A MULTI-LEVEL MODEL FOR NONLINEAR DISPERSIVE WATER WAVES

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Abstract

To improve the accuracy of depth-integrated models for water waves, it is important to express the vertical characteristic of wave motion properly. In the present study, a multi-level model for nonlinear dispersive water waves is derived, in which vertical profile of horizontal velocity is assumed to be a chain of quadratic portions. The properties of the model in dispersion relation and second order nonlinear interactions turned out to converge to that of the Stokes wave theory, with the increasing number of layers.

Introduction

In many coastal projects, depth-integrated horizontal wave models play very important roll in estimating wave deformation. To improve the accuracy of depth-integrated model, it is important to express the vertical characteristic of wave motion properly. In the long wave model such as widely used Boussinesq equations and modified Boussinesq equations by Nwogu(1993), the vertical distribution of horizontal velocities are assumed to be quadratic. The higher order Boussinesq type equations by Kioka・Kashihara(1995), Madsen et.al(1996), Gobbi・Kirby(1996) and so on, have high accuracy in dispersion and nonlinearlity. In these equations, vertical distribution of horizontal velocities are assumed to be bi-quadratic. Different approaches were used by Nadaoka et.al(1994), Nochino(1994) and Isobe(1994), who expressed the vertical characteristic of wave motion as a combination of some components with properly chosen vertical distribution functions. The components are combined by applying the Galerkin method or variational principle to the Euler equation of motion to yield the coupled vibration equation for water waves, which have high accuracy in dispersion and nonlinearlity. As vertical distribution functions, Nadaoka et.al(1994) employed hyperbolic cosine type function, whereas Nochino(1994) used the Legendre's polynomials, and Isobe(1994) chose even-order polynomial functions.

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In the present work, with the objective of expressing the vertical characteristics of wave motion properly, a multi-level model for nonlinear dispersive water waves is derived and analyzed, in which vertical profile of horizontal velocity is assumed to be a chain of quadratic portions

Model Equations

The conceptual diagram of the present multi-level model for nonlinear dispersive water waves is shown in Fig.1. Static water depth is divided into several layers, whose number is N and numberd downward. The upper edge of first layer corresponds to still water level, and lower edge of N-th layer to the bottom. d_n is the thickness of n-th layer and $-h_n$ the depth of lower edge, $\mathbf{u}_n$ are level-averaged horizontal velocities, and $\mathbf{u}_b$ the horizontal velocity at the bottom.

Velocities are assumed to be expressed by (1) and (2). Horizontal velocities vary quadratically over a level, and vertical velocities linearly. For individual layer, this is the same as the standard Boussinesq equations.

$$\mathbf{u}(z) = \mathbf{u}_n + \frac{1}{6}(d_n^{\ 2} - 3(h_n + z)^2)\nabla(\nabla \cdot \mathbf{u}_n) + \frac{1}{2}(d_n - 2(h_n + z))[\sum_{i=n+1}^{N} \nabla(d_i \nabla \cdot u) + \nabla h_n \nabla \cdot \mathbf{u}_n + \nabla(\mathbf{u}_b \cdot \nabla h)] \tag{1}$$

$$w(z) = -\sum_{i=n+1}^{N} d_i \nabla \cdot \mathbf{u}_i - (h_n + z)\nabla \cdot \mathbf{u}_n - \mathbf{u}_b \cdot \nabla h \tag{2}$$

By substituting (1) into depth-integrated continuity equation (3), the continuity equation of multi-level model is obtained as (4).

$$\frac{\partial \eta}{\partial t} + \nabla \int_{-h}^{\eta} \mathbf{u} dz = 0 \tag{3}$$

$$\frac{\partial \eta}{\partial t} + \sum_{i=1}^{N} \nabla(d_i \mathbf{u}_i) + \nabla(\eta \mathbf{u}_1) - \nabla[\frac{1}{2}\eta(d_1 + \eta)\{\sum_{i=1}^{N} \nabla(d_i \nabla \cdot \mathbf{u}_i) + \nabla(\mathbf{u}_b \cdot \nabla h)\}] \tag{4}$$

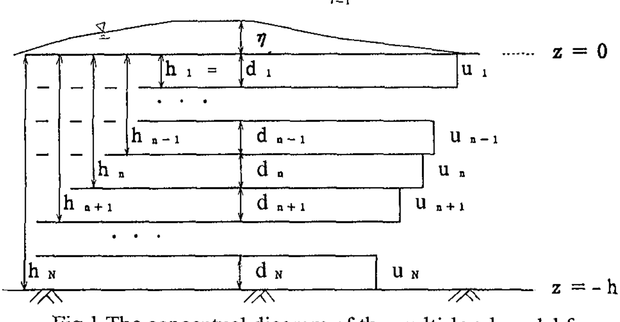

Fig.1 The conceptual diagram of the multi-level model for nonlinear dispersive water waves

Momentum equations are expressed by (6), which are obtained by substituting (1) and (2) to (5), and averaging over individul layer. Equation (5) is the Euler equation of motion modified by using the irrotational condition.

$$\frac{\partial \mathbf{u}}{\partial t}+\frac{1}{2}\nabla(\mathbf{u}_s\cdot\mathbf{u}_s+w_s^2)+g\nabla\eta=-\nabla\int_z^{\eta}\frac{\partial w}{\partial t}dz' \tag{5}$$

$$\begin{aligned}
&\frac{\partial \mathbf{u}_n}{\partial t}+\frac{1}{2}\nabla(\mathbf{u}_s\cdot\mathbf{u}_s+w_s^2)+g\nabla\eta\\
&=\nabla[\eta\sum_{i=1}^{N}d_i\nabla\cdot\frac{\partial \mathbf{u}_i}{\partial t}+\frac{1}{2}(\eta^2\nabla\cdot\frac{\partial \mathbf{u}_1}{\partial t})+\eta\frac{\partial \mathbf{u}_b}{\partial t}\cdot\nabla h]\\
&+\sum_{i=1}^{n-1}\sum_{j=i+1}^{N}\nabla(d_i d_j\nabla\cdot\frac{\partial \mathbf{u}_j}{\partial t})+\sum_{i=1}^{n-1}\frac{1}{2}\nabla(d_i^2\nabla\cdot\frac{\partial \mathbf{u}_i}{\partial t})\\
&+\sum_{i=n+1}^{N}\frac{1}{2}d_n\nabla(d_i\nabla\cdot\frac{\partial \mathbf{u}_i}{\partial t})-\sum_{i=n+1}^{N}(\nabla h_{n-1})d_i\nabla\cdot\frac{\partial \mathbf{u}_i}{\partial t}\\
&+\frac{1}{2}\nabla(d_n^2\nabla\cdot\frac{\partial \mathbf{u}_n}{\partial t})-\frac{1}{6}d_n\nabla(\nabla\cdot\frac{\partial \mathbf{u}_n}{\partial t})\\
&-\frac{1}{2}d_n(\nabla h_n)\nabla\cdot\frac{\partial \mathbf{u}_n}{\partial t}+\sum_{i=1}^{n-1}\nabla(d_i\frac{\partial \mathbf{u}_b}{\partial t}\cdot\nabla h)\\
&+\frac{1}{2}d_n\nabla(\frac{\partial \mathbf{u}_b}{\partial t}\cdot\nabla h)-(\nabla h_{n-1})(\frac{\partial \mathbf{u}_b}{\partial t}\cdot\nabla h)
\end{aligned} \tag{6}$$

In the momentum equations(6), $\mathbf{u}_s$ and w_s are the velocities at the free surface, which are expressed by (7) and (8).

$$\begin{aligned}
\mathbf{u}_s&=\mathbf{u}_1+\frac{1}{6}d_n^2\nabla(\nabla\cdot\mathbf{u}_1)-\frac{1}{2}\eta^2\nabla(\nabla\cdot\mathbf{u}_1)\\
&-(\frac{1}{2}d_1+\eta)[\sum_{i=n+1}^{N}\nabla(d_i\nabla\cdot\mathbf{u}_i)+\nabla(\mathbf{u}_b\cdot\nabla h)]
\end{aligned} \tag{7}$$

$$w_s=-\sum_{i=1}^{N}d_i\nabla\cdot\mathbf{u}_i-\eta\nabla\cdot\mathbf{u}_n-\mathbf{u}_b\cdot\nabla h \tag{8}$$

$\mathbf{u}_b$ the horizontal velocities at the bottom is determined by the relation(9), combined with $\mathbf{u}_N$ the N-th layer-averaged velocity.

$$\mathbf{u}_b=\mathbf{u}_N+\frac{1}{6}d_N{}^2\nabla(\nabla\cdot\mathbf{u}_N)+\frac{1}{2}d_N[\nabla(\mathbf{u}_b\cdot\nabla h)+(\nabla h)\nabla\cdot\mathbf{u}_N] \tag{9}$$

Thus, fundamental system of multi-level model is obtained, which is composed of one continuity equation, layers number of momentum equations and bottom condition. For the momentum equations (6), the convenient form is given by (10), in which the contributions of $\mathbf{u}_i$ to $\mathbf{u}_n$ in linear dispersion terms are shown explicitly.

$$\frac{\partial \mathbf{u}_n}{\partial t} + \frac{1}{2}\nabla(\mathbf{u}_s \cdot \mathbf{u}_s + w_s^2) + g\nabla\eta$$
$$= \nabla[\eta\sum_{i=1}^{N} d_i \nabla\cdot\frac{\partial \mathbf{u}_i}{\partial t} + \frac{1}{2}(\eta^2\nabla\cdot\frac{\partial \mathbf{u}_1}{\partial t}) + \eta\frac{\partial \mathbf{u}_b}{\partial t}\cdot\nabla h]$$
$$+\sum_{i=1}^{N}\alpha_{n,i}\nabla(\nabla\cdot\frac{\partial \mathbf{u}_i}{\partial t}) + \sum_{i=1}^{N}\beta_{n,i}\nabla\cdot\frac{\partial \mathbf{u}_i}{\partial t} + \gamma_n\nabla\cdot\frac{\partial \mathbf{u}_b}{\partial t} + \delta_n\frac{\partial \mathbf{u}_b}{\partial t} \tag{10}$$

where ,coefficients in linear dispersion terms are given as follows.

$$\alpha_{n,i} = \alpha 1_{n,i} + \alpha 2_{n,i} + \alpha 3_{n,i} + \alpha 4_{n,i} \tag{11}$$

$$\alpha 1_{n,i} = \begin{cases} 0 & (n \le 1) \\ \sum_{m=1}^{n-1} d_m d_i & (n > 1, i > n-1) \\ \sum_{m=1}^{i-1} d_m d_i & (n > 1, i \le n-1) \end{cases} \tag{12}$$

$$\alpha 2_{n,i} = \begin{cases} 0 & (n \le 1) \\ \frac{1}{2} d_i^2 & (n > 1, i > n-1) \end{cases} \tag{13}$$

$$\alpha 3_{n,i} = \begin{cases} 0 & (n \ge i) \\ \frac{1}{2} d_n d_i & (n < i) \end{cases} \tag{14}$$

$$\alpha 4_{n,i} = \begin{cases} 0 & (n \ne i) \\ \frac{1}{3} d_i^2 & (n = i) \end{cases} \tag{15}$$

$$\beta_{n,i} = \beta 1_{n,i} + \beta 2_{n,i} + \beta 3_{n,i} + \beta 4_{n,i} \tag{16}$$

$$\beta 1_{n,i} = \begin{cases} 0 & (n \le 1) \\ \sum_{m=1}^{n-1} \nabla(d_m d_i) & (n > 1, i > n-1) \\ \sum_{m=1}^{i-1} \nabla(d_m d_i) & (n > 1, i \le n-1) \end{cases} \tag{17}$$

$$\beta 2_{n,i} = \begin{cases} 0 & (n \le 1) \\ d_i \nabla d_i & (n > 1, i > n-1) \end{cases} \tag{18}$$

$$\beta 3_{n,i} = \begin{cases} 0 & (n \ge i) \\ \frac{1}{2} d_n \nabla d_i - d_i \nabla h_{n-1} & (n < i) \end{cases} \tag{19}$$

$$\beta 4_{n,i} = \begin{cases} 0 & (n \neq i) \\ d_i \nabla d_i - \frac{1}{2} d_i \nabla h_i & (n = i) \end{cases} \tag{20}$$

$$\gamma_n = \sum_{m=1}^{n-1} d_m \nabla h + \frac{1}{2} d_n \nabla h \tag{21}$$

$$\delta_n = \sum_{m=1}^{n-1} \nabla (d_m \nabla h) + \frac{1}{2} d_n \nabla (\nabla h) - \nabla h_{n-1} \nabla h \tag{22}$$

A remarkable feature of multi-level model is applicability to quasi-3 dimensional problem with vertical and horizontal closed boundaries. Vertical closed boundaries can be simply treated by setting the corresponding horizontal velocities to be zero. Horizontal closed boundaries can be treated by following manner(Fig.2). Vertical velocities at the horizontal closed boundaries are set to be zero by condition (23), where $-h_B$ is the depth of horizontal closed boundaries which corresponds to the lower edge of nB-th layer. In this case, the number of equations exceed the number of unknowns. To make the system closed ,a new unknown p_B the pressure under the closed boundary should be introduced.

$$w(-h_B) = -\sum_{i=nB}^{N} d_i \nabla \cdot \mathbf{u}_i - \mathbf{u}_b \cdot \nabla h = 0 \tag{23}$$

By expressing the modified Euler equation (5) using p_B, (24) is obtained for the region under the horizontal closed boundary.

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{1}{2} \nabla (\mathbf{u}_B \cdot \mathbf{u}_B) + \nabla (\frac{p_B}{\rho}) = -\nabla \int_z^{-h_B} \frac{\partial w}{\partial t} dz' \tag{24}$$

Equation(24) is expressed with dependent variables of multi-level model as (25). At the side edge of horizontal closed boundary, variable p_B is connected to that of ordinary region.

$$\begin{aligned} &\frac{\partial \mathbf{u}_n}{\partial t} + \frac{1}{2} \nabla (\mathbf{u}_B \cdot \mathbf{u}_B) + \nabla (\frac{p_B}{\rho}) \\ &= \sum_{i=nB+1}^{n-1} \sum_{j=i+1}^{N} \nabla (d_i d_j \nabla \cdot \frac{\partial \mathbf{u}_j}{\partial t}) + \sum_{i=nB+1}^{n-1} \frac{1}{2} \nabla (d_i^2 \nabla \cdot \frac{\partial \mathbf{u}_i}{\partial t}) \\ &+ \sum_{i=n+1}^{N} \frac{1}{2} d_n \nabla (d_i \nabla \cdot \frac{\partial \mathbf{u}_i}{\partial t}) - \sum_{i=n+1}^{N} (\nabla h_{n-1}) d_i \nabla \cdot \frac{\partial \mathbf{u}_i}{\partial t} \\ &+ \frac{1}{2} \nabla (d_n^2 \nabla \cdot \frac{\partial \mathbf{u}_n}{\partial t}) - \frac{1}{6} d_n \nabla (\nabla \cdot \frac{\partial \mathbf{u}_n}{\partial t}) - \frac{1}{2} d_n (\nabla h_n) \nabla \cdot \frac{\partial \mathbf{u}_n}{\partial t} \\ &+ \sum_{i=1}^{n-1} \nabla (d_i \frac{\partial \mathbf{u}_b}{\partial t} \cdot \nabla h) + \frac{1}{2} d_n \nabla (\frac{\partial \mathbf{u}_b}{\partial t} \cdot \nabla h) - (\nabla h_{n-1})(\frac{\partial \mathbf{u}_b}{\partial t} \cdot \nabla h) \end{aligned} \tag{25}$$

In this manner, the multi-level model proposed in this study can be applied to quasi-3 dimensional problem.

Properties of Model Equations

In this section we will analyze the linear and nonlinear characteristics of the present model. The procedure employed here is fundamentary the same as previous works, for example Madsen et.al(1996).

For simplicity, we treat the one dimensional version of the equations with constant depth, which are expressed by (26), the continuity equation and (27), the momentum equations with the velocities at the free surface expressed by (28) and (29).

$$\frac{\partial \eta}{\partial t}+\sum_{i=1}^{N}\frac{\partial}{\partial x}(d_i u_i)+\frac{\partial}{\partial x}(\eta u_1)-\frac{1}{2}\frac{\partial}{\partial x}[\eta(d_1+\eta)\sum_{i=1}^{N}d_i\frac{\partial^2 u_i}{\partial x^2}] +\frac{1}{6}\frac{\partial}{\partial x}[\eta(d_1^2-\eta^2)\frac{\partial^2 u_1}{\partial x^2}] =0 \tag{26}$$

$$\frac{\partial u_n}{\partial t}+u_s\frac{\partial u_s}{\partial x}+w_s\frac{\partial w_s}{\partial x}+g\frac{\partial \eta}{\partial x} =\frac{\partial}{\partial x}[\eta\sum_{i=1}^{N}d_i\frac{\partial^2 u_i}{\partial t\partial x}+\frac{1}{2}\eta^2\frac{\partial^2 u_1}{\partial t\partial x}]+\sum_{i=1}^{n-1}\sum_{j=i+1}^{N}d_i d_j\frac{\partial^3 u_j}{\partial t\partial x^2} +\frac{1}{2}\sum_{i=1}^{n-1}d_i^2\frac{\partial^3 u_i}{\partial t\partial x^2}+\frac{1}{2}\sum_{i=n+1}^{N}d_n d_i\frac{\partial^3 u_i}{\partial t\partial x^2}+\frac{1}{3}d_n\frac{\partial^3 u_n}{\partial t\partial x^2} \tag{27}$$

$$u_s=u_1+(\frac{1}{6}d_1^2-\frac{1}{2}\eta^2)\frac{\partial^2 u_1}{\partial x^2}-(\frac{1}{2}d_1+\eta)\sum_{i=1}^{N}d_i\frac{\partial^2 u_i}{\partial x^2} \tag{28}$$

$$w_s=-\sum_{i=1}^{N}d_i\frac{\partial u_i}{\partial x}-\eta\frac{\partial u_n}{\partial x}-u_b\frac{\partial h}{\partial x} \tag{29}$$

We look for solutions of the form expressed by (30) and (31), where ε is a small parameter.

$$\eta=\varepsilon\eta^{(1)}\cos(kx-\omega t)+\varepsilon^2\eta^{(2)}\cos 2(kx-\omega t) \tag{30}$$

$$u_n=\varepsilon u_n^{(1)}\cos(kx-\omega t)+\varepsilon^2 u_n^{(2)}\cos 2(kx-\omega t)) \tag{31}$$

Substituting them to (26) and (27) ,the equations for first order of ε are given by (32) and (33).

$$\omega\eta^{(1)}+k\sum_{i=1}^{N}d_i u_i^{(1)}=0 \tag{32}$$

$$\omega u_n^{(1)}-gk\eta^{(1)}+k\omega[(\sum_{i=1}^{n-1}\sum_{j=i+1}^{N}d_i d_j+\frac{1}{2}\sum_{i=1}^{n-1}d_i^2)u_1^{(1)}+\frac{1}{3}d_n^2 u_n^{(1)}]=0 \tag{33}$$

These are homogenious equations and non-trivial solutions require the determinant of the system to vanish to provide the dispersion relation.

In Fig. 2, the dispersion relation of the present model is compared with that of the linear theory. In this case, thickness of n-th layers are set to be n-times of first layer so that deeper layer has larger thickness. With the increasing number of layers, linear dispersion relation of the multi-level model converges to that of linear theory. Four layers are enough for sufficiently accurate dispersion for kh up to 10.

Distribution of $\{u_n^{(1)}\}$ for various values of kh can also be obtained from (32) and (33), which provide the vertical distribution of linear components of velocities according to (1) and (2). In Fig.3, vertical distribution of horizontal velocity $u(z)$ of the present model is compared with that of the linear theory. Fig. 4 shows plots similar to Fig.3 for vertical velocity $w(z)$. Although larger number of layers is required to reproduce the exact solution as kh increase, plots for N=6 and N=8 show very accurate reproduction so that they are hard to distinguish, for the range of kh smaller than 20.

Similary, the equations for second order of ε are given by (34) and (35). From these equations, the second order surface elevation and velocities are obtained, which should be compared with that of Stokes second order wave theory.

$$\omega\eta^{(2)} + k\sum_{i=1}^{N} d_i u_i^{(2)} = \frac{1}{2}k[(1-\frac{1}{6}d_1^2k^2)u_1^{(1)} + \frac{1}{2}d_1k^2\sum_{i=1}^{N} d_i u_n^{(1)}]\eta^{(1)} = 0 \tag{34}$$

$$\begin{aligned}(\omega + \frac{1}{3}kd_n^2)u^{(2)} + k\omega(\sum_{i=1}^{n-1}\sum_{j=i+1}^{N} d_i d_j + \frac{1}{2}\sum_{i=1}^{n-1} d_i^2)u_i^{(2)} - gk\eta^{(2)} \\ = \frac{1}{4}k[(1-\frac{1}{6}d_1^2k^2)u_1^{(1)} + \frac{1}{2}d_1k^2\sum_{i=1}^{N} d_i u_n^{(1)}]^2 \\ -\frac{1}{4}k^3(\sum_{i=1}^{N} d_i u_i^{(1)})^2 - \frac{1}{2}k^2\omega\eta^{(1)}\sum_{i=1}^{N} d_i u_i^{(1)}\end{aligned} \tag{35}$$

The water-surface elevation including second order of ε is expressed by equation(36), where H is wave height. On the other hand, according to Stokes second order theory, it also can be expressed by equation(37).

$$\eta = \frac{H}{2}\cos(kx-\omega t) + \frac{H^2\eta^{(2)}}{4(\eta^{(1)})^2}\cos 2(kx-\omega t) \tag{36}$$

$$\eta = \frac{H}{2}\cos(kx-\omega t) + \frac{H^2}{16}k\frac{\cosh kh(\cosh 2kh+2)}{(\sinh kh^3)}\cos 2(kx-\omega t) \tag{37}$$

The amplitude of second term of equation (36) and (37) are compared in Fig.5, both are plotted being divided by wave number. In this case, wave height is 20% of still water depth. It is seen that the results of the present model converge to that of Stokes wave theory, with the increasing number of layers. Thus, it is confirmed that nonlinear interaction characteristic of the present model is accurate up to the second-order.

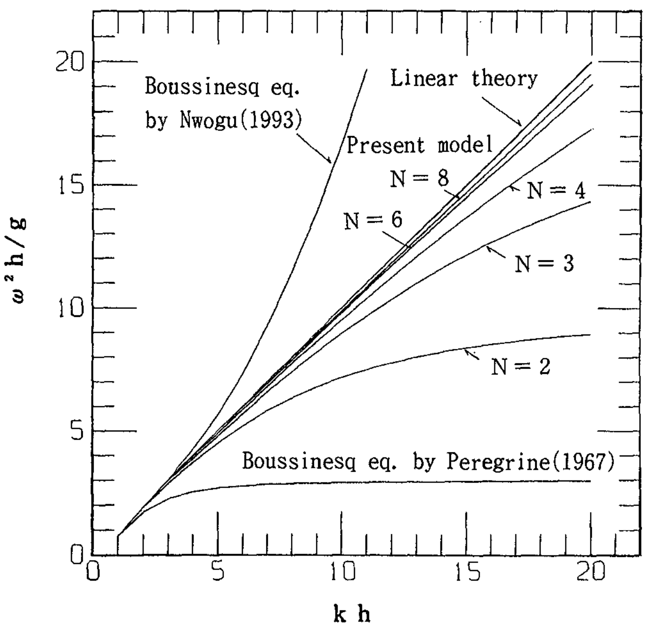

Fig.2 Linear dispersion relation

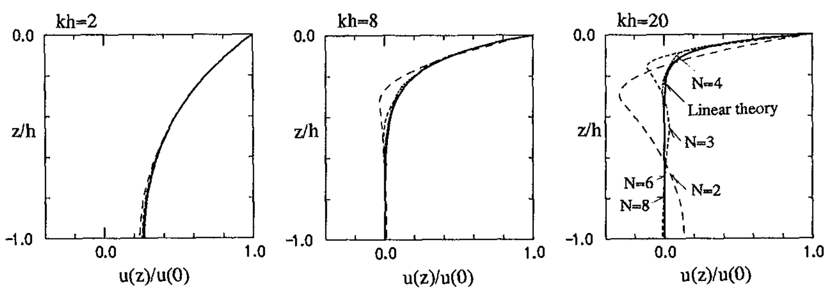

Fig.3 Vertical distribution of linear components of horizontal velocity $u(z)$

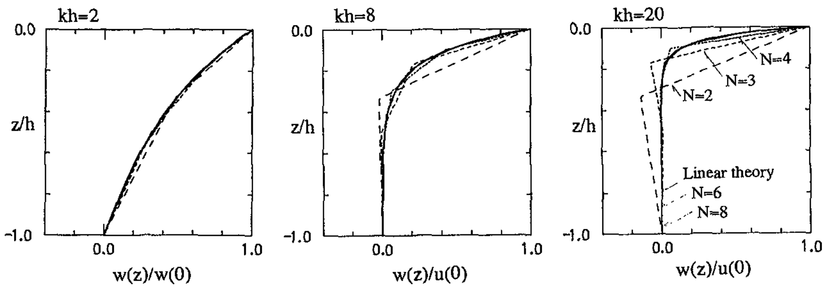

Fig.4 Vertical distribution of linear components of vertical velocity $w(z)$

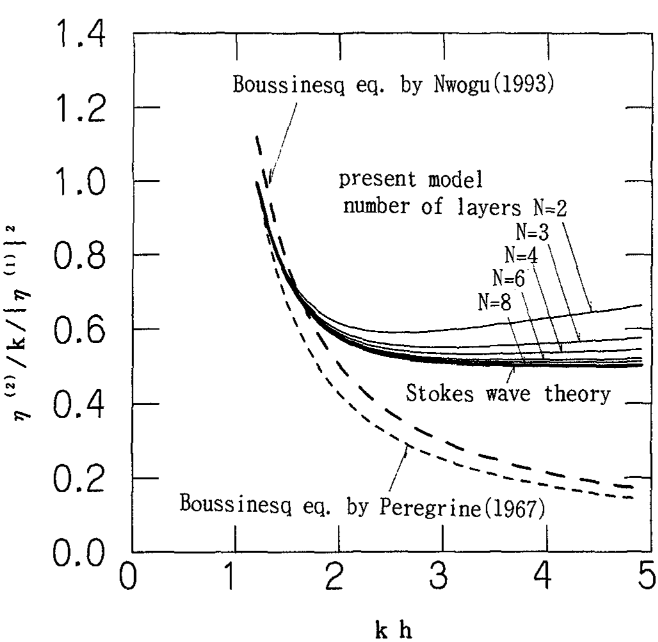

Fig.5 Second-order nonlinear interaction

Numerical Approach

The differential equations are discretized by using a time-centered implicit scheme with variables defined on a space-staggered rectangular grid. The resulting system of difference equations is reduced to a block tridiagonal system, which is solved by generalized Thomas algorithm.

An example of 2-HD simulation with three layers is for oblique wave incidence to sloping beach of 1:20. Regular, unidirectional waves with the period of 10sec and the wave height of 3m are generated at the depth of 24m with the incident angle of 25 deg. In this example, a little simplified version of momentum equation is employed, which are expressed by (38).

$$\frac{\partial \mathbf{u}_n}{\partial t} + (u_n \cdot \nabla) u_n + \frac{1}{d_n} \int_{-h_n}^{-h_{n-1}} w \frac{\partial u}{\partial z} dz + g \nabla \eta$$
$$= \sum_{i=1}^{n-1} \sum_{j=i+1}^{N} d_i d_j \nabla (\nabla \cdot \frac{\partial u_j}{\partial t}) + \sum_{i=1}^{n-1} \frac{1}{2} d_i^2 \nabla (\nabla \cdot \frac{\partial u_i}{\partial t})$$
$$+ \sum_{i=n+1}^{N} \frac{1}{2} d_n d_i \nabla (\nabla \cdot \frac{\partial u_i}{\partial t}) + \frac{1}{3} d_n^2 \nabla (\nabla \cdot \frac{\partial u_n}{\partial t}) + \varepsilon_T (\nabla \cdot \nabla) u_n \tag{38}$$

The third term in left hand side of (38) is expressed by (39) with dependent variables of multi-level model.

$$\frac{1}{d_n} \int_{-h_n}^{-h_{n-1}} w \frac{\partial u}{\partial z} dz = \frac{1}{3} d_n^2 \nabla u_n \nabla (\nabla \cdot u_n) + \frac{1}{2} \nabla (d_n \nabla u_n \sum_{i=n+1}^{N} d_i \nabla u_i)$$
$$+ (\sum_{i=n+1}^{N} d_i \nabla u_i)(\sum_{i=n+1}^{N} d_i \nabla (\nabla \cdot u_i)) \tag{39}$$

In this expression, bottom sloping, vertical acceleration over still water surface, and nonlinearity of vertical momentum equation are neglected. In equation(23), ε_T is the eddy viscosity according to breaking wave propagation model by Katayama and Sato(1993).

Fig.6 depicts a perspective views of calculated wave fields. A vector plot of the layer-averaged velocity of first and third layer is shown in Fig.7. Co-existence of longshore current and undertow is well described by the present model.

Next example is wave diffraction simulation by submerged horizontal plate. In this case, linear and 1-dimensional version of multi-level model with five layers is employed. Fig.8 shows the spatial profile of the propagating wave for different relative submerged depth of horizontal plate. The transmission coefficients K_T agree with the result of simulation by boundary element method, which is shown in brackets. Thus, the applicability of present model to quasi-3 dimensional problem is confirmed.

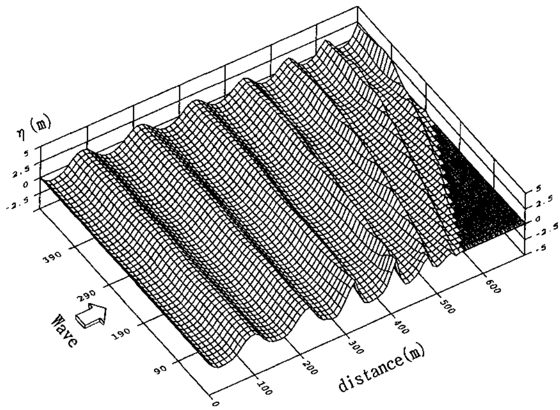

Fig.6 Perspective view of the wave field for oblique wave incidence

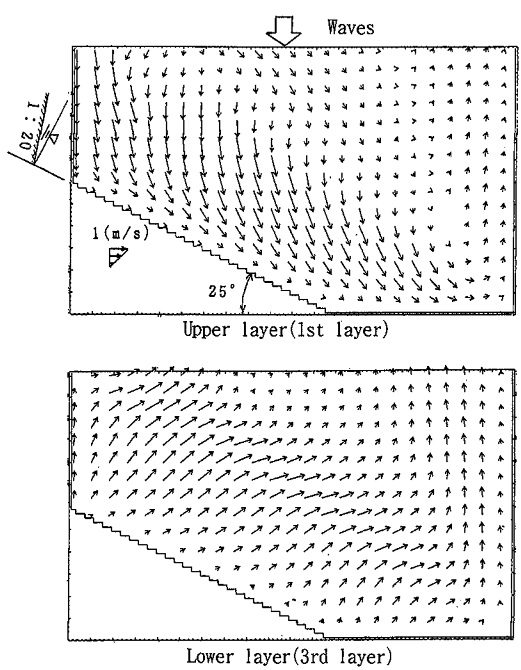

Fig.7 Time-averaged velocity field for oblique wave incidence

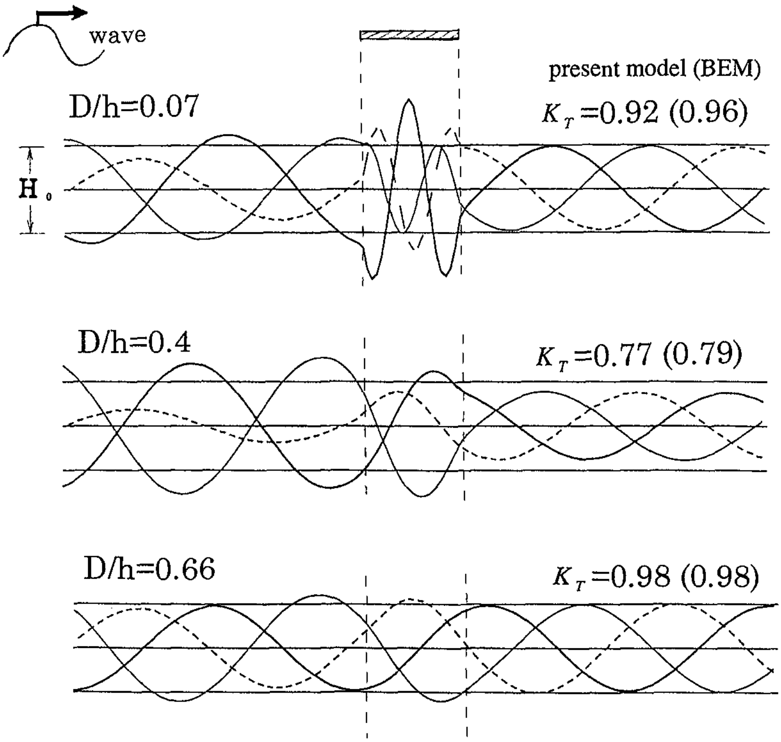

Fig.8 Computation for linear wave diffraction by submerged horizontal plate.

Conclusions

(1)A multi-level model is proposed for computing nonlinear dispersive waves. The accuracy of the model is first examined in terms of the convergence of linear dispersion relation along with quadratic transfer function. It is found that the model gives satisfactory results if the number of layers is properly chosen.
(2)The model is secondly applied to 2-HD wave propagation on sloping beach, and coexisting property of longshore current and undertow can be predicted.
(3)The applicability of the present model to quasi-3 dimensional problem is confirmed by reproducing the diffraction wave field with submerged horizontal plate.

References

Gobbi, M.F. and Kirby, J.T., 1996. A fourth order Boussinesq-type wave model, *Proceedings of 25th International Conference on Coastal Engineering*, pp.1116-1129.

Isobe, M., 1994. Time-dependent mild-slope equations for random waves, *Proceedings of 24th International Conference on Coastal Engineering*, pp.285-299.

Katayama, H. and Sato, S., 1993. A numerical model for simulating random breaking waves based on the Boussinesq equation, *Proceedings of Coastal Engineering*, Vol.40, pp.16-20. (in Japanese)

Kioka, W. and Kashihara, K., 1995. Higher-order Boussinesq equations and their applicability to the wave transformation over a step beach, *Proceedings of Coastal Engineering*, Vol.42, pp.166-170. (in Japanese)

Madsen, P.A., Banijamali, B., Schäffer, H.A. and Sørensen, O.R., 1996. Boussinesq type equations with high accuracy in dispersion and nonlinearity, *Proceedings of 25th International Conference on Coastal Engineering*, pp.95-108.

Nadaoka, K., Beji, S. and Nakagawa, Y., 1994. A fully-dispersive nonlinear wave model and its numerical solutions, *Proceedings of 24th International Conference on Coastal Engineering*, pp.427-441.

Nochino, M., 1994. Coupled vibration equations for irregular water waves, *Proceedings of 24th International Conference on Coastal Engineering*, pp.427-441.

Nwogu, O., 1993. Alternative form of Boussinesq equations for nearshore wave propagation, *Journal of Waterway, Port, Coastal, and Ocean Engineering*, Vol. 119, No.6, pp.618-638.