CHAPTER 323

Analysis of Mud Mass Transport under Waves Using an Empirical Rheological Model

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Abstract

This study is focused on deepening our insight into rheological properties of soft mud under oscillatory forces, on quantifying their effects on the interaction between water waves and mud bed, and especially on the resultant mud mass transport. On the basis of a large number of experiments, an empirical rheological model and formulas for model parameters are proposed. In addition, a simple analytical model for evaluating the temporal change in mud water content ratio under waves is derived. Using these two models, a vertical 2-D numerical model is developed to predict the wave-induced bed mud motion and the resultant mud mass transport velocity. Comparisons between the calculations and measurements are presented.

1. Introduction

Mass transport in a soft mud layer is one of the noteworthy phenomena in a mud bed. Studies on it are of practical importance for both siltation and environmental problems in muddy coasts and esturine areas.

Variety of theories and models have been proposed to describe the wave-mud interaction and relevant phenomena. The theoretical analysis was initiated by Gade (1958), who assumed that the soft mud would behave as a highly viscous fluid and developed an analytical model for the surface wave attenuation in shallow waters. On the same assumption, Dalrymple and Liu (1978) presented a linear theory for water waves propagating in a two-layer viscous fluid system. Instead of viscous fluid assumption, elastic models were proposed by Mallard and Dalrymple (1977) and Dawson (1978), poro-elastic models by Yamamoto

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et al. (1983, 1987) and Yamamoto and Takahashi (1984), visco-elastic models by Hsiao and Shemdin (1980), Macpherson (1980) and Maa and Mehta (1988, 1990), as well as a Bingham fluid model by Mei and Liu (1987). Recent studies include visco-elasto-plastic models by An *et al.* (1991) and Shibayama and Nguyen (1993).

A primary difference among the above models is in the assumptions of rheological properties of mud. Due to the complexity of the mud properties, it is difficult to judge which of them is most applicable to the wave-mud interaction problems under a certain condition. In addition, many of these models have been proposed either without enough experimental proofs or on the basis of experiments with unidirectional rotary viscometers in which the mud motion is considerably different from the real situation in coastal waters.

On the other hand, Hyunh *et al.* (1990) developed a dynamic rotary shear meter and studied the rheological behaviors of mud under various patterns of loading. Their experiments showed that there existed complicated relationships between the shear stress and shear strain or shear rate when the soft mud was subjected to cyclic loading. Following them, Shen *et al.* (1993) conducted similar experiments and proposed empirical models for the rheological properties and for the mud-wave interaction. However, their experiments only under limited conditions are not sufficient to understand and model the general features of mud behaviors under dynamic wave action.

The present study aims at gaining further understanding on the rheological properties of soft mud under the action of oscillatory forces and the interaction between water waves and a mud bed. An empirical rheological model of soft mud under oscillatory loading will be proposed on the basis of large numbers of measurements. Moreover, a simple analytical model for change in the water content ratio under wave action is derived in order to evaluate its effects on the rheological behaviors and on the motion of bed mud. By incorporating the proposed rheological model as well as the water content ratio model with the linearized Navier-Stokes equations, a vertical 2-D numerical model is constituted for simulating wave-mud interaction and the bed mud motion. Finally, calculated mud mass transport velocities and wave height changes are compared with those measured.

2. Rheological Properties of mud

2.1 Experiments and results

In order to clarify the rheological properties of soft mud under waves, laboratory experiments were conducted for more than 800 runs by using a dynamic rotary shear viscometer that can generate oscillatory forcing like waves (See Hyunh, 1991, for details of the setup). Instead of real seabed mud, pure kaolinite mixed with tap water was used for the test. This is because it has rheological properties similar to and less complicated than those of natural mud in actual coastal areas.

A wide range of conditions was covered in the test, in which the water content ratio was changed from 120% to 300%, the period of cyclic loading was from $1s$ to $10s$, and its angular amplitude was from 0.5° to 28°. With a series of combinations of the motion of the viscometer and the water content ratio, the measurements were made to examine influences of the angular amplitude and period of the oscillatory shear forces as well as the water content ratio on the rheological behaviors of the mud.

Figures 1 (a) and 1 (b) are the experimental results showing the effects of the angular amplitude A of cyclic forcing, which were obtained under the fixed period T and water content ratio W. Examples indicating the effects of the cyclic loading period and water content ratio are given in Figs. 2 (a) through 3 (b).

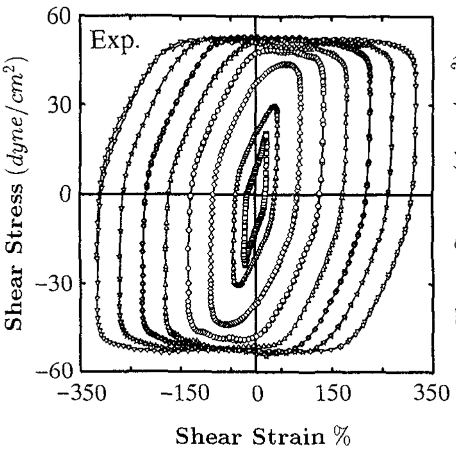

Fig. 1 (a) Shear stress vs. shear strain (effects of the angular amplitude).

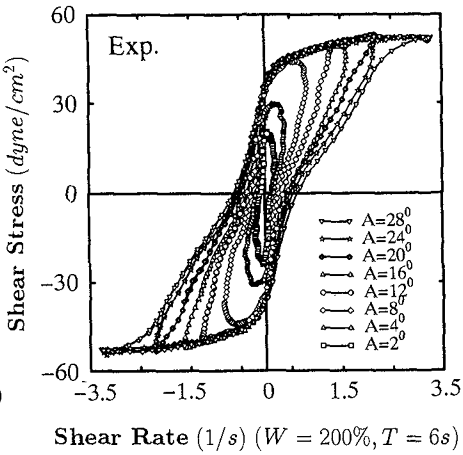

Fig. 1 (b) Shear stress vs. shear rate (effects of the angular amplitude).

From these figures, it can be concluded that under the oscillatory action of external forces, the soft mud is characterized by very complicated visco-elastio-plastic properties. The shear stress of the mud is a function of both the shear strain and shear rate with strong nonlinearity featured by remarkable hysteresisis in their relationships. In addition, the mud properties are significantly influenced by the water content ratio as well as by the period and amplitude of cyclic loading.

2.2 Constitution equation

On the basis of the above experimental results, a constitution equation of the soft mud is constructed for describing its rheological properties under cyclic loading.

Figures 4 (a) through 4 (g) illustrate a basic concept for constructing the constitution equation. Among these figures, Fig. 4 (a) stands for the hysteresis loop of the shear stress τ versus the shear rate γ. It is obvious that this loop can be separated into two parts, namely, a backbone curve shown in Fig. 4 (b) and a

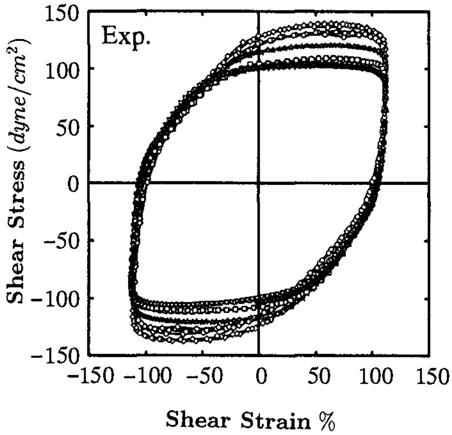

Fig. 2 (a) Shear stress vs. shear strain (effects of the cyclic load period).

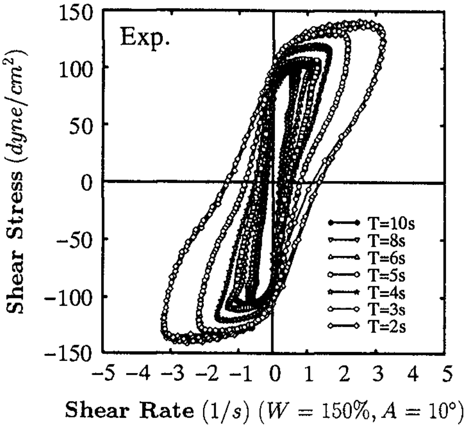

Fig. 2 (b) Shear stress vs. shear rate (effects of the cyclic load period).

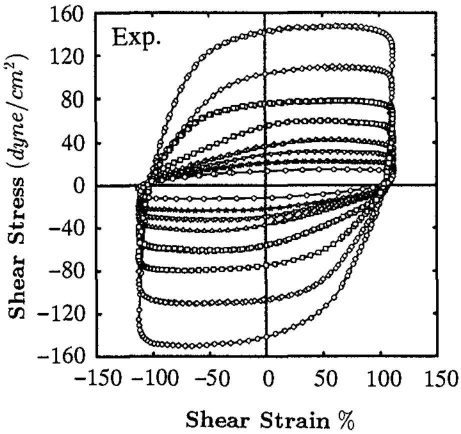

Fig. 3 (a) Shear stress vs. shear strain (effects of the water content).

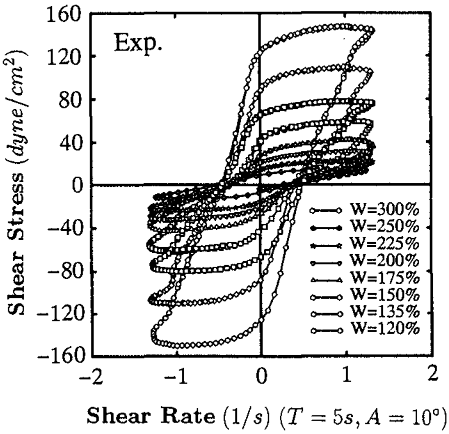

Fig. 3 (b) Shear stress vs. shear rate (effects of the water content).

compensatory curve in Fig. 4 (c). On the other hand, regarding the relationship between the shear stress τ and the shear strain ε shown in Fig. 4 (d), the curves in Fig. 4 (e) and Fig. 4 (f) correspond to Fig. 4 (b) and Fig. 4 (c), respectively, because of the 90° phase lag between the shear strain and shear rate. Namely, combining Fig. 4 (e) and Fig. 4 (f), we obtain Fig. 4 (d) that represents the hysteresis loop of the shear stress versus the shear strain, and Fig. 4 (f) gives the backbone curve for the loop in Fig. 4 (d).

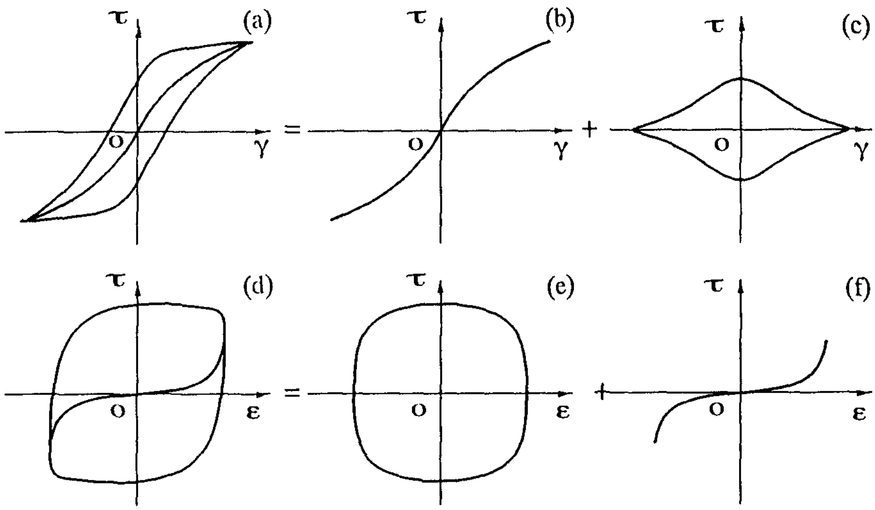

Fig. 4 Definition sketch for the constitution equation.

The concept mentioned above suggests us that if these two backbone curves of the hysteresis loops, Fig. 4 (b) and Fig. 4 (f), are properly determined, the shear stress in mud under waves can be easily obtained through their linear superposition. The constitution equation for the soft mud under the cyclic loading or wave action is thereby written as

$$\tau = \frac{G_0 \varepsilon}{1 - \alpha \mid \varepsilon \mid} + \frac{\mu_0 \gamma}{1 + \beta \mid \gamma \mid} \tag{1}$$

in which G_0 is the initial shear modulus at $\varepsilon = 0$, μ_0 is the initial viscosity at $\gamma = 0$, and α and β are coefficients determining the shapes of the backbone curves. The applicability of the above proposed model is strongly dependent on these four model parameters. With regression analyses, empirical formulas for them are obtained as follows:

$$
\begin{aligned}
\alpha &= \{1.0 - (0.8 - 5.0 \times 10^{-3} \cdot T) \\
&\quad \cdot \exp\left(-\left[0.75 \cdot \tanh\left(T/4\right)\right] \cdot \varepsilon_{\max}\right)\}/\varepsilon_{\max} \\
\beta &= (0.45 + 0.11 \cdot T + 0.015 \cdot T^2) \\
&\quad \cdot \varepsilon_{max}{}^{\left(1.05 - 1.42 \cdot T^{0.2}\right)} \\
G_0 &= (5.6 \times 10^5 \cdot W^{-2.8}) \\
&\quad \cdot \{[-7.26 - 560 \cdot (1.0 + \tanh(-0.58 \cdot T))] \\
&\quad + [81.70 + 707 \cdot (1.0 + \tanh(-0.47 \cdot T))] \\
&\quad \cdot \alpha^{1.71/[1.+9.87 \exp(-0.87 \cdot T)]}\} \\
\mu_0 &= (5.6 \times 10^5 \cdot W^{-2.8}) \cdot \left\{37.5 \cdot \left(1.31^T\right)\right\} \\
&\quad \cdot \ln[24.7 \exp(-0.31 \cdot T \cdot \beta + 1.0\}
\end{aligned}
\tag{2}
$$

where T(s) is the period of the cyclic loading, $\varepsilon_{\max}$ $(0.5 \le \varepsilon_{\max} \le 4.0)$ is the shear strain amplitude, and $W(\%)$ is the water content ratio of the mud. (Here, for convenience, the angular amplitude of cyclic shear load was replaced by the amplitude of shear strain).

Equations (1) and (2) form a thorough rheological model for the soft mud. Their validity was well confirmed both qualitatively and quantitatively through trial hindcasting of experimental data. Examples of the calculated results are shown in Figs. 5, which corresponds to the measurements shown in Fig. 1 in the previous section.

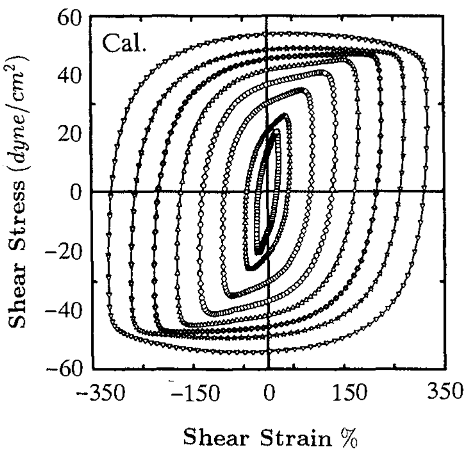

Fig. 5 (a) Shear stress vs. shear strain (effects of the angular amplitude).

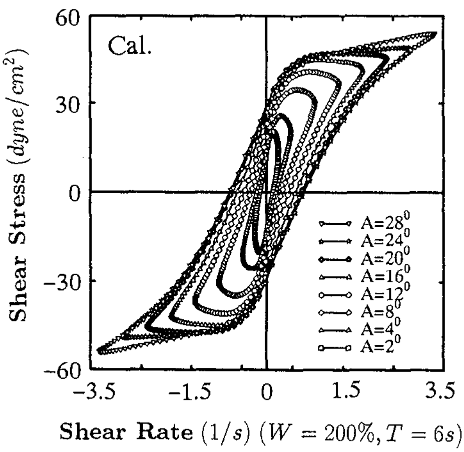

Fig. 5 (b) Shear stress vs. shear rate (effects of the angular amplitude).

3. Water Content Changes in Bed Mud under Waves

The water content ratio is an important factor affecting the rheological properties of the mud. Recent studies revealed that the dynamic change in the water content ratio takes place in a mud bed as a result of wave action. The results of experiments by Shen *et al.* (1993) showed that both the mud self-weight and water waves play significant roles in the variations of water content ratio in mud layers. As compared with the solo consolidation under the self-weight, the wave action causes further complicated change in the water content ratio, including the densification in deeper lower layers of the mud bed and liquefaction or swelling in the near-surface layers. On the basis of these experimental results, a simple analytical model is developed for evaluating the temporal change in water content ratio in bottom mud layers under waves.

3.1 Mechanism of Mud Densification and Liquefaction

Figure 6 illustrates an idealized state of stresses in a mud bed under waves. With consideration for the different effects of wave-induced stresses in mud layers, wave action on a mud bed can be classified into two components which are called "pumping" and "shaking". The pumping means the process of compression and expansion in a mud bed due to the dynamic wave pressure, while shaking refers to the tangential action by the oscillatory bottom shear stresses and the gradient of dynamic pressure in the wave propagation direction.

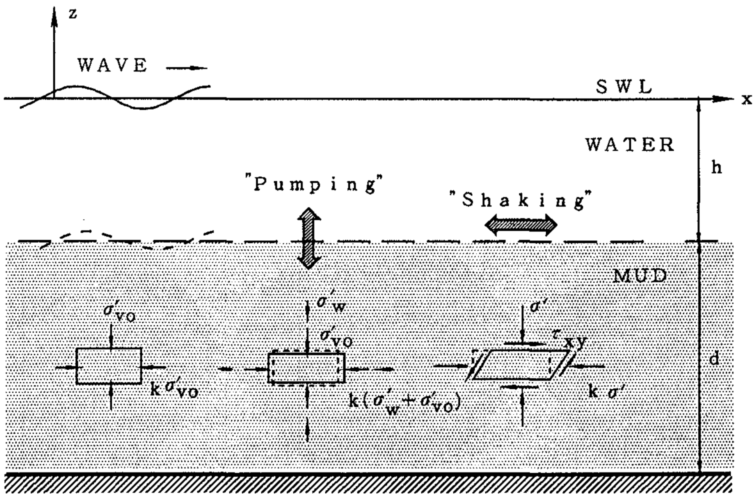

Fig. 6 Idealized stress state in mud bed under waves.

The downward transmission of the pumping and shaking action through the overlying water results in complicated processes in mud layers such as the oscillatory motion of the bed mud, the erosion or settlement of mud particles, and the change of the mud bed structure. Therefore, the water content changes in a mud bed is regarded as the combined results of the pumping and shaking due to waves.

(a) Densification by the Pumping Action of Waves

It is said and often assumed that the effective stress in the underwater deposits remains unchanged even if the water level varies. However, it is not always true under wave action. According to Zen *et al.* (1993), the excess pore pressure and effective stress will be induced whenever the water surface elevation changes so quickly that the associated water pressure on the interface cannot be entirely transmitted into the deposits as the pore pressure. From this fact, the densification of the mud layers can be regarded as the result of the pumping action due to waves.

In this way the pumping action of waves may generally cause oscillatory change of the normal stress in a mud bed. As a result, the excess pore pressure and effective stresses may vary oscillatorily, and the bed mud is thus forced to condense and expand alternatively responding to the up-and-down motion of the water surface. During every wave period, a process of dewatering or densification may occur since the volume compression rate of the mud is bigger than its expansion rate. The cyclic pumping action of waves will thus induce the densification of the bed mud.

(b) Liquefaction Due to the Shaking Action of Waves

A well-known fact as for noncohesive sediment is that shear loading causes the increase in gross sediment volume, which is called dilatancy. In fact, dilatancy is a general soil property both for noncohesive and for cohesive sediments. Therefore, like the liquefaction of noncohesive sediment, the increase of water content ratio in near-surface mud layers can be reasonably interpreted as the dilatancy effect under the cyclic wave action.

This phenomenon can also be regarded as a result of the exchange of water and mud particles. Parchure and Mehta, *et al.* (1985) has found that the erosion of cohesive sediment occurs if the shear force exerted on mud beds is large enough and it takes place much more easily under the cyclic action of waves. This is because the contacts between mud particles are weakened and the shear strength of the mud decreases under the action of external forces in particular under that of oscillatory forces like waves. Thus the mud particles are entrained into the water part and the water particles enter into the new-born pores where the mud particles lodged, resulting in the increase of the water content ratio in near-surface mud layers with the growth of erosion in the mud bed.

3.2 Water Content Change Model

On the basis of the above discussions, the water content change in a mud bed under wave action $e_{\mathrm{s+w}}$ can be divided into three parts, which are the consolidation by the self-weight e_{cs} as well as the densification e_{dw} and liquefaction e_{lw} due to waves. Accordingly, an analytical one-dimensional model is developed for evaluating the vertical distribution of water content ratio in mud layers by the linear superposition of the three parts.

Since the mud densification is similar to the soil consolidation and the liquefaction can be treated as its reversal process, all of these processes can be expressed by the conventional consolidation equation.

A general equation for the consolidation of soft mud was given by Gibson *et al.* (1967) as

$$\frac{\partial e}{\partial t}-\frac{\partial e}{\partial z}\left[\frac{k}{\rho_f(1+e)}\frac{d\sigma'}{de}\frac{\partial e}{\partial z}\right]+(\rho_s-\rho_f)\frac{d}{de}\left[\frac{k}{\rho_f(1+e)}\right]\frac{\partial e}{\partial z}=0 \tag{3}$$

in which e is the void ratio of mud, k is the permeability coefficient, σ' is the effective stress, ρ_f and ρ_s are the densities of the water and mud, and z is the vertical coordinate measured from the rigid bottom elevation. Its linearized form can be written as

$$\frac{\partial e}{\partial t}=C_v\frac{\partial^2 e}{\partial z^2} \qquad \text{for} \quad 0\le z\le z_0 \quad \text{and} \quad t\ge 0 \tag{4}$$

where C_v $(= k/\left[\rho_f(1+e)\right](d\sigma'/de))$ is the coefficient of consolidation, t denotes the time, z_0 is the thickness of mud layer at $t=0$.

For the consolidation due to the self-weight and densification caused by the pumping action of waves, the initial and boundary conditions are given as follows.

1) Assuming that the distribution of void ratio e_0 at $t=0$ is uniform, the initial condition is

$$e(z,0)=e_0=\text{const.} \tag{5}$$

2) On the mud bed surface, the void ratio remains constant because the effective stress is zero, i.e.,

$$e(z_0,t)=e_0 \tag{6}$$

3) At the rigid bottom underlying the mud, the impervious boundary condition is

$$\left(\frac{\partial e}{\partial z}\right)_{z=0}=\beta \tag{7}$$

where β is a constant depending on the void ratio distribution at the ultimate state of consolidation by the self-weight and densification due to waves, and is expressed as

$$\beta=\frac{(\rho_s-\rho_f)}{\lambda}+\frac{p_0}{z_0\lambda} \tag{8}$$

After simple manipulation, the solution of Eq. (4) for the void ratio is obtained as

$$e(z,t)=e_0-\frac{\beta}{z_0}\left[1-\frac{z}{z_0}-2\sum_{n=0}^{\infty}\frac{\cos\frac{m\pi z}{z_0}}{m^2\pi^2}\exp\left(-m^2\pi^2\frac{C_v t}{z_0^2}\right)\right] \tag{9}$$

where $m = 2n + 1/2$, $\lambda = d\sigma'/de$, and p_0 is the amplitude of wave induced bottom pressure.

Regarding the fact that the volume change of pore water in a mud bed compensates that of the mud eroded from the bed, the swelling of mud by the shaking action of waves can be derived as

$$\Delta e\,(z,t) = \Delta e_s \left\{ -\frac{z}{z_0} - \frac{2}{\pi} \sum_{n=1}^{\infty} \left[\frac{\sin \frac{n\pi z}{z_0}}{z_0} \exp\left(\left(\frac{n\pi z_0}{z_0} \right)^2 C_l t \right) \right] \right\} \tag{10}$$

where Δe_s is the change of void ratio on the mud surface expressed as in the above equations, C_l $(= k_l/[\rho_f\,(1+e)]\,(d\sigma'/de))$ is the coefficient of expansion, N is the number of discretized points in the mud bed layers, and Δz is the grid length in the z direction. The volume of mud particles entrained from the mud bed, V_E, can be estimated by using an empirical formulas for the erosion rate E of mud particles (Parture and Mehta, 1985) as follows:

$$V_E = \sum_{k=1}^{K} \left(\frac{E \Delta t}{\gamma_s} \right) = \sum_{k=1}^{K} \left\{ \frac{E_0 \Delta t}{\gamma_s} \exp\left[\alpha\,(\tau_b - \tau_c)^{0.5} \right] \right\} \tag{11}$$

where Δt is time interval, γ_s is the unit weight of the mud, E_0 is the floc erosion rate (g/cm^2/s), α is a model constant, τ_b is the wave-induced shear stress on the mud bed, and τ_c is the shear strength of the mud.

From Eqs. (9) and (10), the change in water content ratio W in the mud bed can be calculated from,

$$W\,(z,t) = e\,(z,t)\,\frac{\gamma_s}{S} \tag{12}$$

in which S is the saturation degree of the mud.

The validity of the above model is examined through comparisons of the computations with the measurements by Shen *et al.* (1993). Figure 7 shows comparisons between the calculations and measurements for the temporal change in water content ratio distributions under the conditions listed in Table 1. A good agreement is seen through the overall depth for each time step except for little discrepancy near the rigid bottom, where accurate measurement is generally difficult. Hence it is concluded that if values of the model parameters are properly determined for a mud material of interest, this model can be used to evaluate the temporal change in water content ratio distributions under waves with an acceptable accuracy.

Table 1 Computation conditions.

W_0 (%)	$k/[\rho_f(1+e)]$ (m^4/kgs)	$k_l/[\rho_f(1+e)]$ (m^4/kgs)	$\lambda = d\sigma'/de$ (kg/m^2)	T (s)	H_0 (cm)
174	3.6×10^{-11}	2.4×10^{-11}	35.0	1.02	4.5

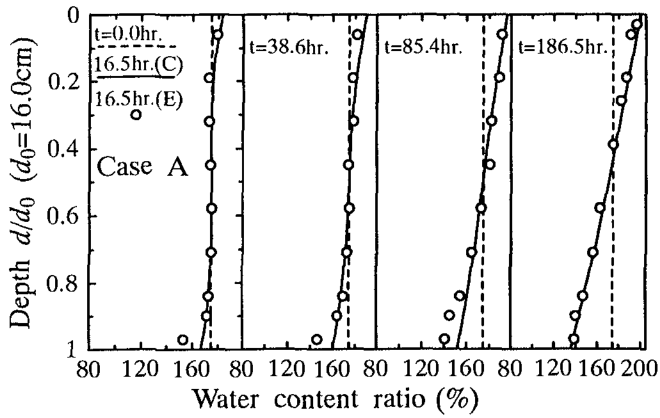

Fig. 7 Comparisons of the change in water content ratio between the calculations and the measurement by Shen *et al.* (1993)

4. Numerical Model of Mud Motion

4.1 Model Equations

A vertical two-dimensional numerical model for the interaction between waves and a mud bed is developed by incorporating the proposed rheological model and the water content ratio model with the Navier-Stokes equations.

On the assumption that the convective acceleration is not significant, the linearized momentum equations and the continuity equation for the incompressible soft mud layer under waves are

$$\rho_m \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} \tag{13}$$

$$\rho_m \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} \tag{14}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{15}$$

in which p is the dynamic pressure, u and w are the velocity components in the horizontal x and vertical z directions, ρ_m the mud density, and τ_{xz} and τ_{zx} are the shear stresses which can be obtained by the proposed rheological model Eqs. (1) and (2).

Equations (13) through (15) give a closed set of governing equations for u, w and p. With the non-slip condition ($u_n = w_n = 0$) at the fixed bottom under the mud layers, the zero shear stresses ($\tau_{xz0} = \tau_{zx0} = 0$) and pressure continuity conditions on the mud surface, and the periodic lateral boundary conditions, this equation system can be solved numerically, for instance, by using the SIMPLEC algorithm.

The coefficient of wave damping along the wave propagating direction, D_H, can be estimated as

$$D_H = -\frac{1}{H}\frac{dH}{dx} = \frac{1}{2EC_gL}\overline{\int_x^{x+L}\int_0^d \tau\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)dzdx} \tag{16}$$

The mass transport velocity of mud, U, is given as the summation of the Stokes drift U_S and the mean Eulerian velocity U_E, which are

$$U_S = \overline{\frac{\partial u}{\partial x}\int_0^t udt'} + \overline{\frac{\partial u}{\partial z}\int_0^t wdt'} \tag{17}$$

$$\mu'\frac{\partial^2 U_E}{\partial z^2} = \frac{\overline{\partial\rho_m u^2}}{\partial x} + \frac{\overline{\partial\rho_m uw}}{\partial z} \tag{18}$$

where μ' is the mean viscosity averaged over one wave period,

$$\mu' = \frac{1}{T}\int_t^{t+T}\frac{\partial\tau}{\partial z}dt = \frac{1}{T}\int_t^{t+T}\frac{\mu_0}{(1+\beta\mid\gamma\mid)^2}dt \tag{19}$$

4.2 Comparisons with Results of Experiment

In order to verify the above-described numerical model, results of calculations are compared with those of experiments by Sakakiyama and Bijker (1989). Figures 8 and 9 give examples of the comparisons with respect to the wave height distribution and the vertical profile of mud mass transport velocity. Agreement is very good for the mass transport velocity distribution, while the wave damping is slightly underestimated by the present model.

To examine the effect of water content ratio on the motion of bed mud, the change of the mud mass transport velocity in response to the temporal change in water content ratio is evaluated as an example. Figure 10 is the predicted mass transport velocity that corresponds to the evolution of water content ratio shown in Fig. 11. It can be seen in these figures that the temporal change in the water content ratio naturally has effect on the mud mass transport but it is not very significant as far as under the conditions in this example.

5. Conclusions

The soft mud is characterized by the very complicated visco-elasto-plastic properties in response to the cyclic action of external forces. In addition, the mud properties are significantly influenced by the water content ratio as well as the period and amplitude of cyclic loading. The proposed empirical rheological model can be regarded more general than most of previous ones, although the formulas for the model parameters may not be directly applicable to actual coastal mud. The temporal change in the mud water content ratio under waves has also been studied and formulated as a simple analytical model. By incorporating these

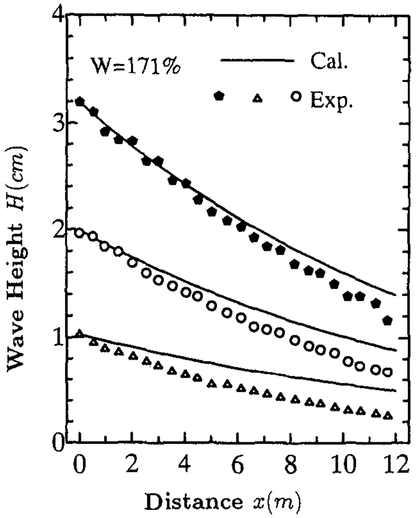

Fig. 8 Comparison between calculated and measured wave damping.

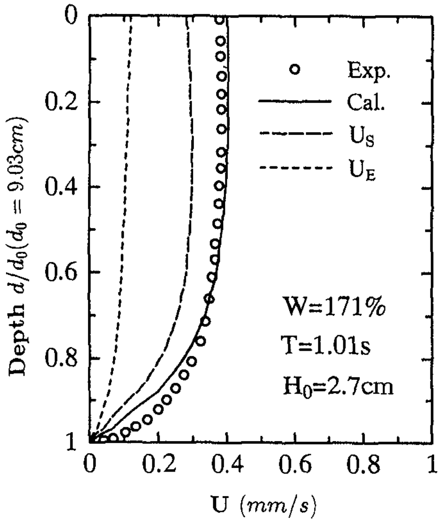

Fig. 9 Calculated and measured mud mass transport velocity.

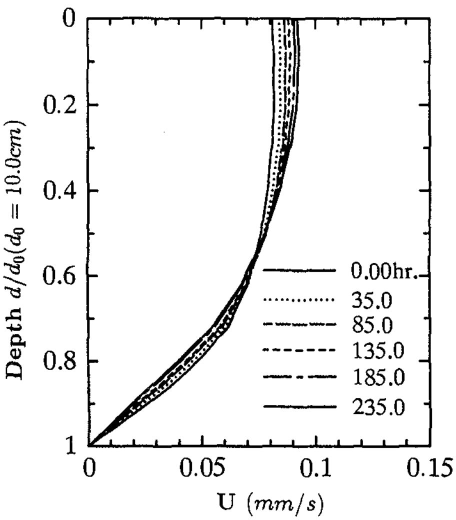

Fig. 10 Temporal changes of mud water content ratio under waves.

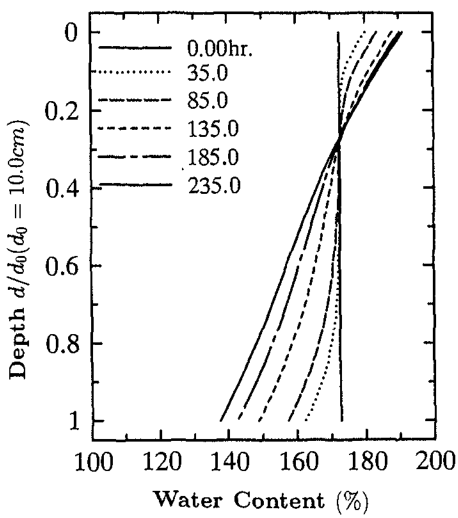

Fig. 11 Evolution of mud mass transport velocity under waves.

models with the Navier-Stokes equations, a vertical two-dimensional numerical model has been constituted to simulate the interaction of waves and mud. It has been verified that the proposed models can reproduce satisfactorily the rheological properties of mud, the temporal change in the water content ratio as well as the mud mass transport velocity under waves.

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