were used, varying in complexity:

1. analytical calculation of shoaling (using conservation of wave energy flux), CERC equation for longshore sediment transport (equation (3));

2. analytical calculation of shoaling (using conservation of wave energy flux), Kamphuis equation for longshore sediment transport;

3. analytical treatment of the shoaling and refraction of waves in CERC equation (equation (8));

4. analytical treatment of shoaling and refraction in Kamphuis equation (analogous to equation (8));

5. shoaling and refraction determined by numerical wave model REFRACT, CERC equation for longshore sediment transport;

6. shoaling and refraction determined by numerical wave model REFRACT, GENESIS equation for longshore sediment transport; and

7. shoaling, refraction, and diffraction determined by numerical wave model REFDIF1, CERC equation for longshore sediment transport.

Figure 6 shows the longshore gradients of longshore sediment transport calculated by the one-line model for one of the laboratory cases. Note the similarity between the results using the numerical and analytical treatment of refraction.

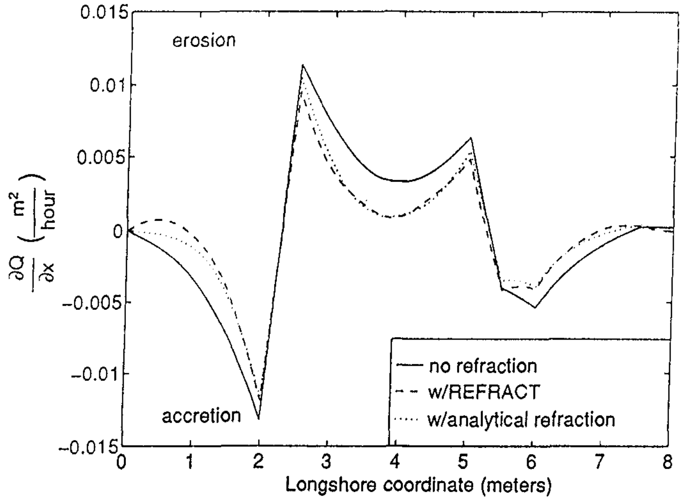

Figure 6. Comparison of Shoreline Modeling Schemes (1, 3, and 5)

Figures 7 compares longshore sediment transport gradients inferred from measurements for a laboratory beachfill to model output using option (1) above. This plot is representative of most comparisons of the one-line model results to lab results: though quantitative agreement is somewhat artificial due to calibration of the longshore sediment transport equation, qualitative agreement is good.

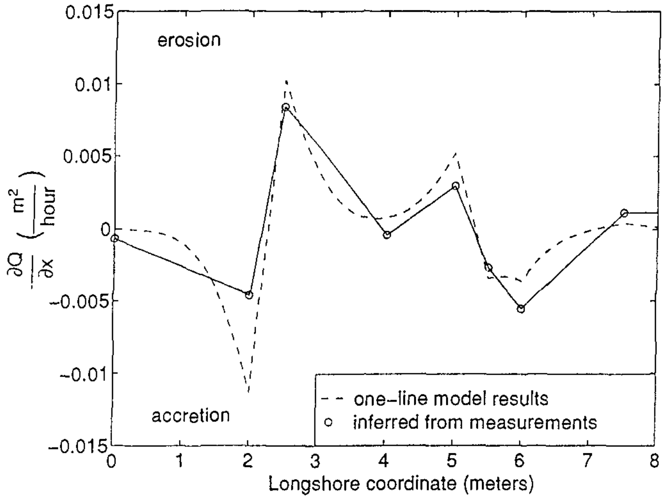

Figure 7. Comparison of Laboratory Results to One-Line Model Results

Using standard "field" values for coefficients of proportionality, none of the longshore sediment transport equations yielded good quantitative agreement. With standard coefficients, the equations typically greatly over-predicted longshore sediment transport. This is almost certainly due to scale effect, and has been observed by other researchers (e.g. Komar and Inman 1970).

The ability of the calibrated numerical one-line model to duplicate laboratory results was determined for each of the variations of the shoreline model. This comparison indicated that the use of the numerical wave models did not provide any added accuracy over method (1) above. On the other hand, use of the longshore sediment transport equation with an analytical treatment of refraction did yield a slight improvement in results.

In this comparison of methods, the Kamphuis equation yielded slightly better qualitative agreement than the other two basic equations for longshore sediment transport. The added term in the GENESIS equation did not result in any improvement over the CERC equation, probably because longshore gradients in wave height were not large in the laboratory.

Comparison to Field Data

Numerical one-line model results were compared to data from a well-monitored beach nourishment project at Folly Beach, South Carolina (Figure 8). The method for analytical treatment of refraction (method 3 above) was used with a standard calibration factor for the CERC equation (K=0.77 using H_{rms}). 2.1 million cubic meters of beachfill material were placed during the first four months of 1993 (Ebersole, et al., 1996). A 25-month period was investigated, from July, 1993 to August, 1995.

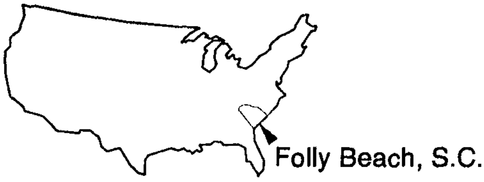

Figure 8. Location of Folly Beach

For the shoreline model initial condition, the "ad hoc transformation" method was used. This method is suggested by Dean and Yoo (1992) for situations "where substantial perturbations (human or natural) have placed the system out of balance." The initial planform used was simply the deviation from an assumed equilibrium planform caused by the placement of the beachfill.

Nine groins located northeast of the Holiday Inn (see Figure 9) were renovated at roughly the same time that the beachfill was placed. These groins were not included in the model results presented in this paper. For the time period modeled, the groins were thought to have a minor impact on shoreline movement; the groins are relatively short and were dry at low tide. Offshore wave data for 1993 and 1994 were obtained from a Wave Information Study (WIS) hindcast database (Brooks and Brandon 1995) for shoreline model input.

Comparison of the one-line model results to measured data is shown in Figure 10. The one-line model provides a reasonably good prediction of erosion and accretion, in both a quantitative and qualitative sense. Close inspection of Figure 10 suggests that the inclusion of the groins in the numerical model may have improved the model's accuracy--erosion was overpredicted by the model inside the groin field, and underpredicted downdrift (southwest) of the field.

Volume calculations indicate that 413,000 m^3, or 19% of the beachfill volume which existed in July 1993 was lost from the nourished region by August, 1995. Volume calculations include both the subaerial and submarine portions of Folly Beach; beachfill material deposited immediately offshore of the beach (above

-3m to -4m NGVD) was not considered a sediment loss, even though, by some measures, this cross-shore sediment transport would decrease the value of the beach. A majority of the 413,000 m^3 should therefore represent a loss of sediment due to longshore gradients in longshore sediment transport. However, other factors such as aeolian sediment transport, transport by tidal currents, and sand deposition well offshore of the beach could have contributed to the loss or gain of sediment. The shoreline model predicted a loss of 302,000 m^3 for the same time period. Had a larger empirical sediment transport coefficient been used, to account for the relatively fine sand at Folly Beach, this volumetric prediction would have been more accurate.

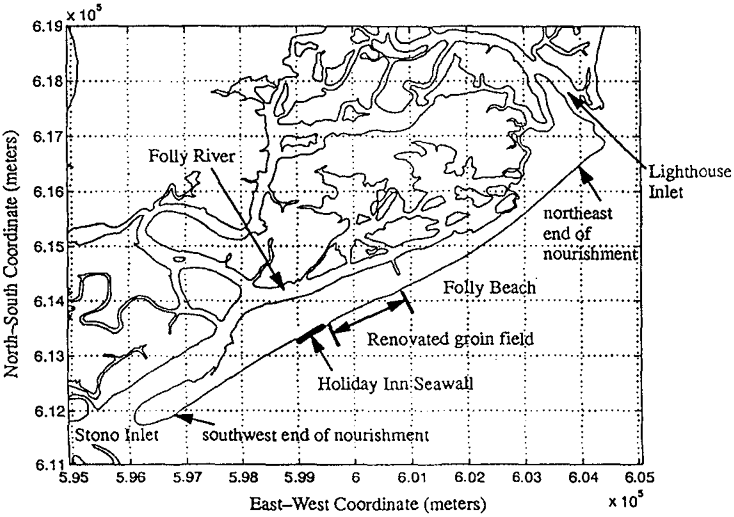

Figure 9. Map of Folly Island, S.C.

Conclusions

The laboratory observations regarding wave height and beachfill geometry shown in Table I agree with conventional wisdom on beach nourishment design. The observed weak dependence of sediment transport (and beachfill lifetime) on beach slope is not widely accepted, though it has been observed by other researchers, such as Kamphuis (1991). On the other hand, the observed positive correlation between wave period and beachfill lifetime is contrary to Kamphuis (1991). This discrepancy merits further study; though shorter waves impact on the shoreline with greater frequency, longer waves possess greater energy.

Results of the laboratory study suggest that relatively small-scale physical models of coastal sediment transport can yield useful qualitative information on shoreline and beach profile changes, though the magnitude of sediment transport rates may be greatly influenced by scale effects. The laboratory models cited in this study evolved similarly to prototype projects.

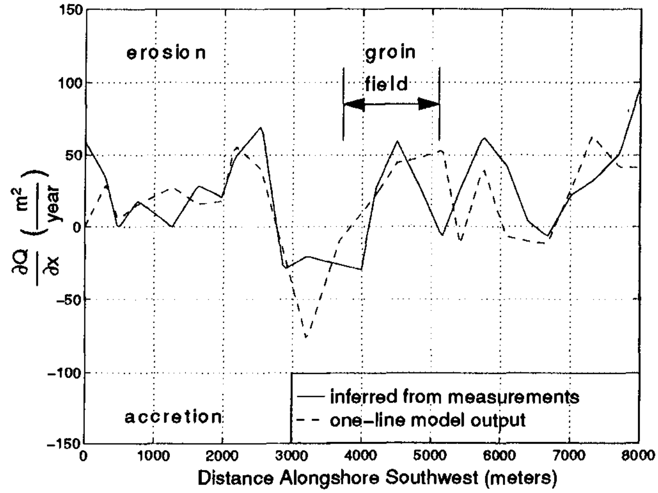

Figure 10. Comparison of One-Line Model Results to Field Measurements

The use of simple, one-line shoreline models in this study indicates that such models are adequate tools for predicting the volumetric redistribution of beachfill material. More rigorous, three-dimensional models of beachfill evolutlon are a worthy goal; however such models are equally, if not more, limited by present uncertainties in prediction of sediment transport.

The CERC equation for longshore sediment transport is adequate for predicting qualitatively the longshore trends of longshore sediment transport at both laboratory and field scale. The equation is a fair predictor of the magnitude of longshore sediment transport at field scale but requires calibration at small scales. Comparison to laboratory data suggests that the Kamphuis equation for longshore sediment transport is slightly more accurate than the CERC equation. Because of the possibilities of measurement error and scale effect with the laboratory study, this finding is inconclusive.

Based on limited laboratory and field-scale beach nourishment modeling, the analytical treatment of refraction seems to be an efficient method of including wave transformation in one-line models. This method may be considered by engineers for use in lieu of more rigorous, numerical approaches to wave transformation. However, in cases where a complex bathymetry exists (and accurate, high resolution survey data are available), a good numerical wave model is likely to give a better representation of the spatial variablilty of the wave climate. But again, the beachfill modeling as a whole is only as good as the "weakest link," so unless longshore sediment transport can be reliably predicted, extensive efforts at wave modeling may be unwarranted.

List of Symbols

The following symbols are used in this paper:

A=cross-sectional area of beach profile;
B=berm height;
C_g=wave group velocity;
C_f=wave celerity at toe of beachfill;
C_*=wave celerity at depth of closure;
D_{50}=representative grain size;
g=acceleration of gravity;
H_s=significant wave height;
h_*=depth to which longshore sediment transport affects beach profile;
K=empirical longshore sediment transport coefficient;
ℓ=length of beach nourishment;
m=beachface slope;
p=sediment porosity;
Q=volumetric longshore sediment transport rate;
SG=sediment specific gravity;
T=wave period;
T_p=wave period at peak of frequency spectrum
t=time;
x=longshore coordinate;
y=cross-shore location of shoreline;
β=local contour or shoreline orientation, relative to North;
κ=ratio of wave height to water depth at breaking;
θ=angle between wave orthogonal and local shore normal.

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