Eddy Viscosity Models for Wave Boundary Layers

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Abstract

Motivated by recent experimental results on wave, current and combined wave-current flows over an artificially rippled bed, the boundary resistance experienced by waves over two-dimensional bottom roughness elements is formulated in terms of a drag law. The resulting empirical relationship for the drag coefficient suggests a flow resistance that is similar in nature to one obtained from a constant, pseudo-laminar eddy viscosity model for the wave boundary layer flow. Analysis of available experimental data on energy dissipation for oscillatory flow over movable rippled beds leads to a constant eddy viscosity model for wave boundary layers above naturally rippled beds. The constant eddy viscosity model is modified to include a near-bottom linear transition to make it zero at the bed. This hybrid eddy viscosity model is shown to capture the essential features of wave boundary layer flows for the full range of bottom roughnesses encountered, i.e. from sand grains to ripples. Application of the hybrid model requires knowledge of the equivalent bottom roughness for which empirical expressions are derived. The implication of the results, obtained here for waves, for combined wave-current boundary layer flows suggests modifications of the Grant-Madsen model that greatly improve this model's ability to predict observed current velocity profiles over rippled bottoms in the presence of waves.

Introduction

In recent papers Mathisen and Madsen (1996a and b, hereafter referred to as MM) reported results from laboratory experiments on currents and waves, separately as well as combined, over a bottom covered by fixed equally spaced 1.5-cm-high triangular two dimensional roughness elements. The major result of their study was that waves and

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currents perpendicular to the axes of their artificial ripples experience the same equivalent bottom roughness, i.e., the equivalent bottom roughness, k_N, was shown to be a function of the bottom roughness characteristics only and independent of the flow. Besides being limited to the particular flow conditions of their experiments, i.e. wave, current and combined co-directional wave-current flows perpendicular to fixed two-dimensional roughness elements, their conclusion of k_N's single-valuedness is limited to analyses of these flows based on a modified version of the Grant-Madsen model (Grant and Madsen, 1979 and 1986; hereafter referred to as GM) since this model was used by MM in their analysis of experimental data.

To make this latter limitation more explicit, and also to introduce concepts and notation to be used in subsequent sections, MM obtained experimental values for the average rate of energy dissipation per unit bottom area, D, from measurements of wave attenuation and expressed D in terms of turbulent wave bottom boundary layer theory,

$$D = \frac{1}{4}\rho\, f_w \cos\varphi\, u_{bm}^3 = \frac{1}{4}\rho\, f_e\, u_{bm}^3 \tag{1}$$

in which ρ denotes the fluid density; f_w is the wave friction factor defined by

$$\frac{\tau_{bm}}{\rho} = u_{*m}^2 = \frac{1}{2} f_w\, u_{bm}^2 \tag{2}$$

where τ_{bm} is the maximum bottom shear stress; and φ is the phase lead of the bottom shear stress relative to the periodic near-bottom wave orbital velocity

$$u_b = u_{bm} \cos\omega t \tag{3}$$

Thus, from their experiments MM obtained values for the energy dissipation factor, $f_e=f_w\cos\varphi$, which, in turn, may be related to the relative bottom roughness, $u_{bm}/(k_N\omega)$, through the use of a theoretical model for turbulent wave boundary layer flows over a rough bottom. In taking this last step, which leads to the determination of the equivalent bottom roughness, k_N, MM found it necessary to modify the GM model in order to obtain the k_N–values that were in agreement with the values obtained from analysis of measured current velocity profiles over the same bottom roughness configuration. This modified version of the original GM-model has been presented by Madsen (1994) and consists of the determination of the wave friction factor, f_w, and the phase lead, φ, by evaluating the bottom shear stress at $z = z_0 = k_N/30$, where z is the vertical distance above the theoretical bed level, instead of in the limit $z \rightarrow 0$.

However, when the modified GM-model is used to predict the structure of the wave orbital velocity profile within the wave boundary layer this is found to be in poor agreement with measurements. In particular, the GM-model's prediction of the wave boundary layer thickness

$$\delta_w = A\frac{\kappa u_{*m}}{\omega} \tag{4}$$

in which κ is von Karman's constant ($=0.4$) and A is a scaling factor given the value of 1 to 2 by GM, is found to be too small by a factor of 2 to 3 when compared with measurements. This disturbing observation has profound implications for the GM-model's ability to predict the velocity profile of currents in the presence of waves, since $z=\delta_w$ is the location where the eddy viscosity is assumed to change from being scaled by the wave-current shear velocity, u_{*m}, to being scaled by the generally much smaller current shear velocity, u_{*c}. Thus, in order to obtain a reasonable agreement between predicted and observed current velocity profiles in the presence of waves, MM found it necessary to introduce an artificially "enhanced" wave boundary layer thickness. This apparent inability of the GM-model to predict the velocity structure as well as the thickness of the wave boundary layer for flows over a rippled bottom motivated the present study.

Drag Law Formulation and its Implications

In an attempt to break with the conventional treatment of wave boundary layer flows based on an eddy viscosity formulation, we express the flow resistance experienced by an oscillatory flow over a bottom covered by two-dimensional ripples in terms of the drag force exerted on the flow by the individual roughness elements. Formally, we may write this drag force per unit length

$$F_D = \frac{1}{2}\rho C_D \eta |u_b| u_b \tag{5}$$

in which η is the height of the ripple, u_b is the wave orbital velocity given by (3), and C_D is a drag coefficient. Assuming this drag force to dominate the flow resistance, i.e. neglecting the contribution of skin friction shear stress acting directly on the bottom between roughness elements (ripple crests), the rate of energy dissipation associated with this drag force is given by $F_D\, u_b$. Since this represents the rate of energy dissipation per roughness element, we obtain the time average rate of energy dissipation per unit area

$$D = \frac{\overline{F_D u_b}}{\lambda} = \frac{2}{3\pi}\rho C_D \frac{\eta}{\lambda} u_{bm}^3 \tag{6}$$

where "overbar" denotes time-averaging and λ the ripple spacing (length). In passing it is noted that the neglect of a potential inertia force has no effect on the calculated average energy dissipation rate since this force would be proportional to $\partial u_b/\partial t$ and hence time-average to zero.

Comparison of the expressions obtained from conventional boundary layer theory, (1), and drag law formulation, (6), shows that

$$C_D = \frac{3\pi}{8}(\lambda/\eta) f_e \tag{7}$$

Thus, the experiments on wave attenuation performed by MM, which provide values of f_e for given λ (10 or 20 cm) and η (1.5 cm), may be used directly in (7) to obtain experimental values for the drag coefficient, C_D, for waves over the artificially rippled bed. In analogy with drag coefficients for circular cylinders, e.g. Sarpkaya and Isaacson (1981), one would expect the drag coefficient to be a function of a Reynolds number, *Re*, and a Keulegan-Carpenter number, *KC*. Recognizing that the roughness elements in the experiments performed by MM were 90° angle iron bars placed on the glass bottom of a flume, i.e. effectively corresponding to half cylinders, the equivalent "diameters" of their two-dimensional roughness elements is 2η. Therefore, one would be seeking an empirical expression of the form

$$C_D = C_D\left(Re = \frac{2\eta u_{bm}}{\nu}, KC = \frac{u_{bm}T}{2\eta} = \pi\frac{A_b}{\eta} \right) \tag{8}$$

where ν is the kinematic viscosity of water and $A_b = u_{bm}/\omega = u_{bm}T/(2\pi)$ is the near-bottom wave orbital excursion amplitude.

For the experiments performed by MM the range of Reynolds Numbers is (3 to 6).10^3, i.e. a range for which *Re*-dependency is expected to be weak. As seen from the values of C_D plotted against η/A_b in Figure 1, the excellent correlation supports this anticipation and results in the empirical drag coefficient relationship given by

$$C_D = C_{Do}\frac{\eta}{A_b} = 9.0\frac{\eta}{A_b} \tag{9}$$

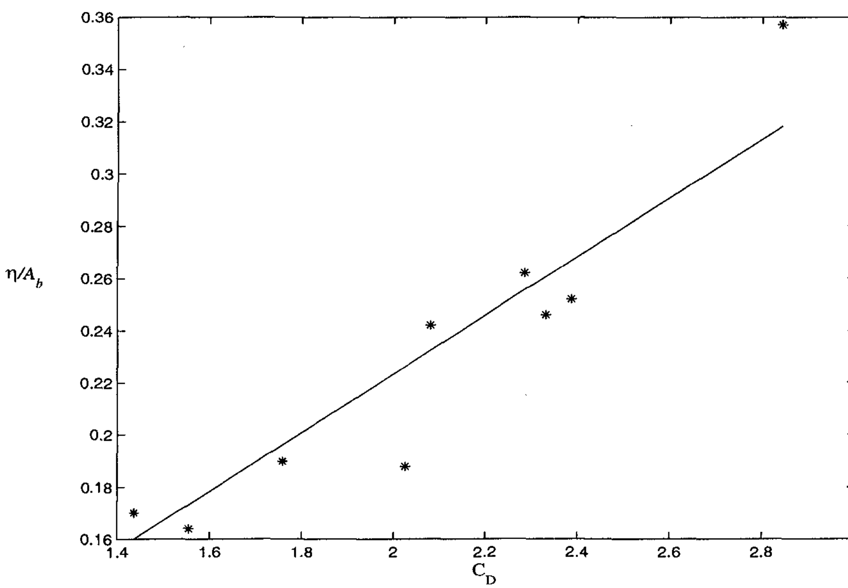

Figure 1: Ratio of the ripple height over the bottom excursion amplitude, η/A_b, as a function of the drag coefficient, C_D. Experimental values (*stars*) and linear fitting (*solid line*). The coefficient of determination is $r^2 = 0.87$.

The limitation of this relationship to the experimental range of η/A_b values from which it is obtained should be kept in mind. Extension beyond this range leads to obvious errors, e.g. $C_D \rightarrow 0$ or ∞ as $\eta/A_b \rightarrow 0$ or ∞. However, for the range covered by (9) the dependency of C_D on η/A_b is similar to the dependency on Keulegan-Carpenter number for circular cylinders and $Re = 10^4$ (Sarpkaya and Isaacson, 1981, Figure 3.15; Salles, 1997).

A Pseudo-Laminar Eddy Viscosity Model

When the empirical drag coefficient relationship, (9), is introduced in the drag law expression for the rate of energy dissipation, (6), and $u_{bm} = A_b\omega$ is utilized one obtains

$$D = \rho\left(\frac{2}{3\pi}C_{Do}\frac{\eta^2}{\lambda}\right)\omega\, u_{bm}^2 \tag{10}$$

This form of the average dissipation rate, in particular, its proportionality to the square of the wave orbital velocity, resembles the expression obtained from a laminar or constant eddy viscosity model of oscillatory boundary layers, e.g. Jonsson (1966)

$$D = \rho\left(\frac{1}{4}\sqrt{2\nu_c/\omega}\right)\omega\, u_{bm}^2 \tag{11}$$

in which ν_c denotes the value of the constant viscosity.

Equating (10) and (11) leads to a functional relationship for an equivalent constant, pseudo-laminar eddy viscosity

$$\nu_c = \left(\frac{8}{3\pi}C_{Do}\right)^2 \pi\frac{\eta^4}{\lambda^2 T} \cong 180\frac{\eta^4}{\lambda^2 T} \tag{12}$$

where C_{Do} = 9.0, obtained from MM's experiments, was introduced. With a constant eddy viscosity, the classical solution of Stokes gives the wave orbital velocity profile as the real part of the complex expression ($i^2 = -1$)

$$u_w = \left(1 - \mathrm{e}^{-(1+i)z/\delta}\right)u_{bm}\mathrm{e}^{i\omega t} \tag{13}$$

with

$$\delta = \sqrt{2\nu_c/\omega} \tag{14}$$

denoting the boundary layer scale.

The wave boundary layer thickness, δ_w, is obtained by requiring the orbital velocity to approach the free stream velocity to within a small, but finite, fraction, ε, of the free stream velocity. Defining δ_w in this manner and making use of (13) and (14) results in

$$\delta_w = \left(-\ln\varepsilon\right)\delta = \begin{Bmatrix}3.0\\4.6\end{Bmatrix}\delta \quad \text{for} \quad \varepsilon = \begin{Bmatrix}5\%\\1\%\end{Bmatrix} \tag{15}$$

Figure 2 demonstrates that the pseudo-laminar wave orbital velocity profile obtained from (13) and (14) with v_c given by (12) is in excellent agreement with the measurements obtained in MM's experiment "a". This agreement includes the prediction of the wave boundary layer thickness, the scale of which is obtained from (14) with v_c given by (12) ($\delta = 7.6\ \eta^2/\lambda = 1.7$ cm). From (15) it follows that δ_w is of the order of 5.1 to 7.8 cm for the λ = 10 cm spacing of the individual roughness elements, in agreement with MM's "enhanced" wave boundary layer thickness of 6.0 cm.

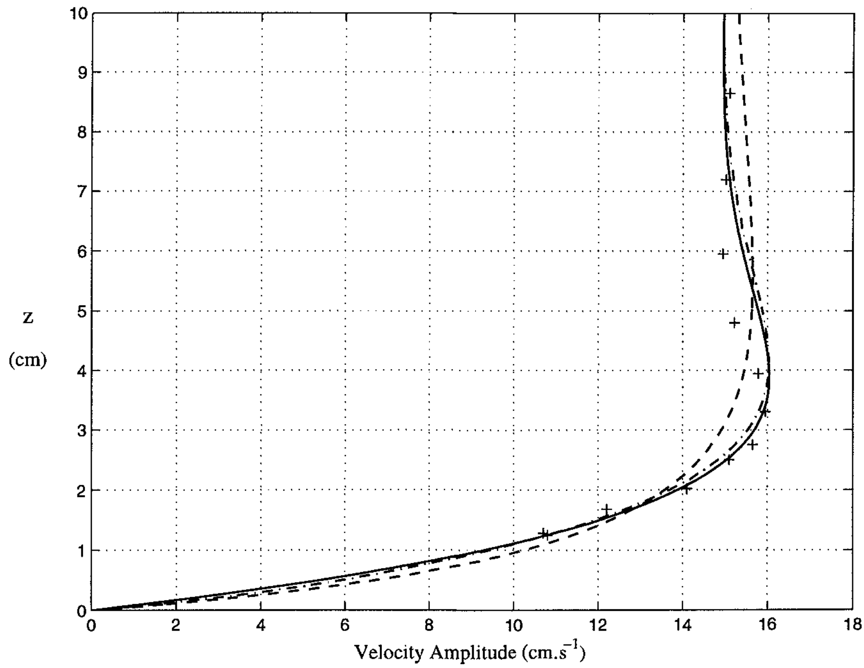

Figure 2: Velocity amplitude profiles obtained from 1) the pseudo-laminar model (*solid line*), 2) the hybrid model (*dash-dotted line*), 3) the GM-modified model (*dashed line*), and 4) MM's experiment "a" measurements above the trough of the ripples (*pluses*).

Thus, the pseudo-laminar model is capable of not only predicting the observed wave attenuation – an ability that is, of course, assured since v_c was derived to produce exactly this result – but also the detailed velocity structure within and the thickness of the wave boundary layer.

Mathisen and Madsen (1996a and b) chose the geometry of their artificial ripples, η = 1.5 cm and λ = 10 cm, in accordance with observed ripple geometry for similar wave conditions to those in their experiments. Differences between the detailed geometry of natural and MM's artificial ripples may, however, prevent the success of the pseudo-laminar model to be transferred to naturally rippled beds. Guided by our results from analysis of MM's data, equating the expressions for the rate of energy dissipation given by (1) and (11), with v_c depending on ripple geometry as suggested by (12), we obtain

$$v_c = v_{co}\frac{\eta^4}{\lambda^2 T} = \frac{1}{4\pi} T f_e^2 u_{bm}^2 \tag{16}$$

Following the suggestion of (16) we plot experimental values of ripple geometry and energy dissipation for experiments with waves over naturally rippled beds in Figure 3, from which we obtain a best fit value of the constant $v_{co} = 95$ in (16), with a standard deviation of 53. Not surprisingly, the eddy viscosity for naturally rippled beds is not as well defined as the one obtained for the artificial ripples. Nevertheless, the result that v_{co} is smaller for natural ripples than for artificial ripples is statistically significant and does make physical sense since one would expect the more rounded crests of the natural ripples to result in a smaller rate of energy dissipation and hence a smaller eddy viscosity.

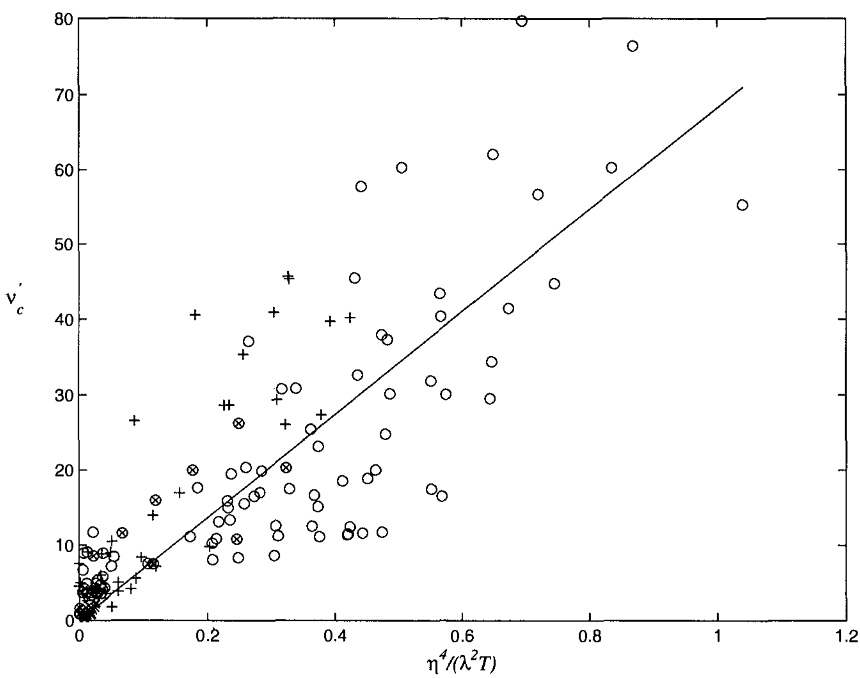

Figure 3: Eddy viscosity $v_c' = (8/3\pi)^2 v_c$ as a function of $\eta^4/(\lambda^2 T)$. Both quantities are in ($cm^2.s^{-1}$). Experimental data from Carstens et al. (1969) (*pluses*), Lofquist (1986) (*circles*), Rosengaus (1987) and Mathisen (1989) (*crosses*). The linear fitting (*solid line*) has a coefficient of determination $r^2 = 0.70$.

From a practical point of view, the difference between the values of v_c for artificial and natural ripples may be less significant, when one considers the fact that the data plotted in

Figure 3 used observed ripple geometry. For application, one would have to use predicted values of η and λ in the empirical expression for ν_c, (16). Our current ability to predict geometry of wave generated ripples is unfortunately such that a potential factor of 2 variability in ν_{co} in (16) is overshadowed by the uncertainty associated with the prediction of η^4/λ^2.

Detailed velocity profile measurements are not available within the wave boundary layers over naturally rippled beds. The similarity of the expressions obtained for ν_c suggests, however, that one may expect the pseudo-laminar eddy viscosity model to afford reasonably accurate predictions of the velocity structure within the wave boundary layer over naturally rippled beds.

A Hybrid Eddy Viscosity Model

The constant eddy viscosity model presented in the preceding section can be considered valid only for the very large bottom roughness values corresponding to rippled beds. It will neither predict a phase lead of the bottom shear stress different from $\pi/4$ or a near-bottom logarithmic wave orbital velocity profile, both of which are experimentally observed features for wave boundary layer flows for smaller values of the relative roughness. These features are, however, predicted by simple wave boundary layer models that employ an eddy viscosity that, near the bottom, increases linearly with distance above the theoretical bottom, e.g. Hsu and Jan (1998).

A hybrid eddy viscosity model that captures the observed features of wave boundary layer flows for small as well as large relative bottom roughness values is chosen as

$$\nu_t = \begin{cases} \kappa u_{*m}(z + z_0), & 0 \le z \le z_m \\ \kappa u_{*m}(z_m + z_0), & z > z_m \end{cases} \tag{17}$$

in which $z_0 = k_N/30$ is the hydraulic bottom roughness, and u_{*m} is defined by (2). In (17) the value of z_m, the elevation above which the eddy viscosity is considered constant, is chosen as

$$z_m = 0.5\frac{\kappa u_{*m}}{\omega} \tag{18}$$

For this choice of z_m the constant value of the eddy viscosity given by (17) is roughly the same as the pseudo-laminar viscosity given by (12) for the experiments of MM. Furthermore, this value of z_m coincides with value chosen by Madsen and Wikramanayake (1991) who showed this choice to lead to acceptable agreement between predicted and observed velocity structures within the wave boundary layer for relative roughness, k_N / A_b, smaller than $O(10^{-1})$.

Solving the linearized boundary layer equation with the eddy viscosity prescribed by (17) and evaluating the maximum bottom shear stress at $z = 0$ determines the relationship between wave friction factor and the relative bottom roughness. Approximating this relationship in a manner analogous to that utilized by Madsen (1994) we obtain explicit expressions for the wave friction factor f_w and the phase angle φ (in degrees)

$$f_w = \exp\left\{8.89(A_b/k_N)^{-0.059} - 10.68\right\}$$
$$\varphi = 38 - 8.3\log\frac{A_b}{k_N} \tag{19}$$

valid for the range $0.2 < A_b/k_N < 10^2$. For larger values of A_b/k_N the formulas given by Madsen (1994) may be used.

From the experimental value of $f_e = f_w \cos\varphi$ obtained by MM the equivalent bottom roughness, k_N, is obtained from (19). The predicted wave orbital velocity profile obtained from the hybrid eddy viscosity model, given by (17) is shown in Figure 2, and is virtually indistinguishable from the velocity profile predicted by the pseudo-laminar model.

The hybrid eddy viscosity model requires knowledge of the equivalent bottom roughness in order to be applied. Thus, to make the model applicable for the prediction of wave boundary layer flows over naturally rippled beds, an empirical relationship for k_N is required. Such a relationship is obtained by assuming it to be of the form

$$k_N = \begin{cases} \alpha\eta \\ \beta(\eta/\lambda)\eta \end{cases} \tag{20}$$

and determine the best fit values of α and β by fitting the experimental data on rate of energy dissipation over naturally rippled beds using the experimentally measured ripple characteristics. This exercise results in values of $\alpha = 12$ and $\beta = 78$, with both formulations providing a fit of equal goodness to the data used.

The experimentally obtained values for f_e are plotted against A_b/k_N, with $k_N = 12\ \eta$, in Figure 4. The relationship, $f_e(A_b/k_N)$, obtained from (19) is shown for comparison, and represents the experimental data with a coefficient of variation of 28%. This accuracy of the hybrid eddy viscosity model is similar to that of the pseudo-laminar model. For the pseudo-laminar model we obtained a coefficient of variation for ν_c of 53/95 = 0.56 which is roughly twice the coefficient of variation for the prediction of f_e since $f_e \propto \sqrt{\nu_c}$ for the pseudo-laminar model. Again, it is emphasized that the accuracy of 28% for the prediction of the energy dissipation factor when k_N is obtained from (20) is likely to be an optimistic value when η is predicted rather than measured. Finally, it should be pointed out that the seemingly very large value of $\alpha = 12$ is obtained as a result of using (19), which is the friction factor relationship obtained by evaluation of the bottom shear stress at $z+z_0 = z_0$. For a model that uses a near-bottom eddy viscosity proportional to z instead of $(z+z_0)$ this corresponds to evaluation of the shear stress at $z=z_0$ instead of in the limit $z \to 0$. The original GM-model, if applied to the data shown in Figure 4 would therefore lead to different best fit constants in (20). In fact, for the GM-model with the shear stress evaluated in the limit $z \to 0$, one obtains values of $\alpha = 4$ and $\beta = 25$, i.e. roughly a factor of three lower than the best fit values obtained for the hybrid eddy viscosity model.

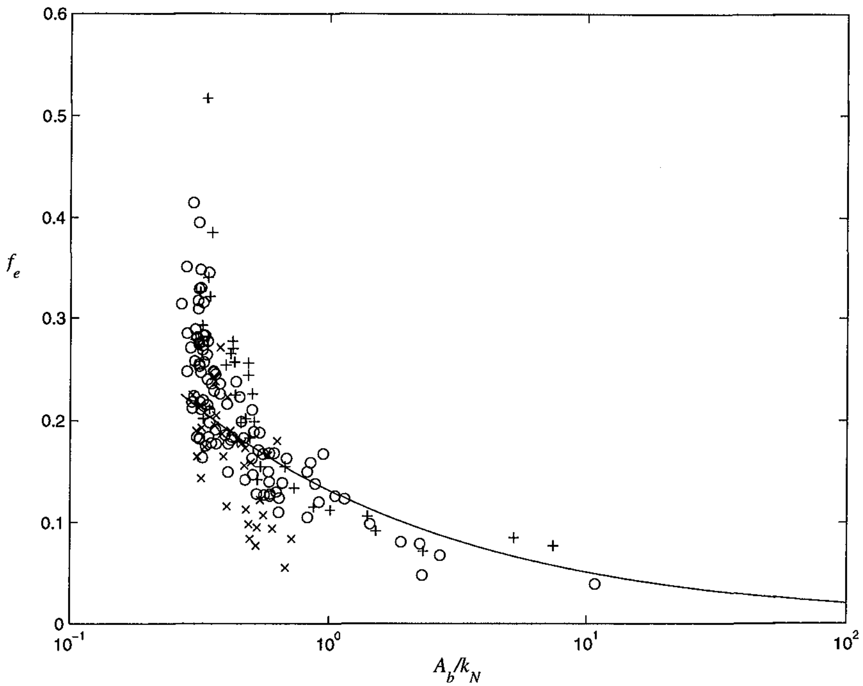

Figure 4: Measured and predicted energy dissipation factors, f_e, as a function of relative roughness, A_b/k_N. Theoretical values using (19) with k_N = 12 η (*solid line*); data from Carstens et al. (1969) (*pluses*), Lofquist (1986) (*circles*), Rosengaus (1987) and Mathisen (1989) (*crosses*).

Modified Grant-Madsen Wave-Current Interaction Model

If one is willing to sacrifice a little accuracy in the prediction of the velocity structure within the wave boundary layer for the sake of simplicity, one may adopt a linearly increasing eddy viscosity throughout the wave boundary layer. In this case the eddy viscosity reduces to the first expression given by (17) and the wave orbital velocity within the wave boundary layer is simply that predicted by Madsen (1994) with the origin of z taken a distance z_0 below his theoretical bed elevation. This effectively changes Madsen's (1994) eddy viscosity length scale from his "z" to the value $z+z_0$ used in (17). Except for this change, all formulas obtained by Madsen (1994) are still valid. To achieve a measure of the accuracy one sacrifices if adopting this simplification, the predicted wave orbital velocity profile of this modified GM-model is show in Figure 2. Obviously, either the pseudo-laminar or the hybrid eddy viscosity models provide a better fit to the experimental data shown in Figure 2 than does the modified GM-model. However, the GM-model does provide a reasonably accurate (better than 10% error) representaion of observations for this large bottom roughness.

Given the modified GM-model's ability to represent the velocity structure within the wave boundary layer, it is extremely surprising that the model's prediction of the wave boundary layer thickness is as poor as it is for the experiments by MM involving artificial ripples. This inability of the modified GM-model was discussed in the Introduction, where it was mentioned that the GM-model estimated the wave boundary layer thickness from (4) with a scaling factor A between 1 and 2, which for MM's experiment "a" shown in Figure 2 gives $\delta_w \approx 2.5$ cm – clearly a value much lower than the wave boundary layer thickness suggested by the data.

This inability of the GM-model to afford an accurate estimate of δ_w and hence, for combined wave-current flows, the elevation at which the current profile exhibits a discontinuous slope in the presence of waves, is actually not an inability of the model but the result of an unfortunate oversight on the part of the model's authors. The estimate of the scaling factor A in (4) being of the order of 1 to 2 was originally derived by Grant and Madsen by considering the elevation above the bottom where the wave orbital velocity had reached the free stream velocity within a relative error of roughly 5%. This, of course, is a completely legitimate criterion to use for the definition of a wave boundary layer thickness. However, GM failed to recognize that their value of A, which they obtained for a relative roughness $k_N / A_b \approx 0.01$, should be considered a function of the relative roughness and not be treated as a generally valid constant. When one corrects this oversight by defining the wave boundary layer thickness as the value of z for which the velocity magnitude is within 5% of the free stream velocity, the resulting value of the scaling factor in (4) may be expressed as

$$A = exp\left\{2.96(A_b/k_N)^{-0.071} - 1.45\right\} \tag{21}$$

For $A_b/k_N = 100$ this formula gives $A = 2.0$ in agreement with the original GM-value. However, for $A_b/k_N = 0.35$, which is representative of the relative roughness values in the MM experiments over artificially rippled beds, $A = 5.7$ is obtained from (21), i.e. an increase by a factor of roughly 2.8 over the value obtained if $A = 2.0$ is assumed independent of the relative roughness.

Use of (21) in conjunction with (4) to obtain the wave boundary layer thickness and modifying the GM-model's prediction of current velocity profiles in the presence of waves accordingly completely removes the need for the artificially "enhanced" wave boundary layer thickness introduced by Mathisen and Madsen. The modified GM-model can be used directly to explain and analyze observations for combined co-directional wave-current flows over a rippled bed so long as it is corrected to account for the wave boundary layer scaling factor's dependency on relative roughness.

Conclusions

From a drag law formulation of the boundary resistance experienced by waves over an artificially rippled bottom, we were led to the establishment of a constant pseudo-laminar eddy viscosity formulation for wave boundary layer flows over rippled bottoms. For naturally rippled bottom, analysis of available experimental data suggests the adoption of a pseudo-laminar eddy viscosity given by

$$\nu_c = \nu_{co} \frac{\eta^4}{\lambda^2 T} \tag{22}$$

in which $\nu_{co} = O(100)$, η and λ are the ripple height and length, respectively, and T is the wave period.

The pseudo-laminar model should be considered limited to the flow conditions corresponding to the existence of two dimensional wave-generated ripples covering the bottom. To establish a model that could be considered generally valid for the range of bottom roughness encountered by waves over a movable bottom, a hybrid eddy viscosity model was developed. For this model, given by (17), the constant eddy viscosity was reduced towards zero through a linear transition immediately above the bottom. The velocity structure within the wave boundary layer predicted by the hybrid eddy viscosity model was shown to be virtually identical to the predictions of the pseudo-laminar model for rippled beds. However, the hybrid model is, in contrast to the pseudo-laminar model, capable of predicting observed velocity features for small values of the bottom roughness. For wave boundary layer flows over naturally rippled bottoms , the bottom roughness, k_N, needed to apply the hybrid eddy viscosity model was found, from laboratory experiments with simple periodic wave motions, to be related to the ripple height through

$$k_N = 12\eta \tag{23}$$

Both the pseudo-laminar and the hybrid eddy viscosity models gave estimates of the wave boundary layer thickness that were in good agreement with observations for artificially rippled beds. An unfortunate oversight in the development and application of the Grant-Madsen model for combined wave-current flows over very rough bottoms was corrected by introducing a roughness-dependent scaling factor, A, given by (21). Following this correction, the artificially "enhanced" wave boundary layer thickness introduced by Mathisen and Madsen (1996b) to explain and analyze their experimental results for combined co-directional wave-current flows over artificially rippled bottoms, was found to be entirely unnecessary. Thus, the applicability of thc corrected Grant-Madsen wave-current interaction model to the prediction of current profiles in the presence of waves over very rough, rippled, bottoms was established. For a rippled bed, it is emphasized, however, that application of the modified GM-model is limited to co-directional wave-current flows. The reason for this limitation is that the bottom roughness has been shown to be the samc for waves and currents only for this type of combined flows over two-dimensional roughness elements.

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