ON THE STATISTICAL VARIABILITY OF WAVE HEIGHT AND PERIOD PARAMETERS

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ABSTRACT

A new method for the determination of short-term variability of some commonly used wave height and period parameters is presented. The new TFD method combines some previously available time and frequency domain approaches, optimizing their application and giving greater accuracy with easy implementation. Short-term variability predicted by the new TFD method is consistently higher than the one obtained with theoretical frequency domain expressions, and proves to be in very good agreement with sampling variability observed in Gaussian, non-periodic and non-deterministic random wave records generated with stable ARMA numerical simulators.

INTRODUCTION

Random nature of ocean waves and their modelling as a stochastic process are basic assumptions commonly used in engineering problems. Although a great deal of effort has been dedicated to describe the statistical behaviour of some important sea state parameters, many problems related to their short-term variability still remain unsolved. Short-term variability can be defined as the statistical sampling variability of any time or frequency domain parameter when it is calculated from a finite length wave record that is supossed to be taken from a theoretically infinite length realization of the underlying stochastic process. Knowledge of short-term variability is important in many practical applications, such as: assessment of wave climate uncertainty; risk-based design and economic analysis of offshore and coastal structures; estimation of maintenance and insurance costs of maritime works; and design of physical or numerical experiments.

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Although continuous measurement of the sea surface movement is no problem with most modern wave recorders -like scalar and directional bouys or coastal and satellite radars-, wave climate non-stationarity makes necessary to divide long wave records into finite length time (space) intervals, if the commonly assumed stationarity (homogeneity) hypothesis is to be fulfilled, at least in a wide sense. Furthermore, many ongoing measurements are being made over short intervals and much existing data are in this form. Sampling variability can be reduced at the expense of a lost of resolution in the time or frequency domain by averaging over time intervals or frequency bands, but it can still be important for some purposes. This means that, even assuming ergodicity, we are forced to face up to sampling variability in practical applications.

One of the main difficulties found in theoretical developments for estimating short-term variability has its origin in the use of a specific wave discretization criterion for the analysis of wave records in the time domain (zero-up-crossing, zero-down-crossing, orbital, etc.). Two basic approaches have been proposed in the last decades to overcome this problem: (1) a numerical approach in the time domain (TD approach) that is based in the numerical simulation of random wave records (e.g., Goda, 1977, 1987); and (2) an analytical approach in the frequency domain (FD approach) that takes advantage of some relations between time domain and frequency domain parameters (e.g., Tucker, 1957; Cavanié, 1979; Giménez et *al.*, 1994b). Some interesting contributions to the theoretical and experimental study of sampling variability of spectral estimates and spectral integrals are discussed in Donelan and Pierson (1983), Young (1986), Elgar (1987) and Forristall *et al.* (1996), among others.

This paper presents a new method for the determination of short-term variability of some widely used wave height and period parameters. The new combined *time and frequency domains method* (TFD method) integrates the two aforementioned approaches, optimizing their application and giving greater accuracy with easy implementation.

VARIABILITY OF WAVE HEIGHT PARAMETERS

Henceforth, we will use the subscript R -in connexion with a time or frequency domain parameter- to point out that it has been calculated from a wave record of duration T_R. Additionally, we will use the subscript r -in connexion with a time domain parameter- to indicate that the orbital criterion for discretizing waves has been applied for the statistical analysis of a wave record. In the orbital criterion a discrete wave is defined by a complete rotation of a sea surface particle around its mean position. The orbital criterion has proved to be more consistent and robust (see Giménez *et al.*, 1994a) and to present less sampling variability (see Giménez *et al.*, 1994b) than the commonly used zero-up-crossing criterion. Furthermore, as shown by Giménez and Sánchez-Carratalá (1997), energy propagation in directional seas is closely related with orbital waves.

Making use of the approximate expression (A.13) obtained in Appendix A for the coefficient of variation of the product of two random variables x and y, we have that when:

$$x=\sqrt{m_{0,R}}\ ;\ y=\frac{H_{1/p,r,R}}{\sqrt{m_{0,R}}} \tag{1}$$

the coefficient of variation of $H_{1/p,r,R}$ is approximately given by:

$$\nu\left[H_{1/p,r,R}\right]\approx\sqrt{\nu^2\left[\sqrt{m_{0,R}}\right]+\nu^2\left[H_{1/p,r,R}/\sqrt{m_{0,R}}\right]} \tag{2}$$

where $\nu[\cdot]$ is an operator denoting the population coefficient of variation of the random variable considered as argument; $H_{1/p,r,R}$ is the mean orbital wave height of the $1/p$ highest waves in a wave record of duration T_R (specifically, when $p=1$ we obtain the mean wave height, $H_{1/1,r,R}=\bar{H}_{r,R}$; and when $p=3$ we obtain the significant wave height, $H_{1/3,r,R}$); and $m_{n,R}$ is the nth order spectral moment of a wave record of duration T_R.

On the one hand, the value of $\nu\left[\sqrt{m_{0,R}}\right]$ in Eq. (2) can be calculated analytically in the *frequency domain* using the following approximate expression derived by Cavanié (1979):

$$\nu\left[\sqrt{m_{0,R}}\right]\approx\sqrt{\frac{1}{4T_R m_0^2}\int_0^{+\infty}S_\eta^2(f)\,df} \tag{3}$$

where f is the cyclic frequency; $S_\eta(f)$ is the variance spectrum of the process; and m_n is the nth order spectral moment of the process. Eq. (3) can be reformulated using the well-known spectral peakedness parameter Q_e proposed by Medina and Hudspeth (1987), so that:

$$\nu\left[\sqrt{m_{0,R}}\right]\approx\sqrt{\frac{Q_e}{8N_{w,r,R}}} \tag{4}$$

where $N_{w,r,R}=T_R/T_{01}$ is the spectral estimation of the number of orbital waves in a wave record of duration T_R; and $T_{01}=m_0/m_1$ is the spectral estimation of the mean orbital wave period of the process, as shown by Giménez *et al.* (1994a).

On the other hand, the value of $\nu\left[H_{1/p,r,R}/\sqrt{m_{0,R}}\right]$ in Eq. (2) should be calculated numerically in the *time domain* using a harmonic DSA random wave numerical simulator (see Tuah and Hudspeth, 1982), as suggested by Sánchez-Carratalá (1995). This kind of simulator has no variability in the frequency domain for a record duration equal to its recycling period, and thus:

$$\nu\left[H_{1/p,r,R}/\sqrt{m_{0,R}}\right]=\nu_{H\text{-}DSA}\left[H_{1/p,r,R}\right]\simeq n_{H\text{-}DSA}\left[H_{1/p,r,R}\right] \tag{5}$$

where $n[\cdot]$ is an operator denoting the sample coefficient of variation of the random variable considered as argument

The value of $n_{H\text{-}DSA}\left[H_{1/p,r,R}\right]$ has been calculated for different wave height parameters ($p=1,3,10$), different record durations ($T_R=2^k\Delta t$, $k=6,7,\ \ ,13$, $\Delta t=1$ s), and different peak enhancement factors of a JONSWAP-type spectrum ($\gamma=1,3\ 3,7$), using samples of 200 wave records generated with an H-DSA(2^{k-1})-FFT numerical simulator, that is, a harmonic wave superposition simulator with 2^{k-1} one-sided DSA frequency components, implemented with an FFT algorithm According to theoretical and numerical evidence (see, e g , Goda, 1987), the results obtained have been fitted with a least squares technique to a function of the following type

$$n_{H\text{-}DSA}\left[H_{1/p,r,R}\right]\simeq kT_R^{-n} \tag{6}$$

Table 1 gives the coefficients $k=k(p,\gamma)$ and $n=n(p,\gamma)$ in Eq (6), obtained for each wave height parameter ($p=1,3,10$) and each peak enhancement factor ($\gamma=1,3\ 3,7$), with T_R in seconds

SEA STATE PARAMETER	$\gamma=1$		$\gamma=3.3$		$\gamma=7$	
	k	n	k	n	k	n
$\bar{H}_{r,R}$	0 8656	0 5279	0 8583	0 5136	0 8191	0 4965
$H_{1/3,r,R}$	1 2644	0 6190	1 2334	0 6152	1 2674	0 6114
$H_{1/10,r,R}$	2 0142	0 5751	2 0054	0 5592	1 6094	0 5125
$\bar{T}_{r,R}$	0 8501	0 5062	0 8078	0 5026	0 7184	0 4940

TABLE 1 Coefficients k and n in Eqs (6) and (11) for different sea state parameters (T_R in seconds)

VARIABILITY OF WAVE PERIOD PARAMETERS

Making use of the approximate expression (A 13) obtained in Appendix A for the coefficient of variation of the product of two random variables x and y, we have that when

$$x=T_{01,R}\ ;\ y=\frac{\bar{T}_{r,R}}{T_{01,R}} \tag{7}$$

the coefficient of variation of $\bar{T}_{r,R}$ is approximately given by:

$$\nu\left[\bar{T}_{r,R}\right] \simeq \sqrt{\nu^2\left[T_{01,R}\right]+\nu^2\left[\bar{T}_{r,R}/T_{01,R}\right]} \tag{8}$$

where $\bar{T}_{r,R}$ is the mean orbital wave period of a wave record of duration T_R; and $T_{01,R}=m_{0,R}/m_{1,R}$ is the spectral estimation of the mean orbital wave period of a wave record of duration T_R.

On the one hand, the value of $\nu\left[T_{01,R}\right]$ in Eq. (8) can be calculated analytically in the *frequency domain* using the following approximate expression derived by Giménez et *al.* (1994b):

$$\nu\left[T_{01,R}\right] \simeq \sqrt{\frac{1}{T_R m_0^2}\int_0^{+\infty}\left(T_{01}f-1\right)^2 S_\eta^2(f)\,df} \tag{9}$$

On the other hand, the value of $\nu\left[\bar{T}_{r,R}/T_{01,R}\right]$ in Eq. (8) should be calculated numerically in the *time domain* using a harmonic DSA random wave numerical simulator, as suggested by Sánchez-Carratalá (1995). This kind of simulator has no variability in the frequency domain for a record duration equal to its recycling period, and thus:

$$\nu\left[\bar{T}_{r,R}/T_{01,R}\right]=\nu_{H\text{-}DSA}\left[\bar{T}_{r,R}\right] \simeq n_{H\text{-}DSA}\left[\bar{T}_{r,R}\right] \tag{10}$$

The value of $n_{H\text{-}DSA}\left[\bar{T}_{r,R}\right]$ has been calculated for different record durations ($T_R=2^k\Delta t$, $k=6,7,\ldots,13$; $\Delta t=1$ s), and different peak enhancement factors of a JONSWAP-type spectrum ($\gamma=1,3.3,7$), using samples of 200 wave records generated with an H-DSA(2^{k-1})-FFT numerical simulator. According to theoretical and numerical evidence (see, e.g., Goda, 1987), the results obtained have been fitted with a least squares technique to a function of the following type:

$$n_{H\text{-}DSA}\left[\bar{T}_{r,R}\right] \simeq k\,T_R^{-n} \tag{11}$$

Table 1 gives the coefficients $k=k(\gamma)$ and $n=n(\gamma)$ in Eq. (11), obtained for each peak enhancement factor ($\gamma=1,3.3,7$), with T_R in seconds.

Table 2 gives a comparison between short-term variability predicted by the FD approach and the new TFD method, for each peak enhancement factor ($\gamma=1,3.3,7$), and for a record duration of about 100 waves. According to these results, sampling variability of wave height and period parameters included in this study, is systematically underestimated by the presently available frequency domain expressions

SEA STATE PARAMETER	$\frac{v_{TFD}[x_R]}{v_{FD}[x_R]}-1$ (%)		
	$\gamma=1$	$\gamma=3.3$	$\gamma=7$
$\bar{H}_{r,R}$	10 7	7 6	5 9
$H_{1/3,r,R}$	7 0	4 1	3 0
$H_{1/10,r,R}$	28 7	21 5	17 4
$\bar{T}_{r,R}$	56 6	54 3	56 7

TABLE 2 Comparison between short-term variability predicted by the FD approach and the new TFD method for different sea state parameters (T_R in seconds) ($N_{w,r,R}\simeq 100$)

CONTRAST WITH NUMERICAL SIMULATIONS

The new TFD method herein presented has been compared with sampling variability estimates obtained by numerical simulation, in order to test its performance Samples of 100 wave records with about 1000 waves each one (N=8192, $\Delta t=1$ s), corresponding to different sea states characterized by a JONSWAP-type spectrum ($\gamma=1,3$ 3,7), have been generated using an AR(100)-RT(0 05) numerical simulator, that is, a digital linear filter with 100 autoregressive parameters, fitted with the robust technique proposed by Medina and Sánchez-Carratalá (1991), with only a 0 05% of white noise in the target spectrum Digital filters obtained with the robust technique are always stable, and present extremely low fitting errors according to the hierarchic criteria introduced by Medina and Sánchez-Carratalá (1988) for qualifying ARMA representations of ocean wave spectra

Figs 1, 2, 3 and 4 show the evolution of the coefficient of variation of $\bar{H}_{r,R}$, $H_{1/3,r,R}$, $H_{1/10,r,R}$, and $\bar{T}_{r,R}$, respectively, as a function of the number of waves $N_{w,r,R}$, for different peak enhancement factors ($\gamma=1,3$ 3,7). The continuous thin line represents the sampling variability predicted by the new TFD method according to Eqs (2) or (8), while the dashed thin lines are an estimation of the corresponding 95% confidence intervals The thick line represents the sampling variability obtained from a sample of numerically simulated wave records, showing the overall good performance of the new TFD method for predicting sampling variability of wave height and period parameters in random Gaussian seas.

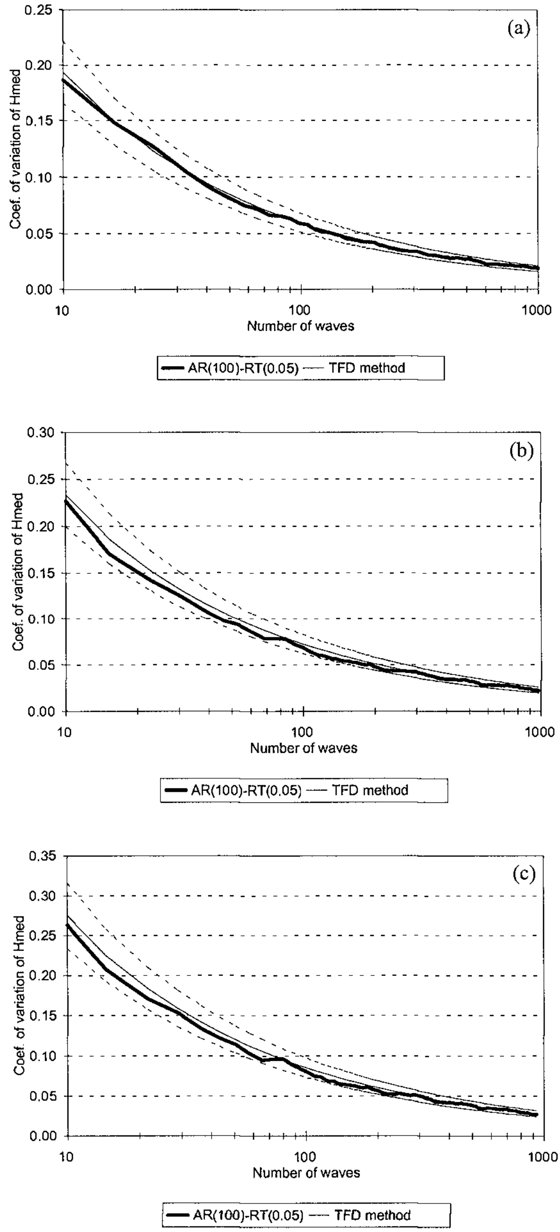

FIGURE 1. Evolution of $\nu\left[\bar{H}_{r,R}\right]$ as a function of $N_{w,r,R}$; $\Delta t=1$ s; JONSWAP target spectrum ($H_{m_0}=4.0$ m; $f_p=0.1$ Hz; $\sigma_a=0.07$; $\sigma_b=0.09$; $f_{min}=0.5f_p$, $f_{max}=6.0f_p$).
(a) $\gamma=1$; **(b)** $\gamma=3.3$; **(c)** $\gamma=7$.

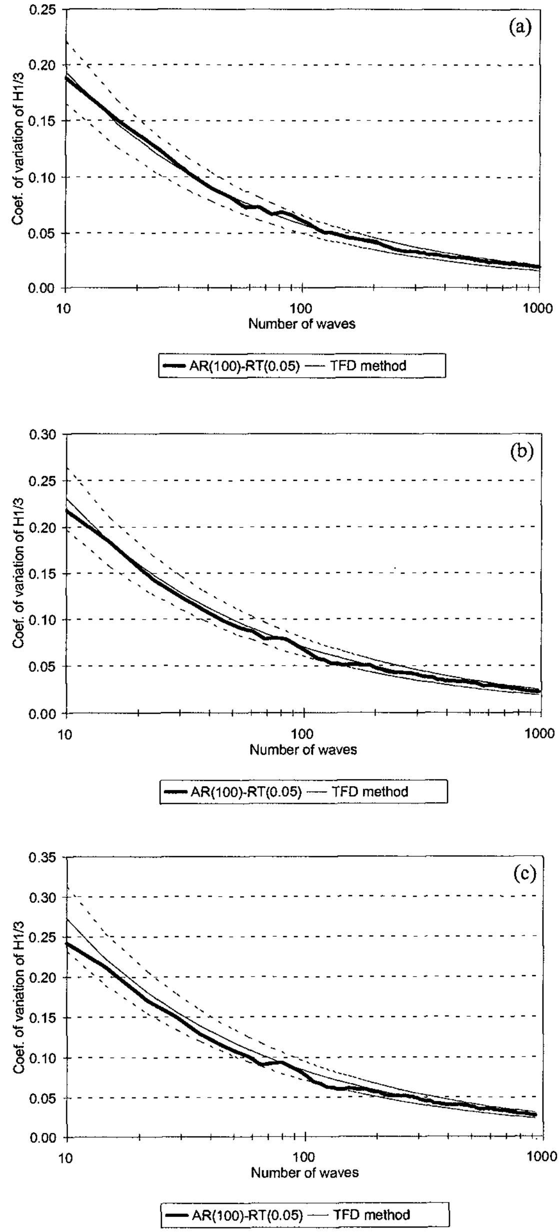

FIGURE 2. Evolution of $\nu\left[H_{1/3,r,R}\right]$ as a function of $N_{w,r,R}$; $\Delta t=1$ s; JONSWAP target spectrum ($H_{m_0}=4.0$ m; $f_p=0.1$ Hz; $\sigma_a=0.07$; $\sigma_b=0.09$; $f_{min}=0.5f_p$, $f_{max}=6.0f_p$).
(a) $\gamma=1$; **(b)** $\gamma=3.3$; **(c)** $\gamma=7$.

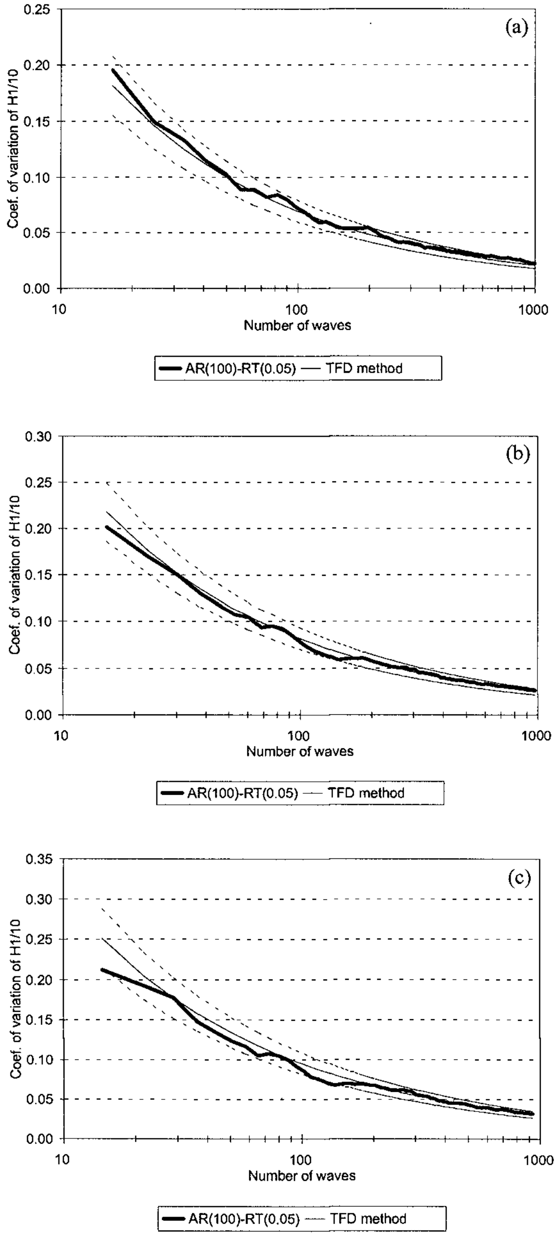

FIGURE 3. Evolution of $\nu\left[H_{1/10,r,R}\right]$ as a function of $N_{w,r,R}$; Δt=1 s; JONSWAP target spectrum (H_{m_0}=4.0 m; f_p=0.1 Hz; σ_a=0.07; σ_b=0.09; f_{min}=0.5f_p, f_{max}=6.0f_p).
(a) γ=1; **(b)** γ=3.3; **(c)** γ=7.

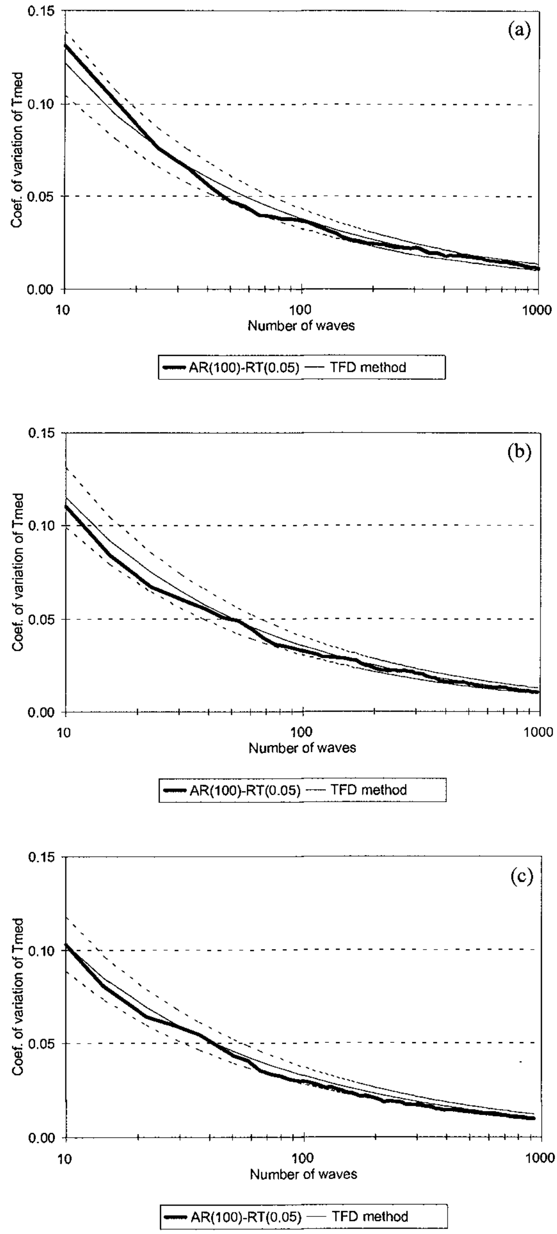

FIGURE 4. Evolution of $\nu\left[\bar{\bar{T}}_{r,R}\right]$ as a function of $N_{w,r,R}$; $\Delta t=1$ s; JONSWAP target spectrum ($H_{m_0}=4.0$ m; $f_p=0.1$ Hz; $\sigma_a=0.07$; $\sigma_b=0.09$; $f_{min}=0.5f_p$, $f_{max}=6.0f_p$).
(a) $\gamma=1$; **(b)** $\gamma=3.3$; **(c)** $\gamma=7$.

CONCLUSIONS

A new method for the determination of short-term variability of some commonly used wave height and period parameters has been developed as a combination of some previous time and frequency domain approaches. Short-term variability predicted by the new *time and frequency domains method* (TFD method) is consistently higher than the one obtained with theoretical frequency domain expressions. The difference is specially notorious for $H_{1/10,r,R}$ (17 ~ 29%) and $\overline{T}_{r,R}$ (54 ~ 57%).

Results obtained with the new TFD method have proved to be in very good agreement with sampling variability observed in Gaussian, non-periodic and non-deterministic random wave records generated with stable ARMA numerical simulators, thus constituting a straightforward and reliable alternative for the prediction of short-term variability of many time domain parameters.

APPENDIX A. Coefficient of variation of the product of two random variables

Let x and y be two random variables distributed in the interval $[0,+\infty[$ with a joint probability density function $p(x,y)$. The aim of this Appendix is to obtain an approximate expression for the coefficient of variation of the product xy.

Let a be a random variable defined as the following function of x and y:

$$a=a(x,y)=xy \tag{A.1}$$

The first and second order partial derivatives of a with respect to x and y are:

$$\begin{cases} \dfrac{\partial a}{\partial x}=y \quad ; \quad \dfrac{\partial a}{\partial y}=x \\ \dfrac{\partial^2 a}{\partial x^2}=0 \quad ; \quad \dfrac{\partial^2 a}{\partial y^2}=0 \quad ; \quad \dfrac{\partial^2 a}{\partial x \partial y}=1 \end{cases} \tag{A.2}$$

A Taylor series expansion of a around the point (x_0,y_0) gives:

$$\begin{aligned} a=(a)_0&+\frac{1}{1!}\left(\frac{\partial a}{\partial x}\right)_0(x-x_0)+\frac{1}{1!}\left(\frac{\partial a}{\partial y}\right)_0(y-y_0)+ \\ &+\frac{1}{2!}\left(\frac{\partial^2 a}{\partial x^2}\right)_0(x-x_0)^2+\frac{1}{2!}\left(\frac{\partial^2 a}{\partial y^2}\right)_0(y-y_0)^2+\frac{1}{1!1!}\left(\frac{\partial^2 a}{\partial x \partial y}\right)_0(x-x_0)(y-y_0)+\dots \end{aligned} \tag{A.3}$$

On substituting (A.2) in (A.3) for $x_0=E[x]$ and $y_0=E[y]$, we obtain the following exact expression for a:

$$a = E[x]\,E[y] + E[y]\,(x - E[x]) + E[x]\,(y - E[y]) + (x - E[x])\,(y - E[y]) =$$
$$= E[x]\,E[y]\left[1 + \frac{(x - E[x])}{E[x]} + \frac{(y - E[y])}{E[y]} + \frac{(x - E[x])\,(y - E[y])}{E[x]\,E[y]}\right] \tag{A.4}$$

where $E[x]$ is the expected value of the random variable x.

Hence, the mean value of a is:

$$E[a] = \int_0^{+\infty}\int_0^{+\infty} a\,p(x,y)\,dx\,dy =$$
$$= E[x]\,E[y]\left(1 + \frac{C[x,y]}{E[x]\,E[y]}\right) = E[x]\,E[y]\,(1 + c\,[x,y]\,\nu[x]\,\nu[y]) \tag{A.5}$$

where $C[x,y]$ and $c[x,y]$ are, respectively, the covariance and the normalized covariance of the random variables x and y; and $\nu[x]$ is the coefficient of variation of the random variable x, given by:

$$\nu[x] = \frac{\sigma[x]}{E[x]} \tag{A.6}$$

where $\sigma[x]$ is the standard deviation of the random variable x.

Let b be a random variable defined as the following function of x and y:

$$b = b(x,y) = x^2 y^2 \tag{A.7}$$

that is, $b = a^2$.

The first and second order partial derivatives of b with respect to x and y are:

$$\begin{cases} \dfrac{\partial b}{\partial x} = 2xy^2 \;;\; \dfrac{\partial b}{\partial y} = 2x^2 y \\ \dfrac{\partial^2 b}{\partial x^2} = 2y^2 \;;\; \dfrac{\partial^2 b}{\partial y^2} = 2x^2 \;;\; \dfrac{\partial^2 b}{\partial x \partial y} = 4xy \end{cases} \tag{A.8}$$

A Taylor series expansion of b around the point (x_0, y_0) gives:

$$b=(b)_0+\frac{1}{1!}\left(\frac{\partial b}{\partial x}\right)_0(x-x_0)+\frac{1}{1!}\left(\frac{\partial b}{\partial y}\right)_0(y-y_0)+$$
$$+\frac{1}{2!}\left(\frac{\partial^2 b}{\partial x^2}\right)_0(x-x_0)^2+\frac{1}{2!}\left(\frac{\partial^2 b}{\partial y^2}\right)_0(y-y_0)^2+\frac{1}{1!1!}\left(\frac{\partial^2 b}{\partial x\partial y}\right)_0(x-x_0)(y-y_0)+\ldots \tag{A.9}$$

On substituting (A.8) in (A.9) for $x_0=E[x]$ and $y_0=E[y]$, we obtain the following approximate expression:

$$b\simeq E^2[x]E^2[y]+2E[x]E^2[y](x-E[x])+2E^2[x]E[y](y-E[y])+$$
$$+E^2[y](x-E[x])^2+E^2[x](y-E[y])^2+4E[x]E[x](x-E[x])(y-E[y])=$$
$$=E^2[x]E^2[y]\left[1+2\frac{(x-E[x])}{E[x]}+2\frac{(y-E[y])}{E[y]}+\right.$$
$$\left.+\frac{(x-E[x])^2}{E^2[x]}+\frac{(y-E[y])^2}{E^2[y]}+4\frac{(x-E[x])(y-E[y])}{E[x]E[y]}\right] \tag{A.10}$$

Hence, the mean value of b is:

$$E[b]=\int_0^{+\infty}\int_0^{+\infty}b\,p(x,y)\,dx\,dy\simeq$$
$$\simeq E^2[x]E^2[y]\left(1+\frac{\sigma^2[x]}{E^2[x]}+\frac{\sigma^2[y]}{E^2[y]}+4\frac{C[x,y]}{E[x]E[y]}\right)= \tag{A.11}$$
$$=E^2[x]E^2[y]\left(1+v^2[x]+v^2[y]+4c[x,y]\,v[x]\,v[y]\right)$$

Now from (A.5) and (A.11) we find that the square of the coefficient of variation of a is given by:

$$v^2[a]=\frac{\sigma^2[a]}{E^2[a]}=\frac{E[b]-E^2[a]}{E^2[a]}\simeq$$
$$\simeq\frac{E^2[x]E^2[y]\left(v^2[x]+v^2[y]+2c[x,y]\,v[x]\,v[y]-c^2[x,y]\,v^2[x]\,v^2[y]\right)}{E^2[x]E^2[y]\left(1+2c[x,y]\,v[x]\,v[y]+c^2[x,y]\,v^2[x]\,v^2[y]\right)} \tag{A.12}$$

Assuming that $v[x]\ll 1$, $v[y]\ll 1$ and $c[x,y]\ll 1$, we obtain the following approximate expression for the coefficient of variation of the product xy:

$$v[xy]\simeq\sqrt{v^2[x]+v^2[y]} \tag{A.13}$$

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