

## CHAPTER 270

### Movable Bed Roughness in the Flow of Irregular Waves and Currents over Movable Beds

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#### Abstract

The bed roughness  $k_s$  in the flow of irregular waves and currents over a movable bed is studied on the basis of measured current profiles and the model of You (1994). It is found that the movable bed roughness  $k_s$  is affected by both waves and currents and that the existing formulae derived in purely oscillatory flow generally fail to predict  $k_s$  in the flow of waves and currents. A new formula is proposed to calculate  $k_s$  in the flow of irregular waves and currents over a movable bed. The present bed roughness formula together with You's (1994) model gives better prediction of current shear velocity  $\bar{u}_{cw}^*$  and especially apparent roughness  $z_1$  than the existing formulae derived in purely oscillatory flow.

#### Introduction

The bed roughness  $k_s$  is an important input parameter in the modelling of coastal processes, but usually unknown in the coastal zone where the seabed is often movable owing to irregular waves and currents. The movable bed roughness  $k_s$  in purely oscillatory flow has been studied by many investigators (Van Rijn, 1982, Grant and Madsen, 1982; Nielsen, 1983; Raudkivi, 1988), but little investigated in the flow of irregular waves and currents. Consequently, the formulae derived in purely oscillatory flow have been often applied to estimate the movable bed roughness  $k_s$  in

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in the flow of irregular waves and currents. For example, Coffey (1987) used the formulae of Nielsen (1983), Van Rijn (1982), and Grant and Madsen (1982) to calculate  $k_s$  at  $\theta = 0^\circ$ ,  $90^\circ$  and  $180^\circ$ , but did not explain why the different formulae were applied for the different angles, where  $\theta$  is the angle between the wave propagation and a current.

Van Kampen and Nap (1988) found that the movable bed roughness  $k_s$ , estimated at  $\theta = 0^\circ$  or  $180^\circ$  from their experimental data was very large and could not be predicted by the existing formulae. In the sequential experiment of Havinga (1992), however,  $k_s$  estimated at  $\theta = 60^\circ$  or  $120^\circ$  was found to be very small and could not be predicted by the existing formulae either.

Drake *et al.* (1992) used the model of Grant and Madsen (1979) to study the movable bed roughness  $k_s$  in the flow of irregular waves and currents on the basis of measured current profiles. A simple empirical formula was suggested to be  $k_s = 28\eta^2 / \lambda - 0.14\theta$ , where  $0 \leq \theta \leq 90^\circ$ . Unfortunately, the model of Grant and Madsen (1979) together with the empirical formula still failed to predict the apparent roughness  $z_1$  as shown in Figure 4 of Drake *et al.* (1992).

More recently, Mathisen and Madsen (1996) studied the fixed bed roughness in steady flow, oscillatory flow and combined wave-current flow, respectively. It was found that  $k_s$  experienced by a pure current was equal to that by waves alone or by waves and currents. However, this is not valid for a movable bed simply because the movable bed form, which is suggested to a main contributor to  $k_s$ , is found to be quite different in steady flow compared to that in a combined wave-current flow (Arnot and Southard, 1990; Havinga, 1992).

In the present study, the movable bed roughness  $k_s$  in the flow of irregular waves and currents over a movable bed is studied on the basis of measured current profiles and the calibrated model of You (1994). A practical application is also given to calculate  $k_s$  in the coastal zone.

#### Model of Wave-Current Flows over Fixed Beds

In steady flow over a fully rough fixed bed, the current profile is logarithmic

$$\bar{u} = \frac{\bar{u}_c^*}{\kappa} \ln \frac{z}{z_0} \quad (1)$$

in which  $\bar{u}_c^*$  is the current shear velocity,  $\kappa$  is the von Karman constant,  $k_s = 30z_0$  and  $k_s$  is the bed roughness. When waves are superimposed on a current, a logarithmic current profile near the bed can still be found and similarly expressed as

$$\bar{u} = \frac{\bar{u}_{cw}^*}{\kappa} \ln \frac{z}{z_1} \quad (2)$$

but with  $\bar{u}_{cw}^* > \bar{u}_c^*$  and  $z_1 > z_o$  as first studied experimentally by Bijker (1967) and recently by Havinga (1992) and Klopman (1994).

Many models of combined wave-current flows over fixed beds have been developed to calculate the current shear velocity  $\bar{u}_{cw}^*$  and the apparent roughness  $z_1$  in Eq.(2). A review of existing models of combined wave-current flows refers to You (1992). You (1994, 95a) compared the four analytical models of Christoffersen and Jonsson (1985), Coffey and Nielsen (1986), Sleath (1991) and You (1994) with the laboratory measurements of van Doorn (1991, 82) and Kemp and Simons (1982). It was concluded that the model of You (1994) was simpler and gave better agreement with the experimental data than the others. In the present study, the model of You (1994) is chosen to study the movable bed roughness  $k_s$  in the flow of irregular waves and currents.

In the model of You (1994), the input parameters were chosen to be the wave parameters ( $A$ ,  $\omega$ ), a reference current velocity  $\bar{u}_r$  at an arbitrary level, and the bed roughness  $k_s$ . For irregular waves, significant wave height  $H_s$  and period  $T_s$  were suggested to calculate  $A$  and  $\omega$  using linear wave theory. The current shear velocity  $\bar{u}_{cw}^*$  in Eq.(2) was explicitly calculated by

$$\bar{u}_{cw}^* = 0.5u_w^* \left\{ \log_a^b + \left[ (\log_a^b)^2 + 1.6 \frac{\bar{u}_r}{u_w^*} \log_a^a \right]^{0.5} \right\} \quad (3)$$

in which the wave shear velocity  $u_w^*$  was evaluated as

$$u_w^* = \sqrt{0.5f_w} A \omega \quad \text{and} \quad f_w = 0.108 \left( \frac{k_s}{A} \right)^{0.343} \quad (4)$$

and the parameters  $a$  and  $b$  were defined by

$$a = \frac{30e\delta_1}{k_s}, \quad b = \frac{2\delta_1}{z_r} \quad \text{and} \quad \delta_1 = \frac{0.5\kappa u_w^*}{\omega} + 0.2z_o. \quad (5)$$

The apparent roughness  $z_1$  in Eq.(2) was also calculated explicitly by

$$z_1 = 2\delta_1 \left( \frac{30e\delta_1}{k_s} \right)^{\frac{\bar{u}_{cw}^*}{u_w^*}}. \quad (6)$$

The derivations of Eqs.(3)-(6) refer to You (1994). Alternatively, when the current shear velocity  $\bar{u}_{cw}^*$  and the apparent roughness  $z_1$  are measured, the movable bed roughness  $k_s$  in the flow of waves and currents can be determined by the model.

### Movable Bed Roughness

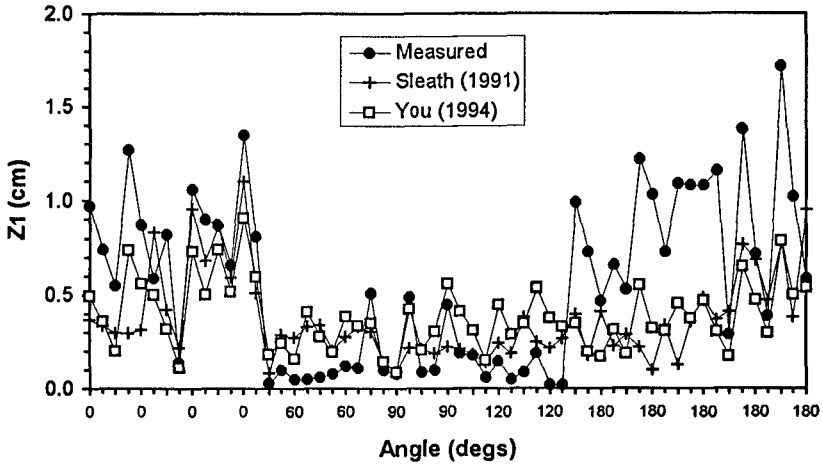
The formulae derived in purely oscillatory flow have been often used to calculate the movable bed roughness  $k_s$  in the flow of waves and currents (Grant Williams, 1984; Coffey, 1987; Drake et al, 1992)

$$k_s = C \eta^2 / \lambda + f(\Theta) \quad (7)$$

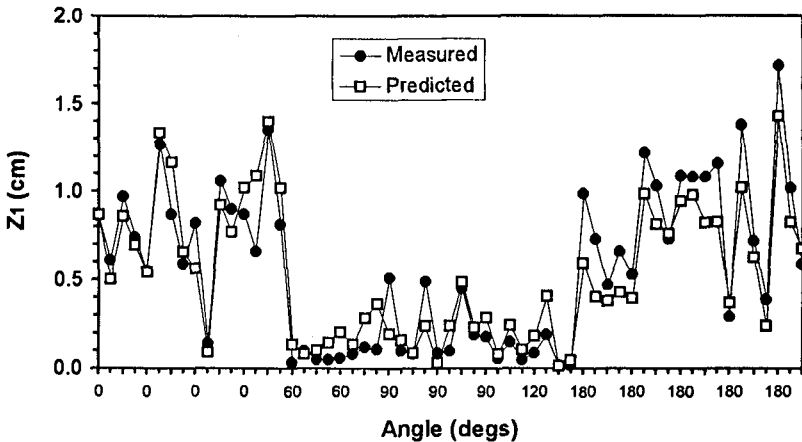
in which  $\eta$  and  $\lambda$  are the ripple height and length,  $f(\Theta)$  is an additional bed roughness owing to the near-bed sediment transport, and  $C$  is constant and was chosen to be 8, 16 and 28 by Nielsen (1983), Raudkivi (1988) and Grant and Madsen (1982), respectively. Since no direct measurements of  $k_s$  are available in the flow of waves and currents over movable beds, a direct comparison of Eq.(7) with experimental data on  $k_s$  becomes impossible. When the current shear velocity  $\bar{u}_{cw}^*$  and apparent roughness  $z_1$  are measured, however, Eq.(7) may be verified indirectly by using a calibrated model of combined wave-current flows. That is, if the calibrated model together with Eq.(7) can agree well with the measured  $\bar{u}_{cw}^*$  and  $z_1$ , it may be concluded that Eq.(7) gives good estimation of  $k_s$ .

In the present study, the laboratory experimental data from Kaaij and Nieuwjaar (1987), Van Kampen and Nap (1988) and Havinga (1992) are used to study the movable bed roughness  $k_s$  in the flow of irregular waves and currents. The irregular waves in the three sequential experiments were generated by a directional wave generator. The wave spectrum was JONSWAP. The significant wave heights  $H_s$  ranged from 7.5 to 18cm, the peak wave period  $T_p$  was about 2.5s, and the depth-averaged current velocity  $\langle \bar{u} \rangle$  varied from 10 to 40cm/s. The current profiles and bed forms in the absence and presence of irregular waves were measured. The movable roughness  $k_s$  calculated from Eq.(7) was found to be generally dominated by the first term in Eq.(7).

In Figure 1, the models of Sleath (1991) and You (1994) together with Grant and Madsen's (1982) formula of  $k_s = 28\eta^2 / \lambda$  are used to calculate the apparent roughness  $z_1$  and compared with the experimental data. It can be seen that the models generally underestimate  $z_1$  at  $\theta = 0^\circ$  and  $180^\circ$ , but overpredict  $z_1$  at  $\theta = 60^\circ$  and  $120^\circ$ . This indicates that the bed roughness  $k_s$  used in the models has been underestimated at  $\theta = 0^\circ$  and  $180^\circ$ , but overcalculated at  $\theta = 60^\circ$  and  $120^\circ$ . The models would not give better prediction of  $z_1$  by adjusting the constant  $C$  or the second term in Eq.(7).



**Figure 1.** Comparison of the apparent roughness  $z_1$  measured by Kaaij and Nieuwjaar (1987), Van Kampen and Nap (1988) and Havinga (1992) with those calculated by the models of Sleath (1991) and You (1994) with  $k_s = 28\eta^2 / \lambda$ . The ripple height  $\eta$  and length  $\lambda$  were directly measured in the experiments.



**Figure 2.** Comparison of the apparent roughness  $z_1$  measured by Kaaij and Nieuwjaar (1987), Van Kampen and Nap (1988) and Havinga (1992) with those calculated by You's (1994) model with the input of  $k_s$  estimated from Eq.(8). The bed roughness  $z_0$  in steady flow alone, the depth-averaged current velocity  $\langle \bar{u} \rangle$  and the nearbed wave velocity amplitude  $A\omega$  in Eq.(8) were measured in the experiments.

In other words, the use of the other former formulae (eg Nielsen, 1983; Raudkivi, 1988) in the models would not give better prediction of  $z_1$  in Figure 1.

As a first approximation, the movable bed roughness  $k_s$  in the flow of irregular waves and current flow may be simply structured as

$$k_s = 30z_o \left( 1 + \frac{A\omega}{\langle \bar{u} \rangle} \right) \quad (8)$$

in which  $A$  is the nearbed semi-excursion of wave orbital motion,  $\omega$  is the angular frequency,  $\langle \bar{u} \rangle$  is the depth-averaged current velocity, and  $z_o$  is the movable bed roughness in steady flow alone. In the field, a current velocity  $\bar{u}_A$  averaged from the bed to a level  $h_A$  is suggested to replace  $\langle \bar{u} \rangle$  in Eq.(8). It can be seen from Eq.(8) that the movable bed roughness  $k_s$  in the flow of waves and currents is larger than that in steady flow alone. The increase of  $k_s$  in the presence of waves may be understood to result from the appearance of wave-generated sand ripples and the increase of the nearbed sediment transport. It is shown in Figure 2 that the model of You (1994) together with Eq.(8) gives satisfactory agreement with the measured apparent roughness  $z_1$ . The three variables  $z_o$ ,  $\langle \bar{u} \rangle$  and  $A\omega$  in Eq.(8) were directly measured in the experiments. It may be concluded here that the new formula expressed by Eq.(8) gives good estimation of  $k_s$  in the flow of waves and currents.

It should be mentioned here that the formula suggested by Eq.(8) is only valid for a movable bed. For a fixed bed, it is often assumed that the bed roughness in steady flow is equal to that in oscillatory flow alone or in a combined wave-current flow. This has also been studied quantitatively by Mathisen and Madsen (1996).

### Practical Applications

The bed roughness in the flow of waves and currents can't be obtained from Eq.(8) unless all the variables in Eq.(8) are known. The movable bed roughness  $k_s$  in the field can be determined from Eq.(8) when  $\bar{u}_{cw}^*$ ,  $z_1$  and  $A\omega$  are measured.

The current velocity  $\bar{u}_A$ , which is depth averaged from  $z_o$  to a level  $h_A$  in steady flow alone, can be calculated by

$$\bar{u}_A = \frac{\bar{u}_c^*}{\kappa} \ln \frac{h_A}{e z_o}. \quad (9)$$

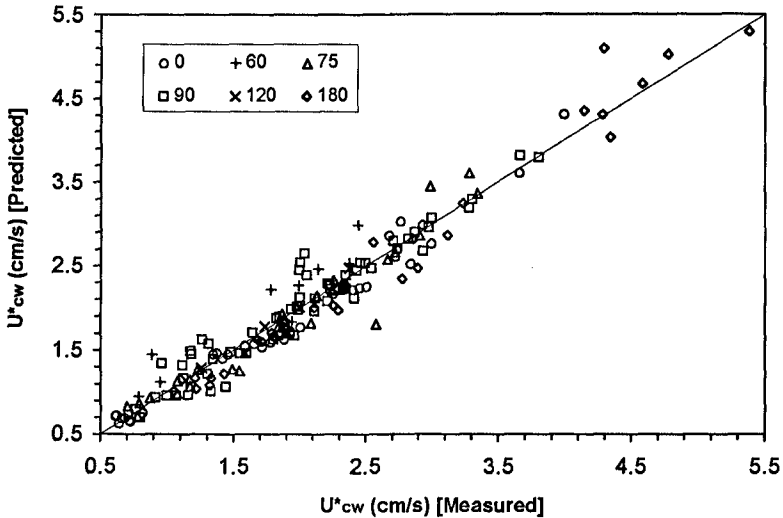
In the presence of waves,  $\bar{u}_A$  can also be approximately estimated from

$$\bar{u}_A = \frac{\bar{u}_{cw}^*}{\kappa} \ln \frac{h_A}{e z_1} \quad (10)$$

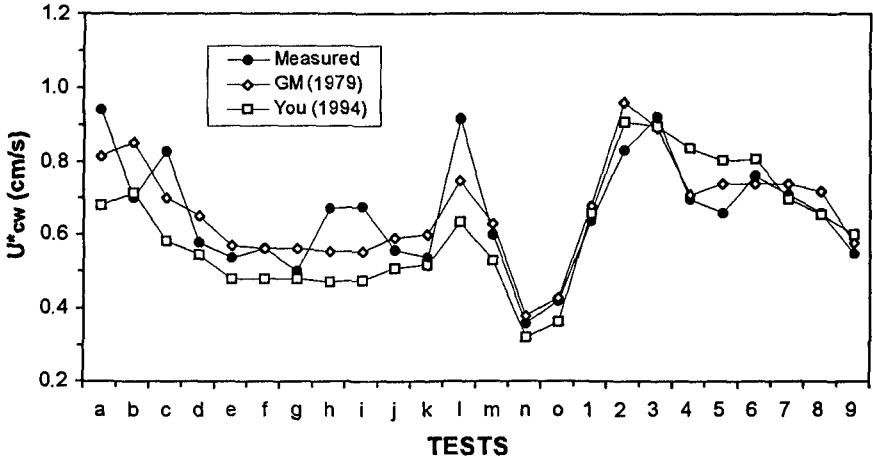
in which  $h_A$  is an arbitrary level and is chosen to be about 1.0m above the seabed in the present study. On the other hand, the relationship between  $\bar{u}_c^*$  and  $\bar{u}_{cw}^*$  was derived by You (1995b) as

$$\bar{u}_{cw}^* = \bar{u}_c^* + K \times A \omega \quad (11)$$

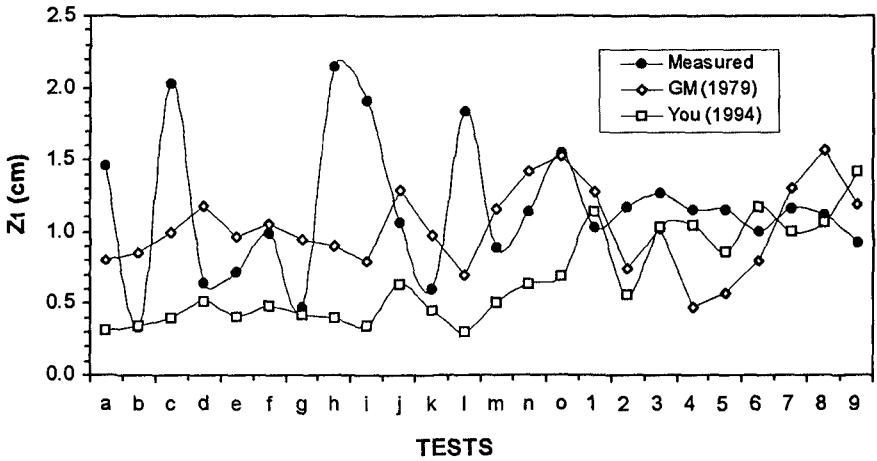
in which  $K$  is constant and was determined to be 0.026 and 0.020 for the laboratory and field data, respectively. For irregular waves, significant wave height  $H_s$  and period  $T_s$  were suggested to calculate  $A \omega$  in Eq.(11) using linear wave theory. The comparison of Eq.(11) with available experimental data is shown in Figure 3. Therefore, the movable bed roughness  $k_s$  in the coastal zone can be explicitly calculated from Eqs.(8)-(11). The depth-averaged current velocity  $\langle \bar{u} \rangle$  in Eq.(8) should be replaced by  $\bar{u}_A$  in Eq.(10).



**Figure 3.** Comparison of Eq.(11) with the available experimental data from Bijker (1967) [ $\theta = 75^\circ$  and  $90^\circ$ , regular waves], Kemp and Simons (1982, 1983) [ $\theta = 0^\circ$  and  $180^\circ$ , regular waves], Visser (1986) [ $\theta = 90^\circ$ , regular waves], Kaaij and Nieuwjaar (1987) [ $\theta = 0^\circ$  and  $180^\circ$ , irregular waves], Van Kampen and Nap (1988) [ $\theta = 0^\circ$  and  $180^\circ$ , irregular waves], Simons *et al.* (1988) [ $\theta = 0^\circ$ , regular waves], and Havinga (1992) [ $\theta = 60^\circ$ ,  $90^\circ$  and  $120^\circ$ , irregular waves]. After You (1995b).

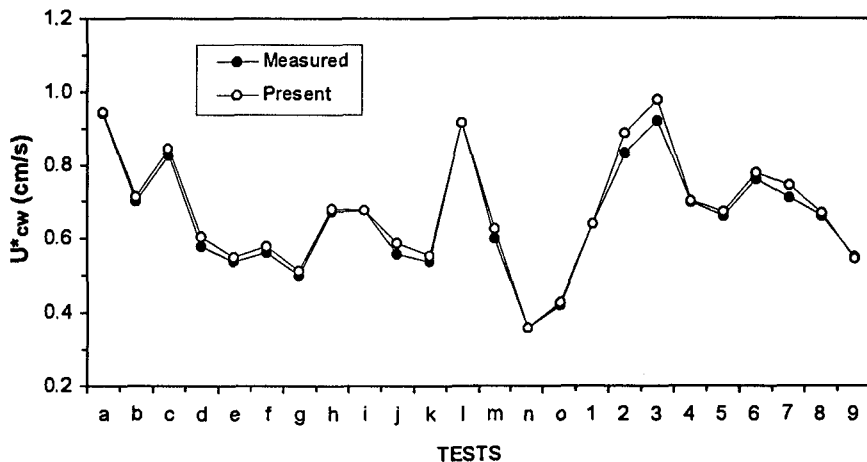


**Figure 4.** Comparison of the current shear velocities  $\bar{u}^*_{cw}$  calculated by the models of Grant and Madsen (1979) and You (1994) with the field measurements of Grant and Williams (1984, 85). The movable bed roughness of  $k_s=6.0\text{cm}$  used in the models was calculated from Grant and Madsen's (1982) formula by Grant and Williams (1984, 85).

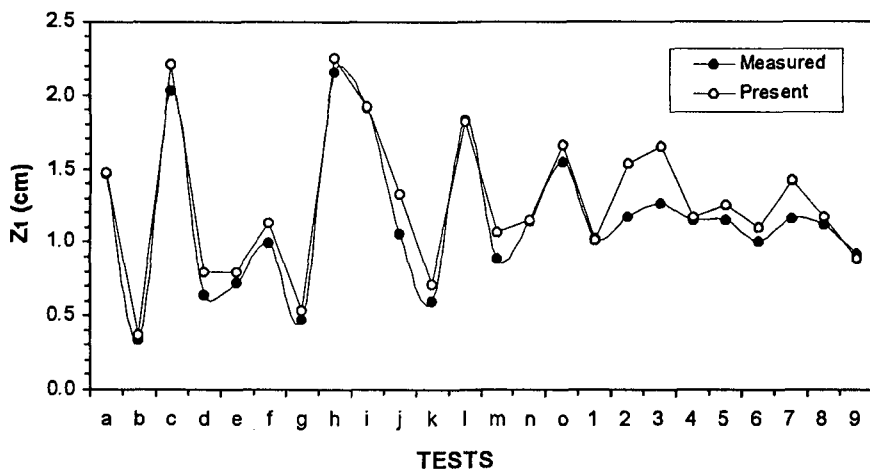


**Figure 5.** Comparison of the apparent roughness  $z_1$  calculated by the models of Grant and Madsen (1979) and You (1994) with the field measurements of Grant and Williams (1984, 85). The bed roughness of  $k_s=6.0\text{cm}$  used in the modes was estimated from Grant and Madsen's (1982) formula by Grant and Williams (1984, 85).





**Figure 6.** Comparison of the current shear velocities calculated by the model of You (1994) with the field measurements of Grant and Williams (1984, 85). The bed roughness  $k_s$  used in the model is calculated from Eqs.(8)-(11).



**Figure 7.** Comparison of the apparent roughness  $z_1$  calculated by the model of You (1994) with the field measurements of Grant and Williams (1984, 85). The bed roughness  $k_s$  used in the mode is calculated from Eqs.(8)-(11).

Grant and Williams (1984, 85) provided high-quality field measurements of current profiles of combined wave-current flows over a movable seabed. In Figures 5 and 6, the models of Grant and Madsen (1979) and You (1994) are used to calculate the current shear velocity  $\bar{u}_{cw}^*$  and apparent roughness  $z_1$  and compared with the field measurements. The bed roughness of  $k_s = 6.0\text{cm}$  used in the two models was estimated from Grant and Madsen's (1982) formula by Grant and Williams (1984, 85). It can be seen that the two models generally fail to predict the apparent roughness  $z_1$  even though the models give good agreement with the measured current shear velocities  $\bar{u}_{cw}^*$ . In Figures 6 and 7, however, the model of You (1994) gives satisfactory agreement with the measured  $\bar{u}_{cw}^*$  and  $z_1$  when the bed roughness  $k_s$  is calculated from Eqs.(8)-(11). The depth-averaged current velocity  $\langle \bar{u} \rangle$  in Eq.(8) should be replaced by  $\bar{u}_A$  in Eq.(10), and  $h_A = 1.0\text{m}$ .

### Conclusions

The movable bed roughness in the flow of irregular waves and currents are studied based on the simple model of You (1994) and measured current profiles. A new formula expressed by Eq.(8) is proposed to calculate  $k_s$  in the flow of irregular waves and current. The present bed roughness formula together with the model of You (1994) gives better agreement with the measured apparent roughness  $z_1$  than the former formulae derived in purely oscillatory flow. The field measurements of current profiles in the presence of irregular waves can be also predicted well with the model of You (1994) and the new bed roughness formula.

### References

- Arnot, R. W and Southard, J. B (1990). Exporatory flow duct experiments on combined flow bed configurations and some implications for interpreting storm even stratification. *Sedimentary Petrology*, 60: 211-219.
- Bijker, E. W (1967). Some considerations about scales for coastal models with movable beds. Delft Hydraulics Lab, Pub No. 50.
- Christoffersen, J. B and Jonsson, I. G (1985). Bed friction and dissipation in a combined current and wave motion. *Ocean Eng.*, 12: 387-423.
- Coffey, F. C (1987). Current profiles in the presence of waves and the hydraulics roughness of natural sand beaches. Ph.D thesis, Dept of Geography, University of Sydney, Australia.
- Coffey, F. C and Nielsen, P (1986). The influence of waves on current profiles, *Proc 20th Int Conf Coastal Eng.*, Taipei, p. 82-96.
- Drake, D. E, Cacchinone, D. A and Grant, W. D (1992). Shear stress and bed roughness estimates for combined wave and current flows over a rippled bed. *J Geophys Res.*, 97: 2319-2326.

- Grant, W. D and Madsen, O. S (1979). Combined wave current interaction with a rough bottom. *J Geophys Res.*, 84: 1797-1808.
- Grant, W.D and Madsen, O.S (1982). Movable bed roughness in unsteady oscillatory flow. *J Geophys Res.*, 87: 469-481.
- Grant, W. D and Williams, A. J (1984). Bottom stress estimates and their prediction on the Northern California Continental Shelf during CODE-1: the importance of wave-current interaction. *J Phys Oceanogr.*, 14: 506-727.
- Grant, W. D and Williams, A. J (1985). Reply to Huntley's Comments. *J Phys Oceanogr.*, 15: 1219-1228.
- Havinga, F. J (1992). Sediment concentrations and sediment transport in case of irregular non-breaking waves with a current. Draft Thesis, Faculty of Civil Engineering, Delft University of Technology.
- Kemp, P. H and Simons, R. R (1982). The interaction between waves and a current: waves propagating with the current. *J Fluid Mech.*, 116: 227-250.
- Kemp, P. H and Simons, R. R (1983). The interaction between waves and a current: waves propagating against the current. *J Fluid Mech.*, 130: 73-89.
- Klopman, G (1994). Vertical structure of the flow due to waves and currents, Delft Hydraulics Laboratory, Report H840, Part II.
- Mathisen, P. P and Madsen, O. S (1996). Waves and currents over a fixed rippled bed: 1. Bottom roughness experienced by waves in the presence and absence of currents. *J Geophys Res.*, 101: 16,533-16,542.
- Mathisen, P. P and Madsen, O. S (1996). Waves and currents over a fixed rippled bed: 2. Bottom and apparent roughness experienced by currents in the presence of waves. *J Geophys Res.*, 101: 16,543-16,550.
- Raudkivi, A, J (1989). The roughness height under waves. *J. Hydraulic Res*, 26: 569-584.
- Simons, R. R, Kyriacou, A, Soulsby, R. L, Davis, A. G (1988). Predicting the nearbed turbulent flow in waves and currents. *IAHR Symp Mah Modelling of Sediment Transport in the Coastal Zone*, Copenhagen, pp.33-47.
- Nielsen, P (1983). Analytical determination of nearshore wave height variation due to refraction, shoaling and friction. *Coastal Eng.*, 7: 233-251.
- Sleath, J. D (1991). Velocities and shear stress in wave-current flows. *J Geophys Res.*, 96: 15237-15244.
- van Doorn, Th (1981). Experimental investigation of near-bottom velocities in water waves without and with a current. Delft Hydraulics Laboratory, Report No M143.
- van Doorn, Th (1982). Experimental investigation of the velocity field in the turbulent bottom boundary layer in an oscillatory water tunnel. Delft Hydraulics Laboratory, Report No M1562-b (in Dutch).

- van Kampen, H. F. A and Nap, E. N (1988). Sediment concentrations and sediment transport in case of irregular non-breaking waves with a current. Faculty of Civil Engineering, Delft University of Technology.
- van Rijn, L.C (1982). The prediction of bedforms and alluvial roughness. *In Mechanics of Sediments Transport*, Proc Euromech 156, pp. 133-135.
- You, Z. J (1992). Oscillatory boundary layers with and without currents. Ph.D thesis. School of Civil Engineering, University of New South Wales.
- You, Z. J (1994). A simple model for current velocity profiles in combined wave-current flows. *Coastal Eng.*, 23: 289-304.
- You, Z. J (1995a). A simple model for current velocity profiles in combined wave-current flows: reply to the comments by P. Nielsen. *Coastal Eng.*, 26: 101-104.
- You, Z. J (1995b). Increase of current bottom shear stress due to waves. *Coastal Eng.*, 26: 291-295.